ON PARTIALLY COMPOSED PROPERTY OF GENERALIZED LEXICOGRAPHIC PRODUCT GRAPHS

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Overview

1. Introduction
   - Basic Definitions
   - Graphical Properties
   - Vertex Properties
   - Generalized Partially Composed Vertex Properties
   - Example of Generalized Partially Composed Vertex Properties
2. Main Results
   - $m_{\mathcal{P}}$ and $M_{\mathcal{P}}$
   - Applications

♦ Introduction ♦

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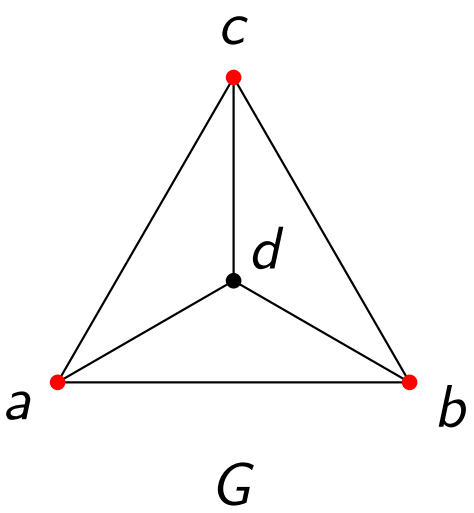

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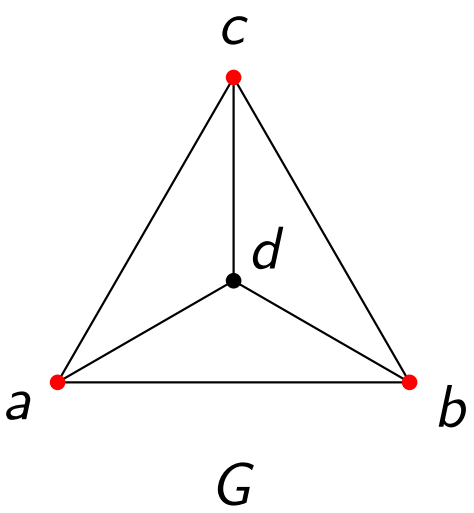

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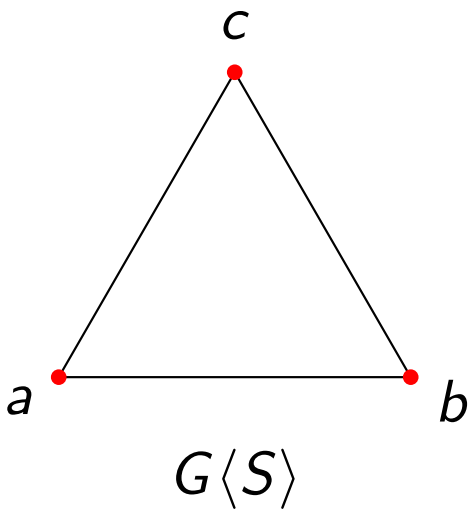

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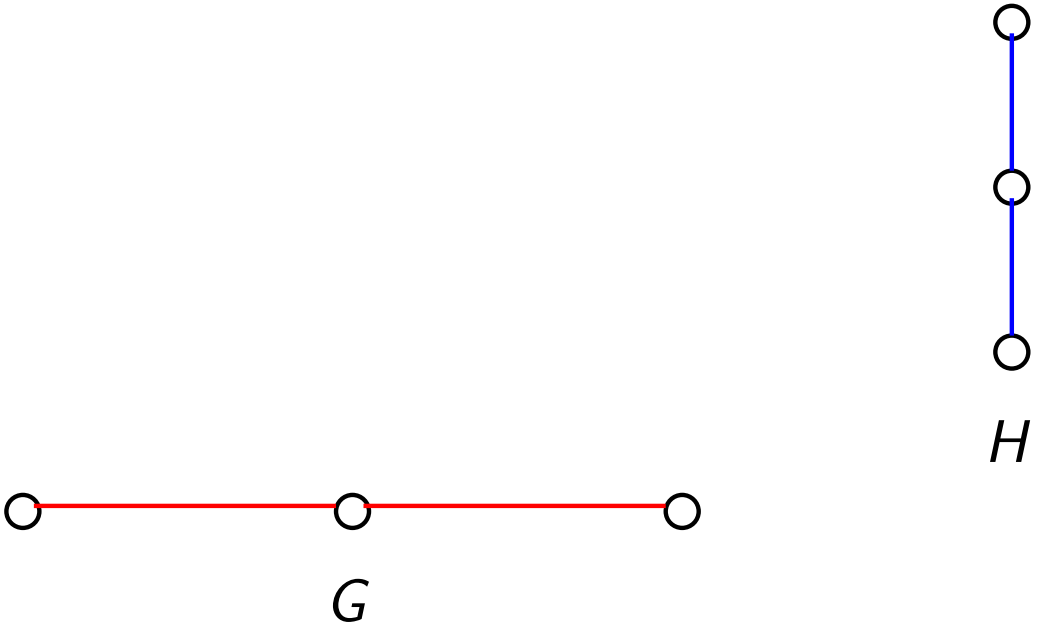

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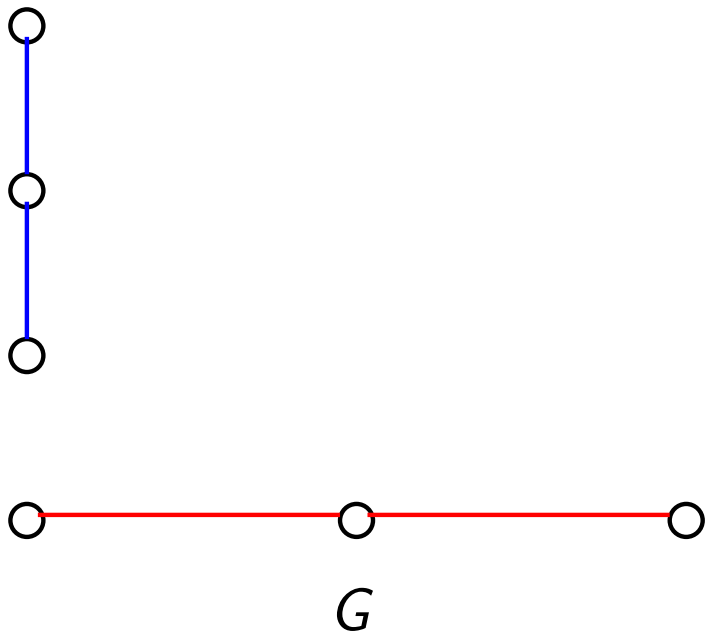

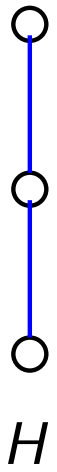

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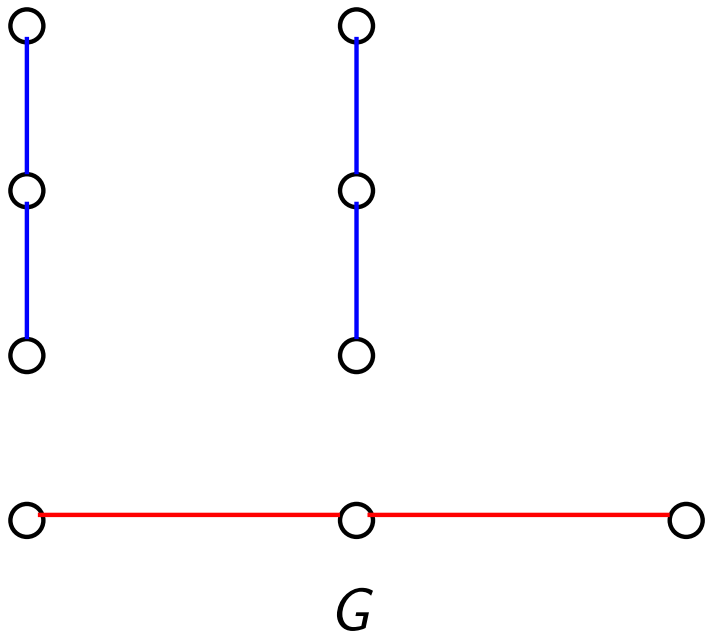

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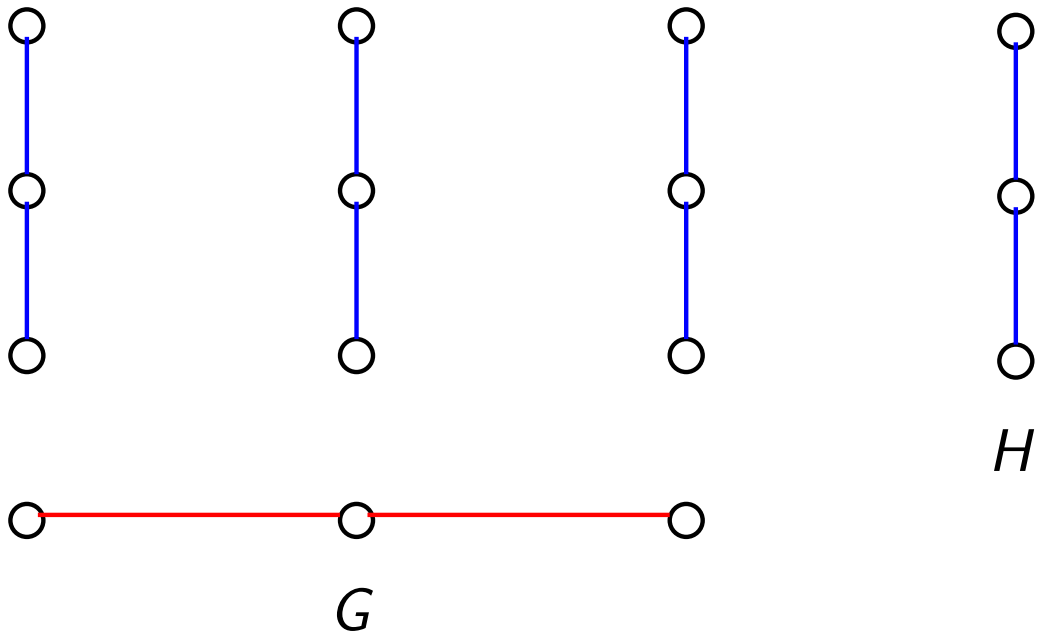

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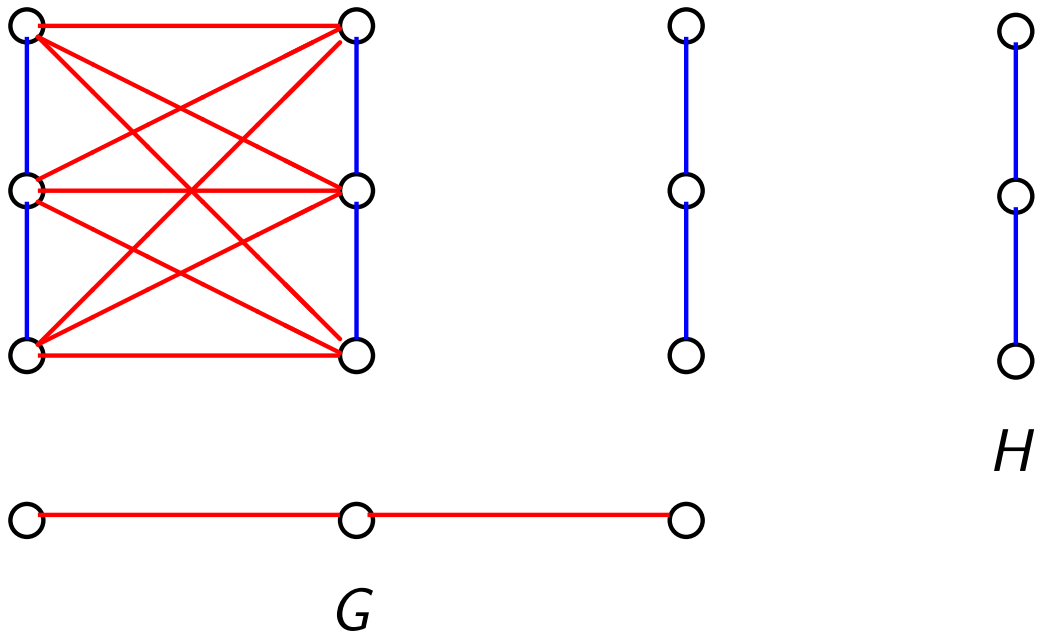

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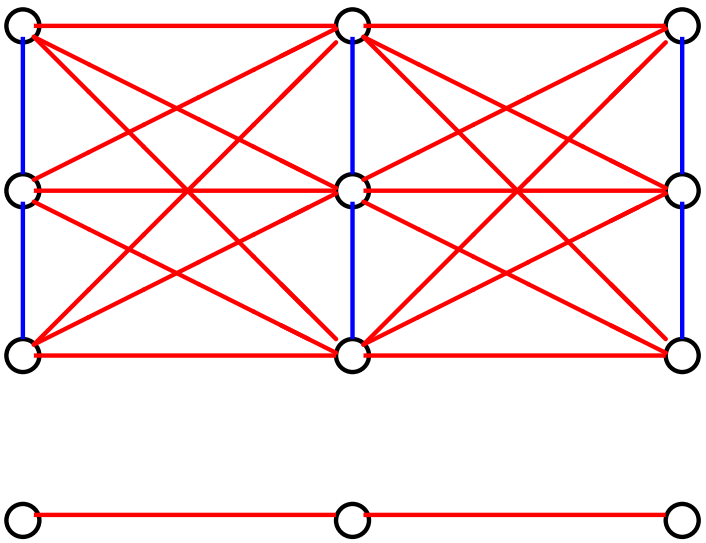

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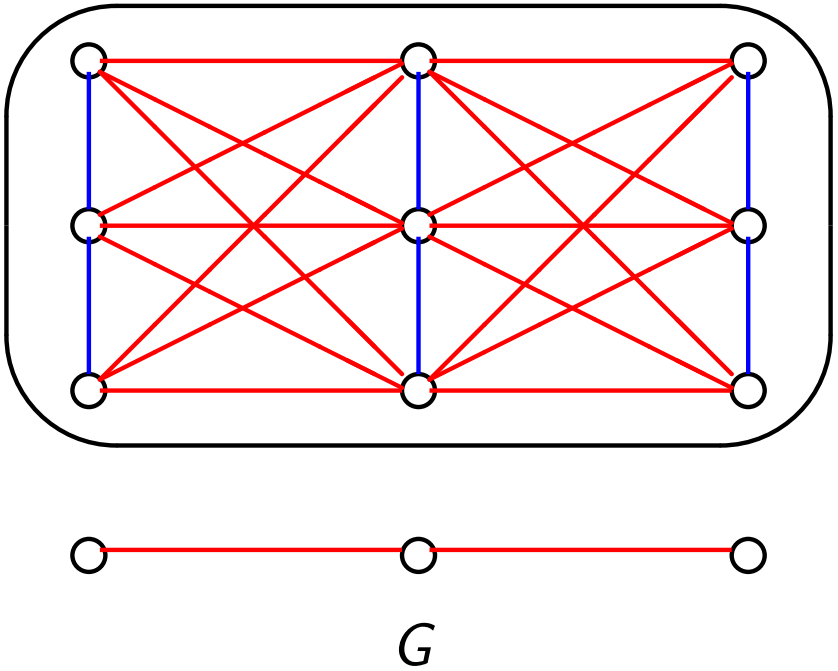

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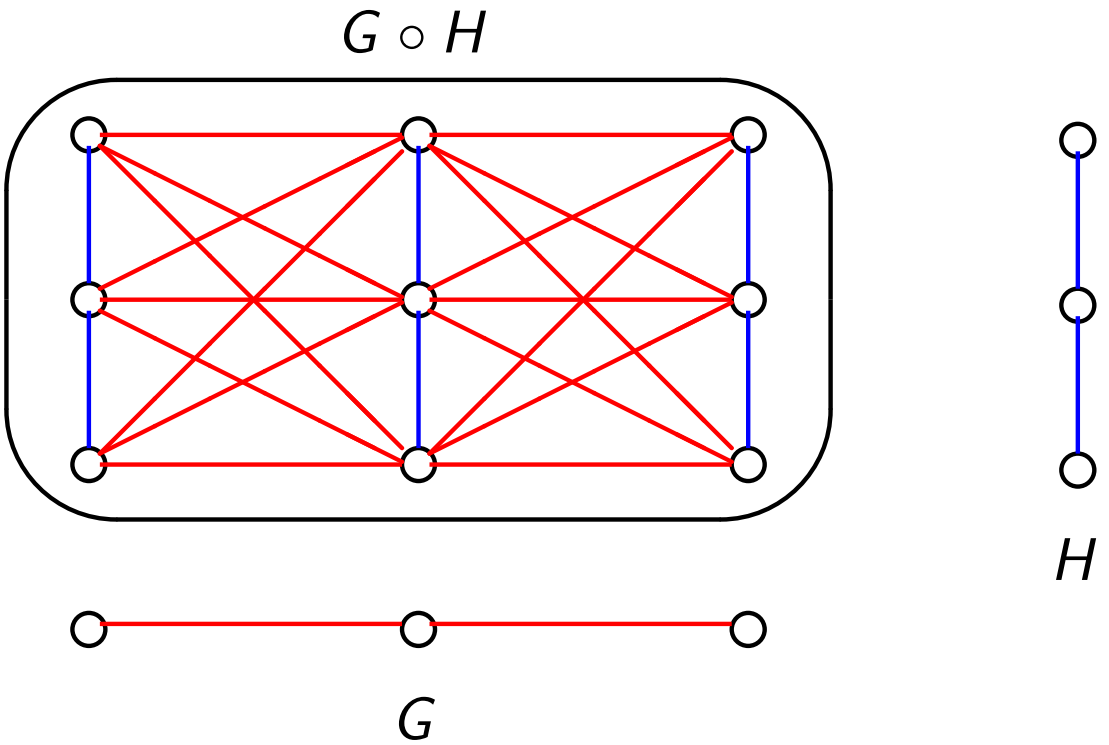

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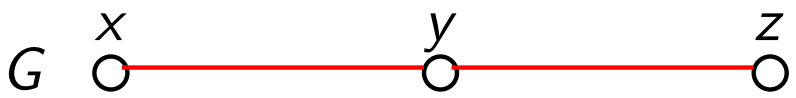

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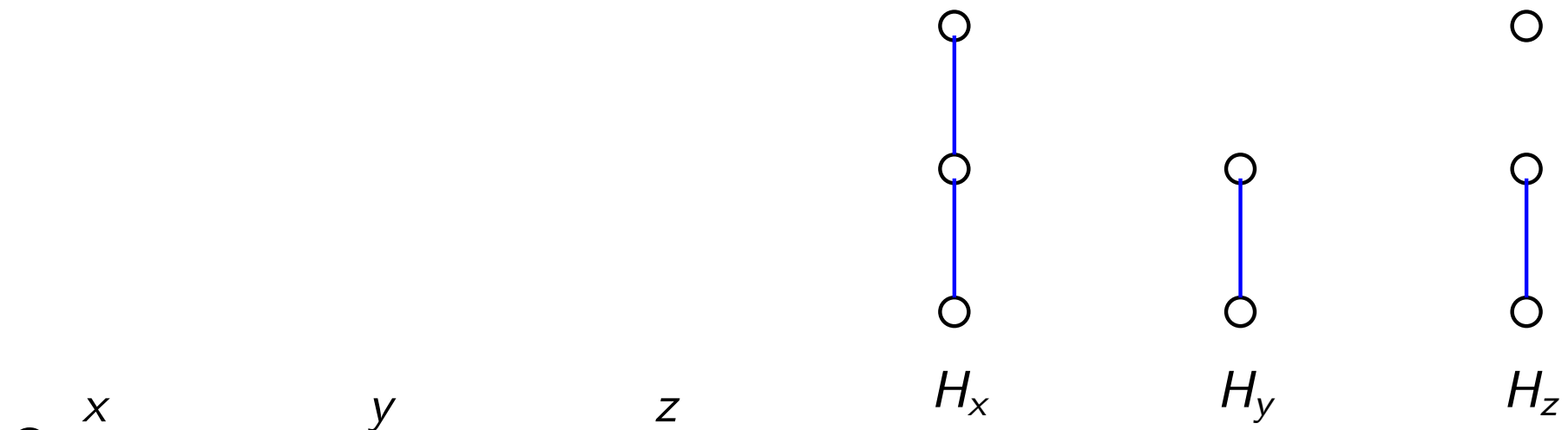

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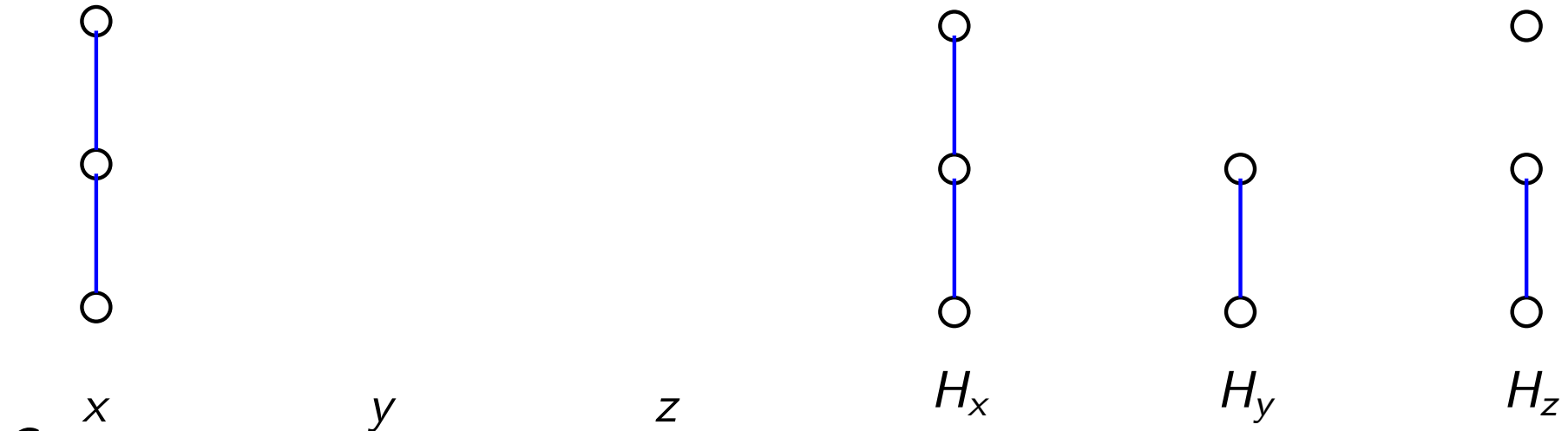

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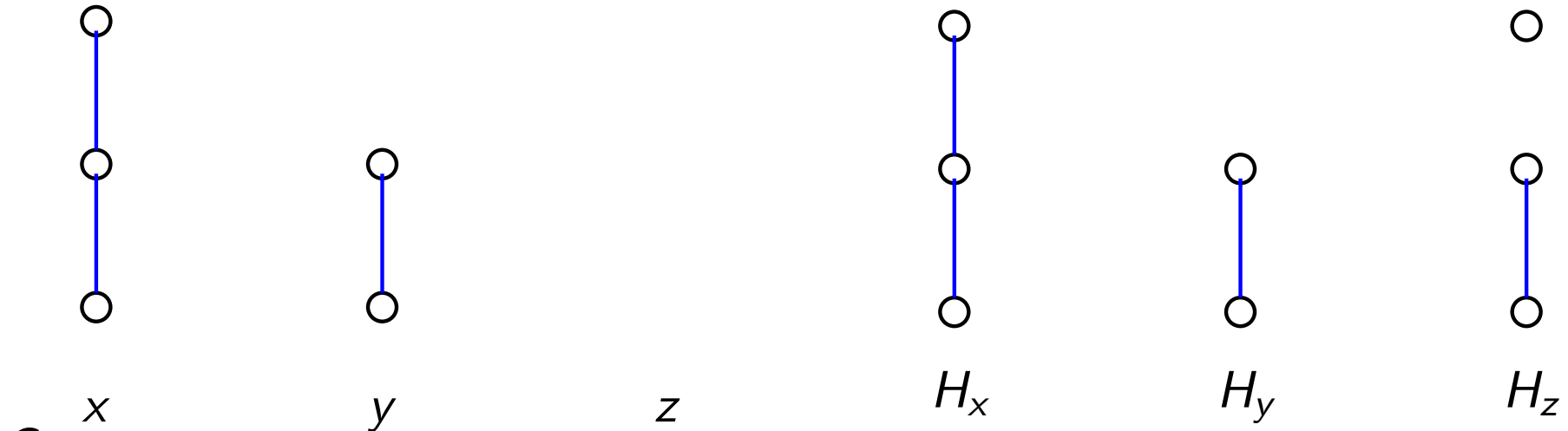

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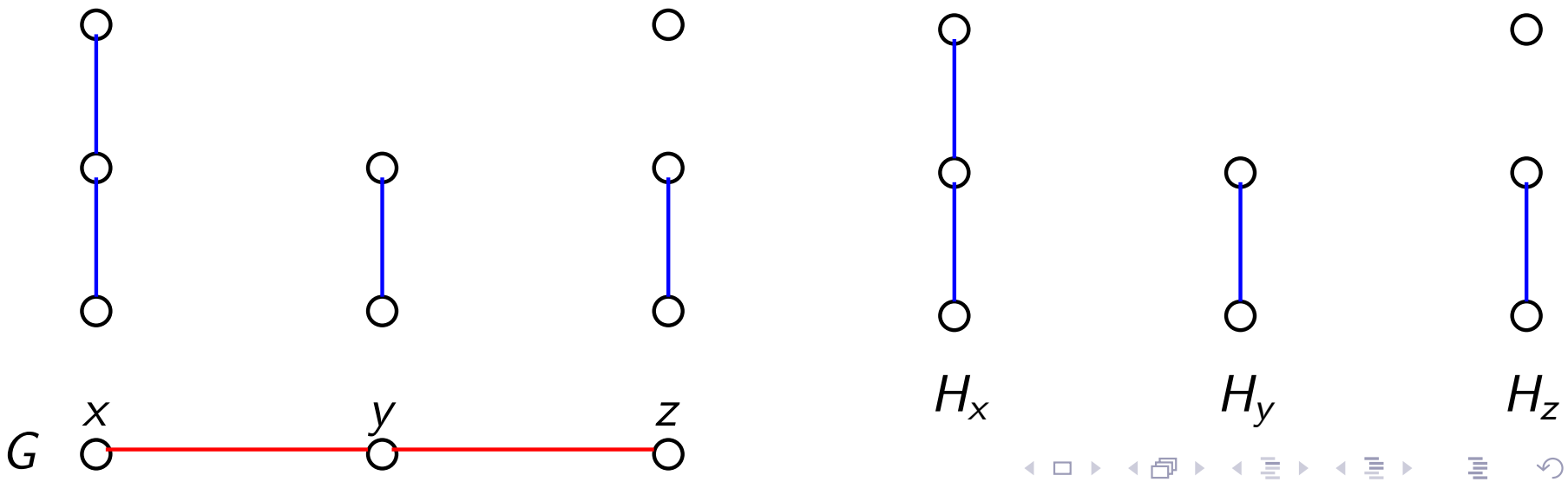

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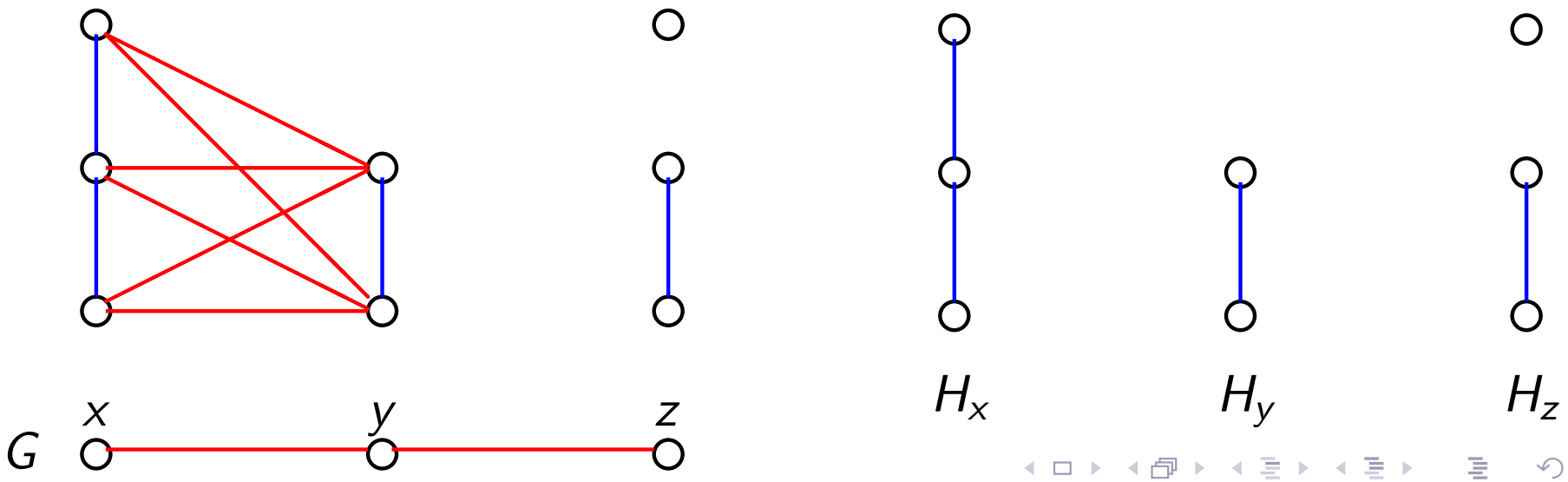

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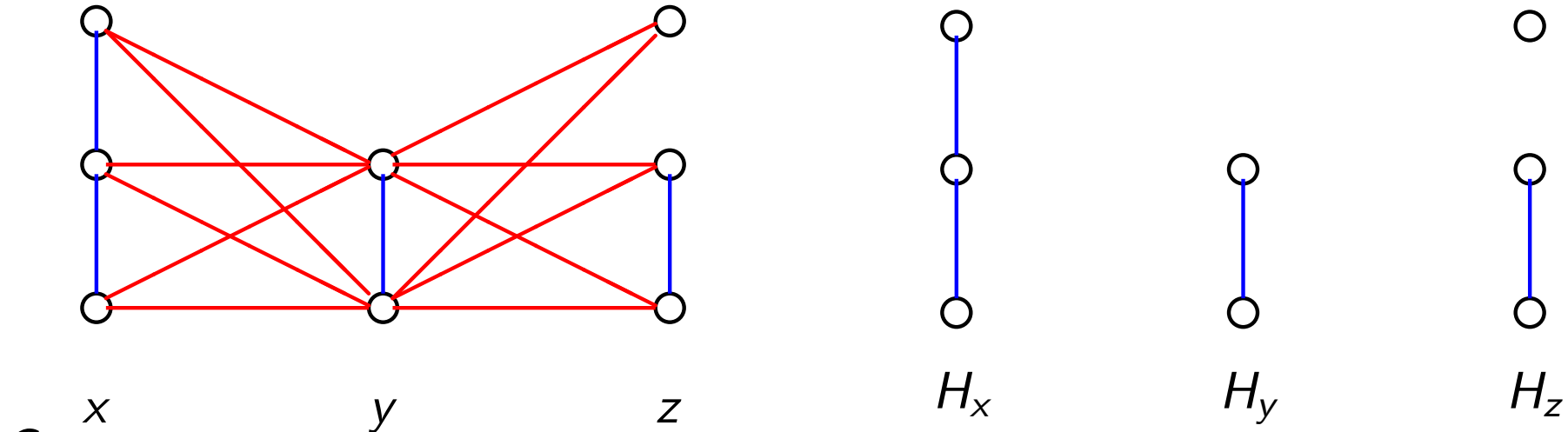

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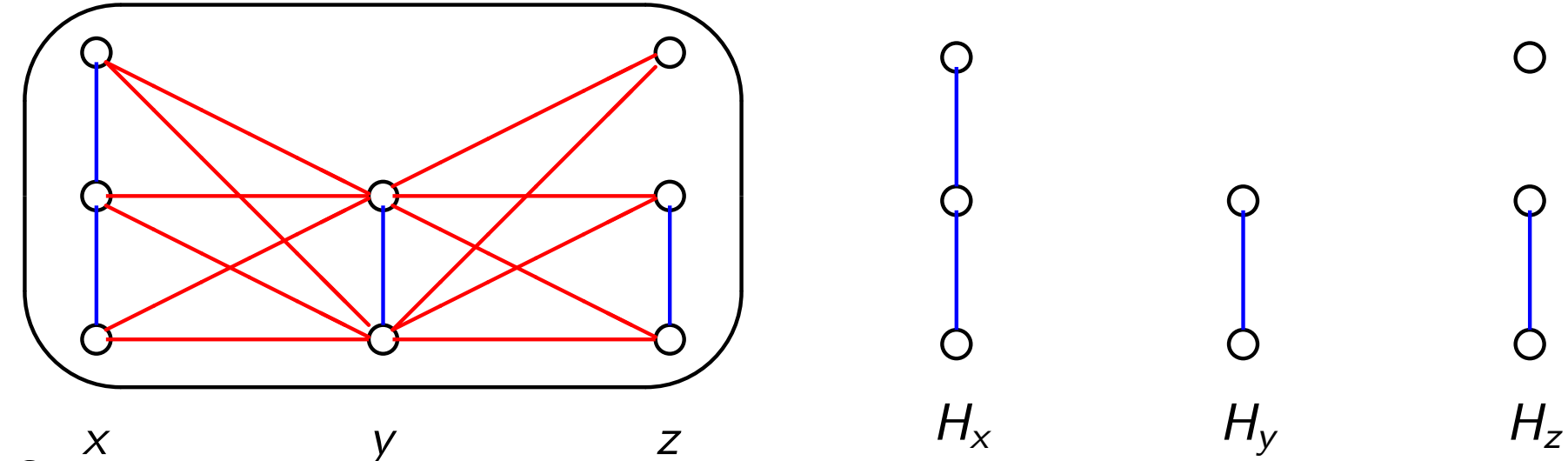

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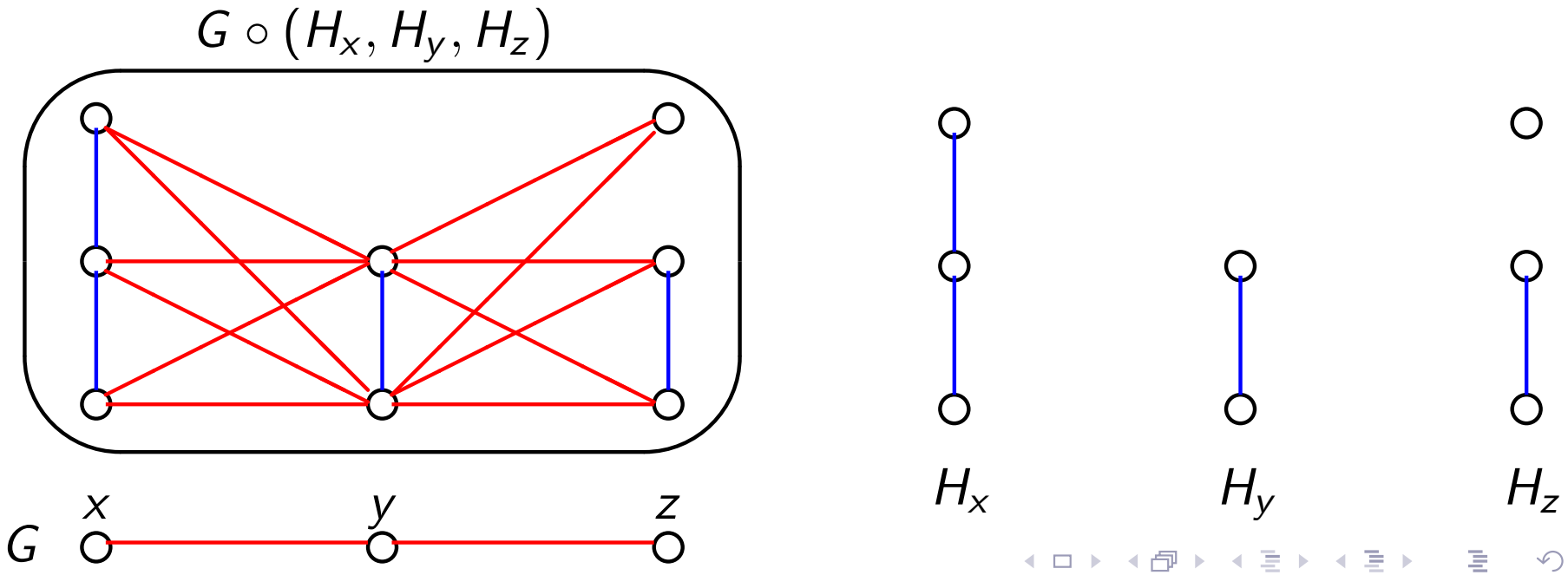

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- If S is a $\mathcal{P}$-set of a graph G such that $|S| = M_{\mathcal{P}}(G)$ or $|S| = m_{\mathcal{P}}(G)$, we say that S is an *$M_{\mathcal{P}}$-set* or an *$m_{\mathcal{P}}$-set* of G, respectively.

Generalized Partially Composed Vertex Properties

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- $\mathcal{P}$ is said to be *generalized partially composed* by $\mathcal{P}_1$ and $\mathcal{P}_2$ if it satisfies: for any $G \in \mathcal{I}_1, H_x \in \mathcal{I}_2$ and a nonempty subset S of $V(G \circ (H_x)_{x \in V(G)})$, we have

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Generalized Partially Composed Vertex Properties

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Generalized Partially Composed Vertex Properties

$V(G \circ (H_x)_{x \in V(G)})$

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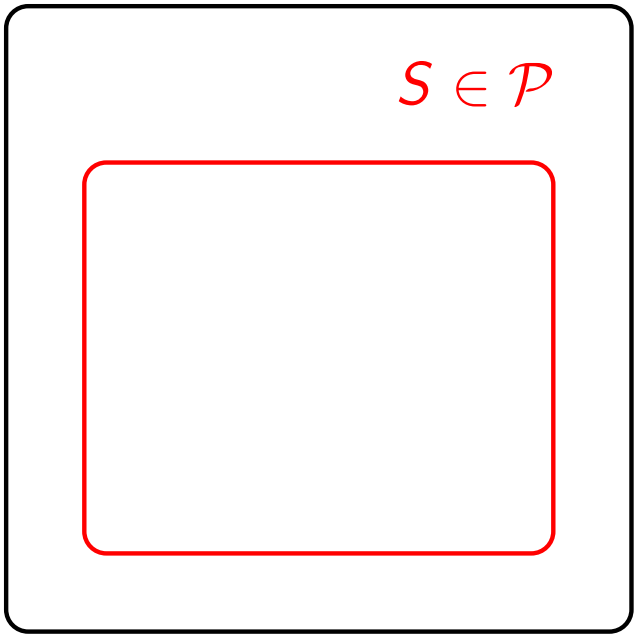

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Generalized Partially Composed Vertex Properties

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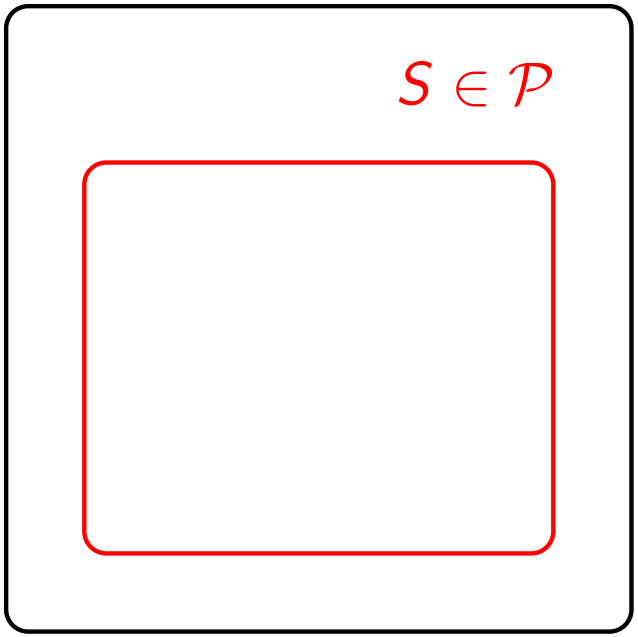

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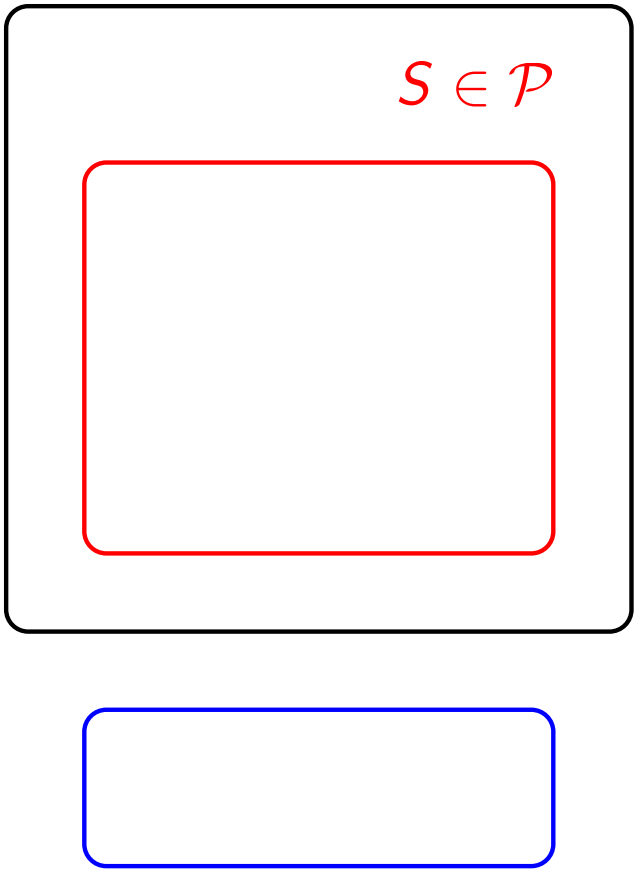

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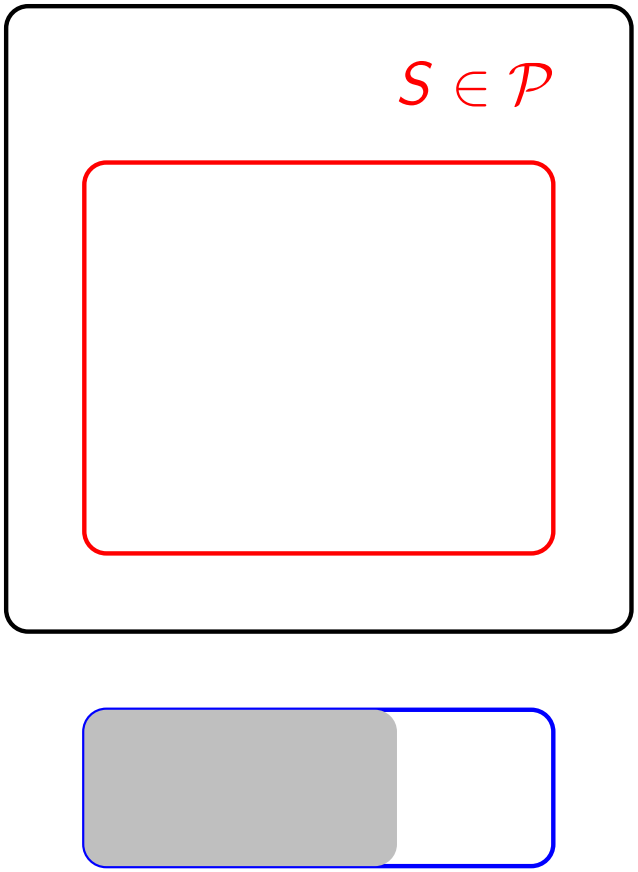

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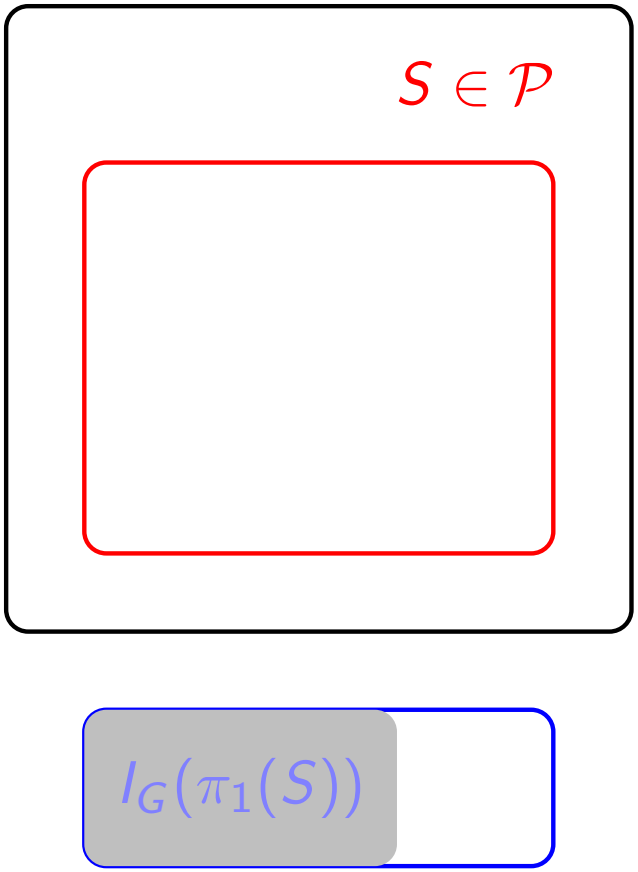

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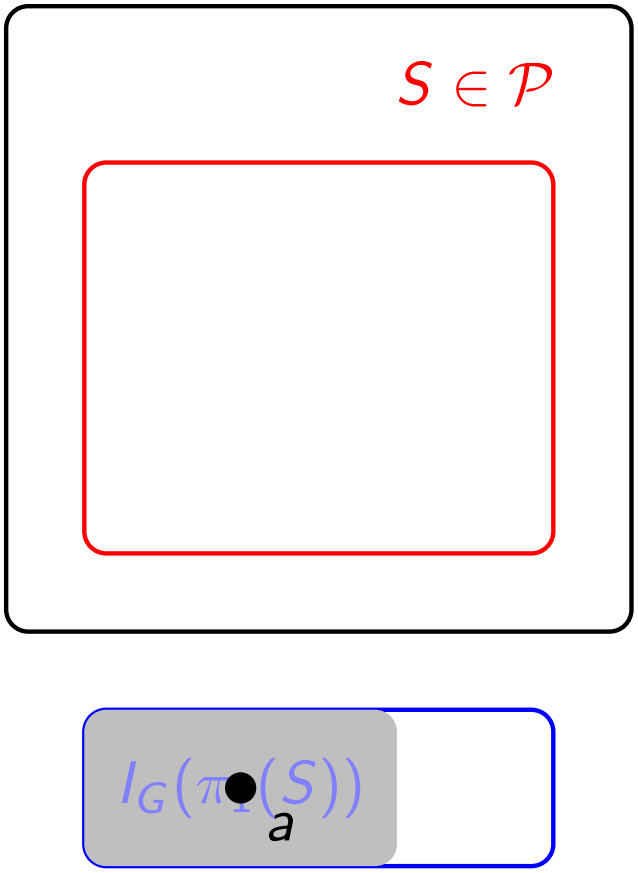

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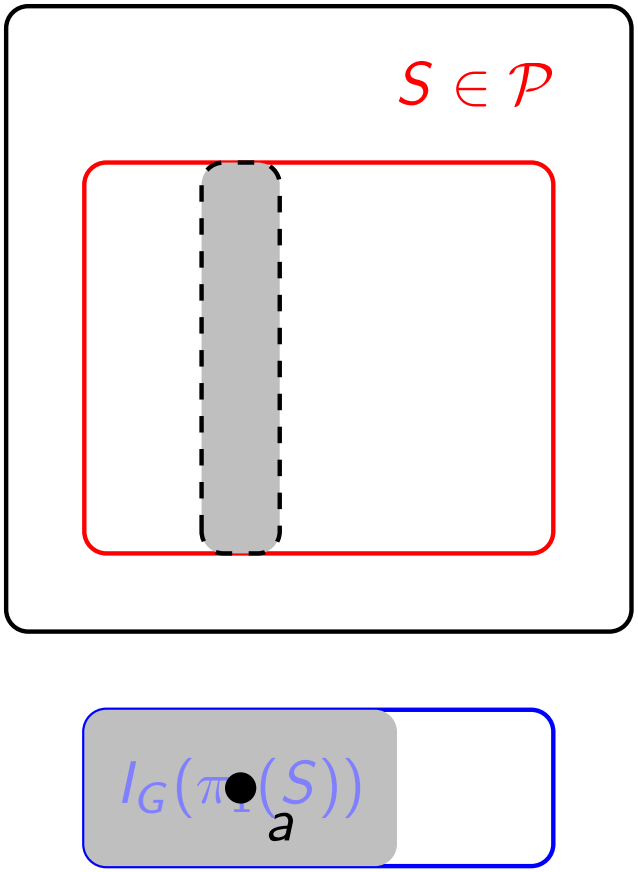

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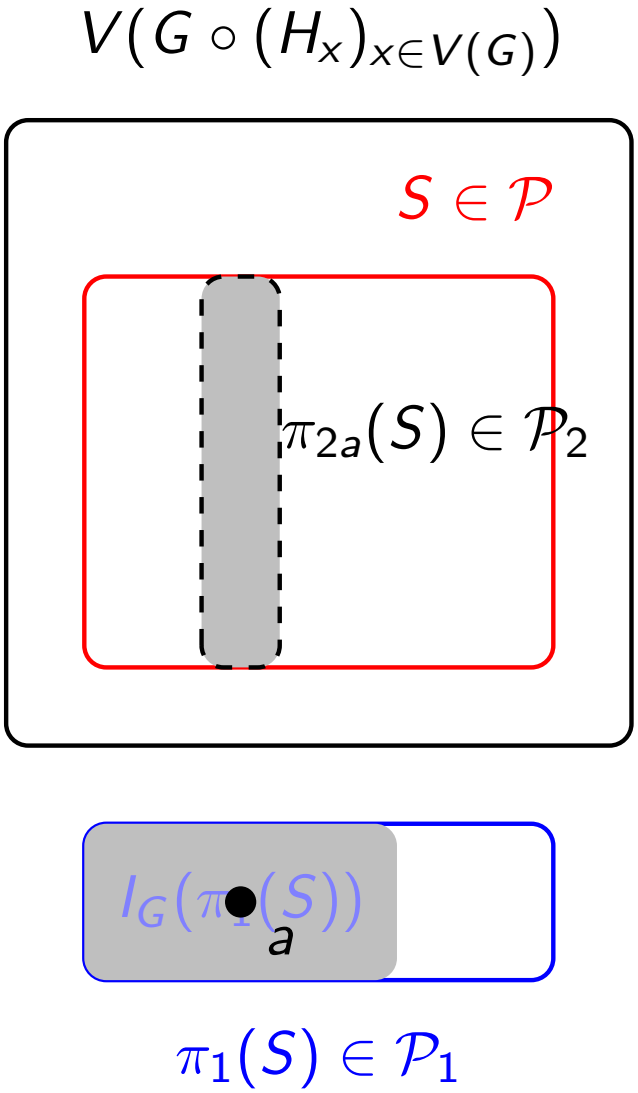

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Example of Generalized Partially Composed Vertex Properties

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Definition of Dominating Sets

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♦ Main Results ♦

$m_{\mathcal{P}}$ and $M_{\mathcal{P}}$

Theorem

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such that $\mathcal{P}$ is generalized partially composed by $\mathcal{P}_1$ and $\mathcal{P}_2$.
Further, let $G \in \mathcal{I}_1$ and $H \in \mathcal{I}_2$. Then

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For graphical properties $\mathcal{I}_1$ and $\mathcal{I}_2$,
let $\mathcal{P}_1$ be a vertex property appearing in $\mathcal{I}_1$,
$\mathcal{P}_2$ be a vertex property appearing in $\mathcal{I}_2$ and
$\mathcal{P}$ a vertex property appearing in $\mathcal{I}_1 \circ \mathcal{I}_2$
such that $\mathcal{P}$ is generalized partially composed by $\mathcal{P}_1$ and $\mathcal{P}_2$.
Further, let $G \in \mathcal{I}_1$ and $H \in \mathcal{I}_2$. Then

1. $m_{\mathcal{P}}(G \circ (H_x)_{x \in V(G)}) =$
   $\min \left\{ |J_G(S)| + \sum_{a \in I_G(S)} m_{\mathcal{P}_2}(H_a) : S \text{ is a } \mathcal{P}_1\text{-set of } G \right\}$

$m_{\mathcal{P}}$ and $M_{\mathcal{P}}$

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2. $M_{\mathcal{P}}(G \circ (H_x)_{x \in V(G)}) =$
   $\max \left\{ \sum_{a \in J_G(S)} |V(H_a)| + \sum_{a \in I_G(S)} M_{\mathcal{P}_2}(H_a) : S \text{ is a } \mathcal{P}_1\text{-set of } G \right\}$.

Applications

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For simple graph G and H we have

- Domination Number
  - $\gamma(G \circ (H_x)_{x \in V(G)}) =$
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- Total Domination Number
  - $\gamma_t(G \circ (H_x)_{x \in V(G)}) =$
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Applications

For simple graph G and H we have

- Independent Domination Number
  - $\gamma_i(G \circ (H_x)_{x \in V(G)}) =$
    $\min \left\{ \sum_{a \in I_G(S)} \gamma_i(H_a) : S \text{ is an independent dominating set of } G \right\}.$

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$m_{\mathcal{P}}(G \circ (H_x)_{x \in V(G)}) =$
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For simple graph G and H we have

- Independent Domination Number
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$m_{\mathcal{P}}(G \circ (H_x)_{x \in V(G)}) =$
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Applications

For simple graph G and H we have

- Independent Domination Number
  - $\gamma_i(G \circ (H_x)_{x \in V(G)}) =$
    $\min \left\{ \sum_{a \in I_G(S)} \gamma_i(H_a) : S \text{ is an independent dominating set of } G \right\}.$
- Connected Domination Number
  - $\gamma_c(G \circ (H_x)_{x \in V(G)}) = \gamma_c(G)$ if $\gamma_c(G) \geq 2$.

THANK YOU

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