

Sandpiles : hourglasses 1D (Ice, Parallel, Symmetric, Kadanoff)

IPM Lattice isomorphic to the partitions of n .

[Brylawski, 1973]

SPM Fixed point in $\mathcal{O}(n^{3/2})$ sequential steps.

[Goles, Kiwi, 1993]

PSPM Deterministic in $\mathcal{O}(n)$ steps with signals.

[Durand-Lose, 1998]

SSPM $\lfloor \sqrt{n} \rfloor$ Fixed points shapes.

[Formenti, Masson, Pisokas, Phan, 2006]

PSSPM Fixed points shapes.

[P, Pham, Phan, 2011]

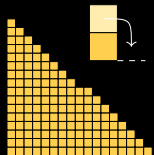
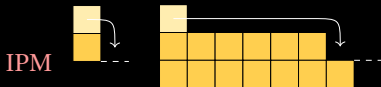
Linear CFG Lattice.

[Goles, Morvan, Phan, 2002]

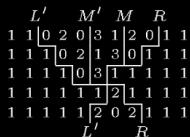
KSPM Fixed point waves.

[Formenti, P, Remila, 2013]

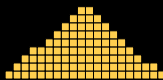
$$5 = 4 + 1 = 3 + 2 = 3 + 1 + 1 = 2 + 2 + 1 \\ = 2 + 1 + 1 + 1 = 1 + 1 + 1 + 1$$



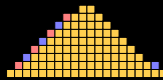
SPM



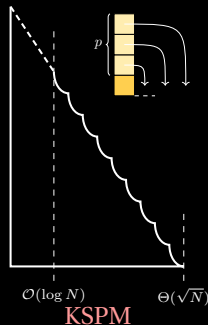
PSPM



SSPM

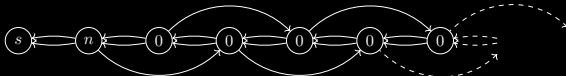


PSSPM



KSPM

Linear CFG



Sandpiles : hourglasses 2D



Results.

Convergence of rescaled stable configuration $(1_n)^\circ$.

Fractal solutions of the sandpile continuum limit.

Partial Differential Equations – Discrete potential theory

Viscosity solutions of nonlinear elliptic obstacle problem

Superharmonic functions – Apollonian circle packing

Pegden, Smart: “Convergence of the Abelian sandpile”, 2011. arXiv:1105.0111

Levine, Pegden, Smart: “Apollonian structure in the Abelian sandpile”, 2012. arXiv:1208.4839

Levine, Pegden, Smart: “The Apollonian structure of integer superharmonic matrices”, 2013. arXiv:1309.3267

Levine, Peres: “Laplacian growth, sandpiles and scaling limits”, 2016. arXiv:1611.00411

Pegden, Smart: “Stability of patterns in the Abelian sandpile”, 2017. arXiv:1708.09432