How We Solve CG:SHOP Problems

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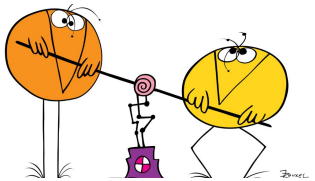
+ All Shadoks team members



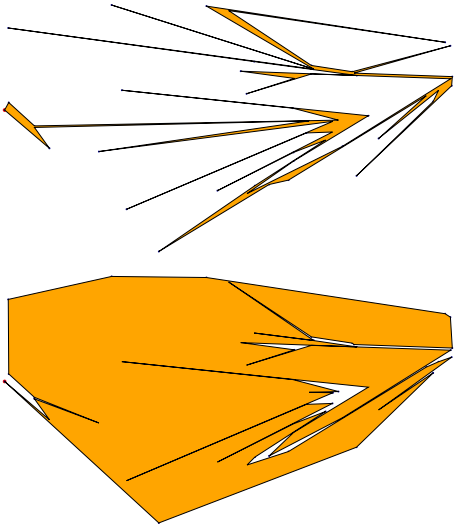
CG:SHOP Competition

- Part of SoCG (International Symposium on Computational Geometry)
- Annual event that started in 2018–2019
- Hard geometric optimization problems
- Different problem each year
- ~ 200 instances given
- ~ 3 months to compute solutions
- Send our solutions (not the code)
- Score based on the quality of the solutions
- Top teams invited to publish in SoCG proceedings and ACM Journal of Experimental Algorithmics or Computing in Geometry and Topology

Minimum (or Maximum) Area Polygon:

- Input: A set of points $S \subset \mathbb{R}^2$
- Output: A simple polygon with vertex set S
- Goal: Minimize (or maximize) the area

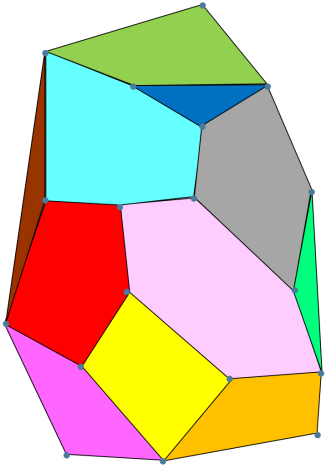
- Related to Euclidean TSP
- Two categories: minimization, maximization
- We got 2nd place
- Techniques: greedy and local search



Minimum Convex Partition:

- Input: A set of points $S \subset \mathbb{R}^2$
- Output: A simple partition of the convex hull of S into convex regions with vertex set S
- Goal: Minimize the number of regions

- We got 4th place
- Used integer programming



11 convex regions

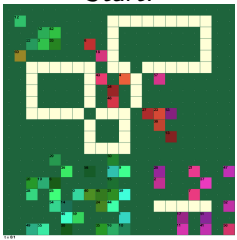
CG:SHOP 2021 (Robots)

Coordinated Motion Planning:

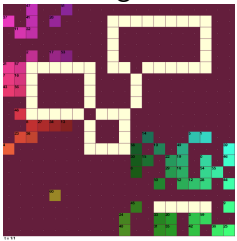
- Input: Sets $S, T \subset \mathbb{Z}^2$ of start and target locations for n robots and possibly a set of obstacles
- Output: A sequence of movements for all robots from start to target avoiding collisions
- Goal: Minimize the total time (makespan) or the total number of movements (energy)

- 1st place in makespan category, 3rd place in energy category
- Used storage network and conflict optimizer

Start:



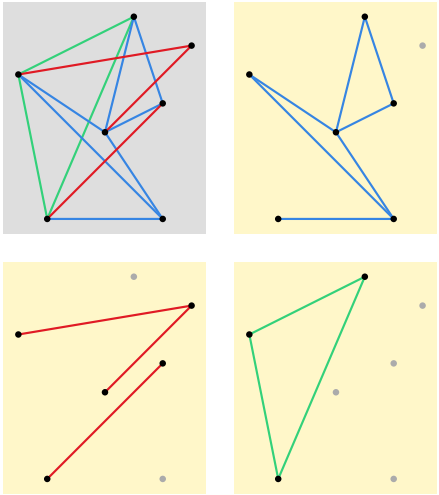
Target:



Partition Into Plane Graphs:

- Input: A graph G embedded in the plane with straight edges
- Output: A partition of G into plane graphs
- Goal: Minimize the number of partitions (colors)

- We won 1st place
- Best solution of all teams to all instances
- Optimal solution to at least 23
- Reused conflict optimizer



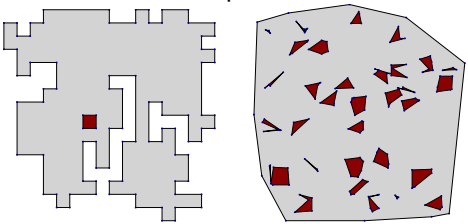
CG:SHOP 2023 (ConvexCover)

Convex Covering:

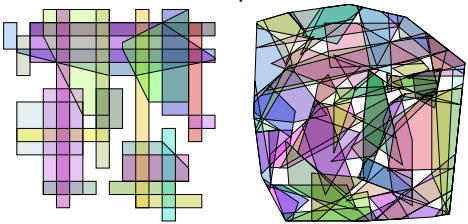
- Input: A polygon with holes P
- Output: A collection of convex polygons whose union is P
- Goal: Minimize the number of convex polygons

- We won 2nd place
- Best solution among all teams to 128 of 206 instances
- Used integer programming and simulated annealing

Input



Output



Knapsack Translational Packing:

- Input: A convex polygon (*container*) and a multi-set of polygons with *values* (*items*)
- Output: A translation of some items that form a packing inside the container
- Goal: Maximize the sum of the values in the output

- We won 1st place
- Used greedy, local search, and integer programming

Initial Solution and Optimized Solution

- Our strategy in every competition:
  1. Find initial feasible solutions
  2. Improve them
- We've seen two types of problems:
  - The quality of the initial solution is irrelevant and the improvements are major
  - The quality of the initial solution is essential and the improvements are minor
- Hard to tell them apart
- If the solution to a subproblem can be incorporated into an existing solution, it is easier to improve

Initial

- Greedy heuristics often produce good initial solutions:
  1. Choose an element to add to the solution
  2. Choose how to incorporate this element
  3. Repeat
- Good data structures to implement it efficiently
- Three examples:
  - `Packing`
  - `Coloring`
  - `MaxPolygon`

Greedy Maximum Area Polygon

Simple idea:

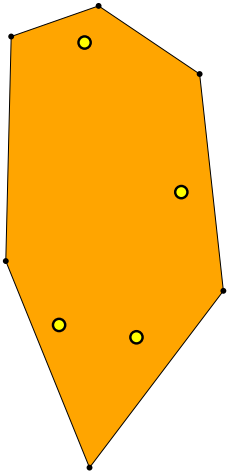
- Start with the convex hull
- Choose the valid vertex-edge pair that decreases the area the least

Issues:

- Long edges block many future vertex-edge pairs
- Stall: May have no valid vertex-edge pair

Solution:

- Penalize long edges and favor breaking long edges: $weight(p_1, p_2, q) = area(p_1p_2q) + \alpha(\|qp_1\| + \|qp_2\| - \|p_1p_2\|)$ for small α
- Add random noise to circumvent stalls



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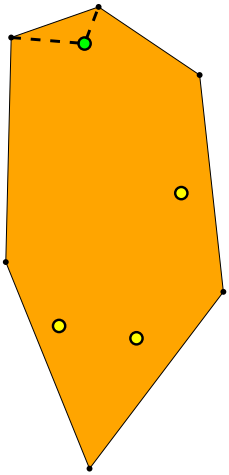

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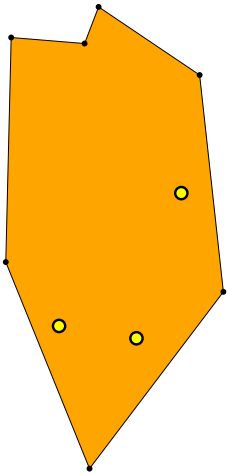

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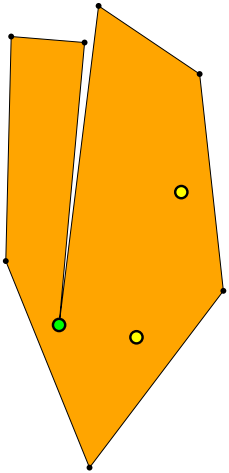

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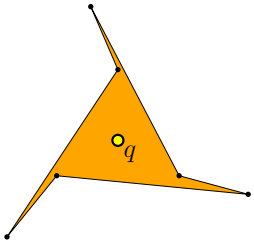

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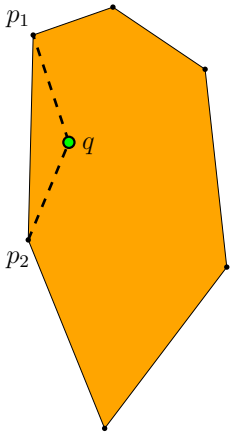

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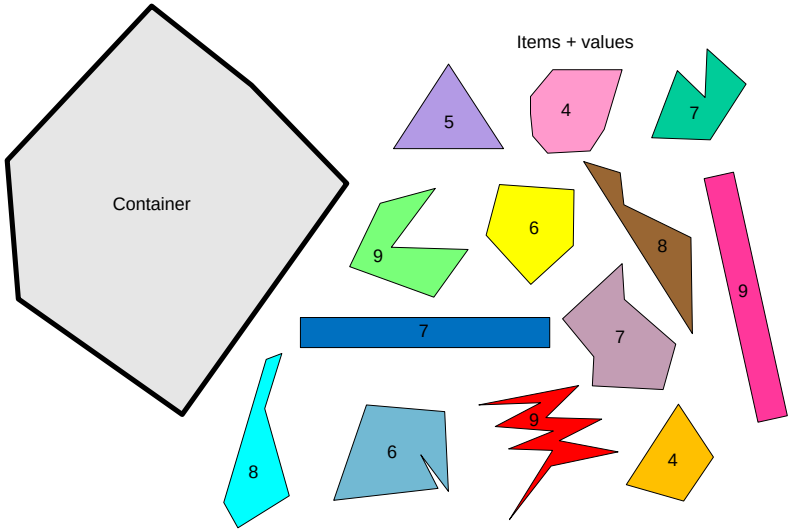

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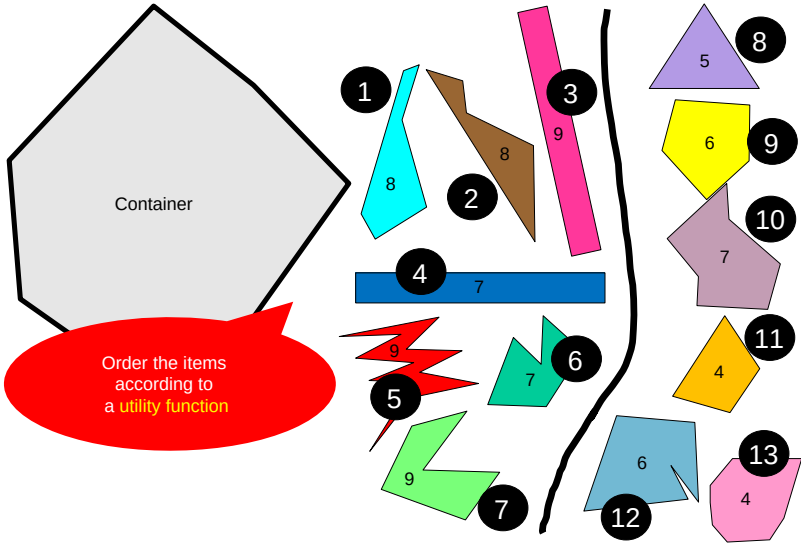

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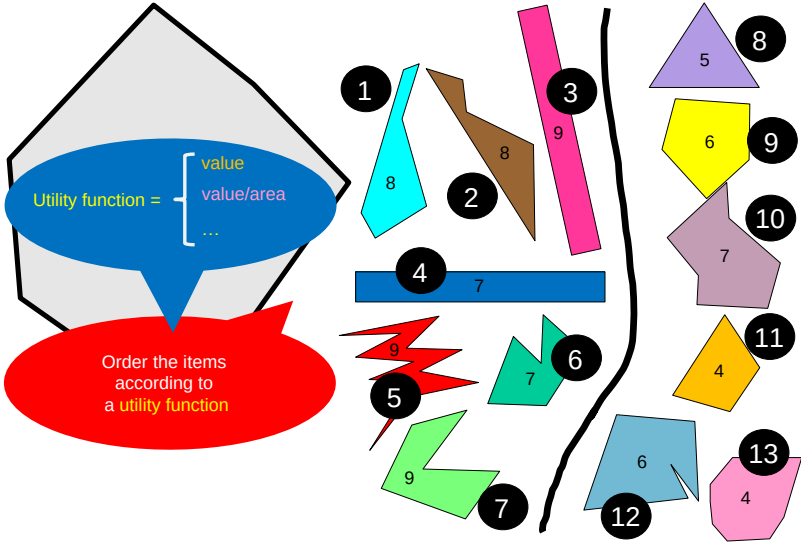

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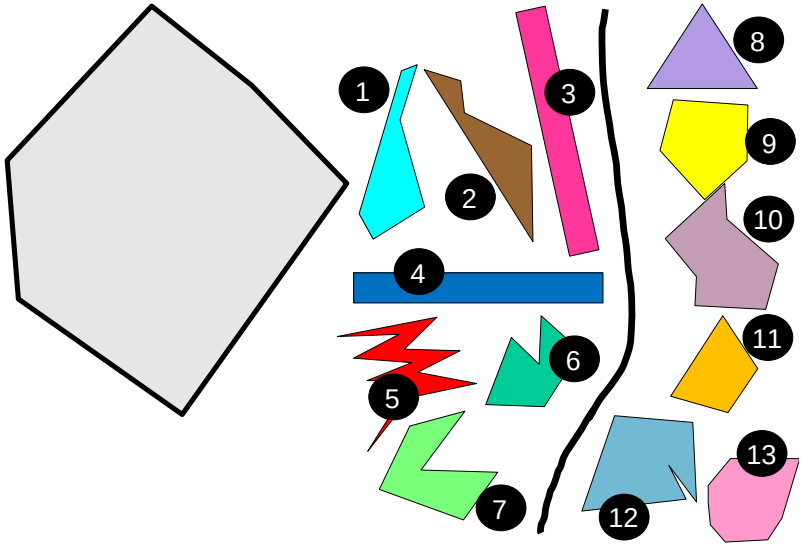

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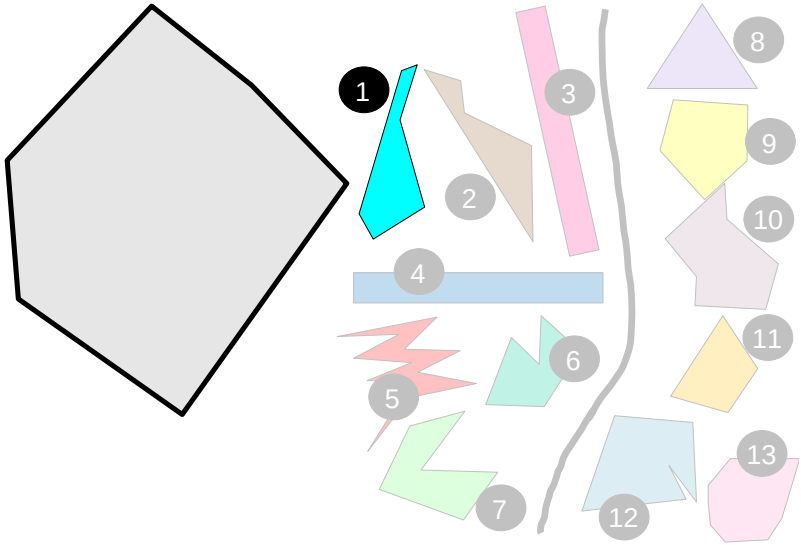

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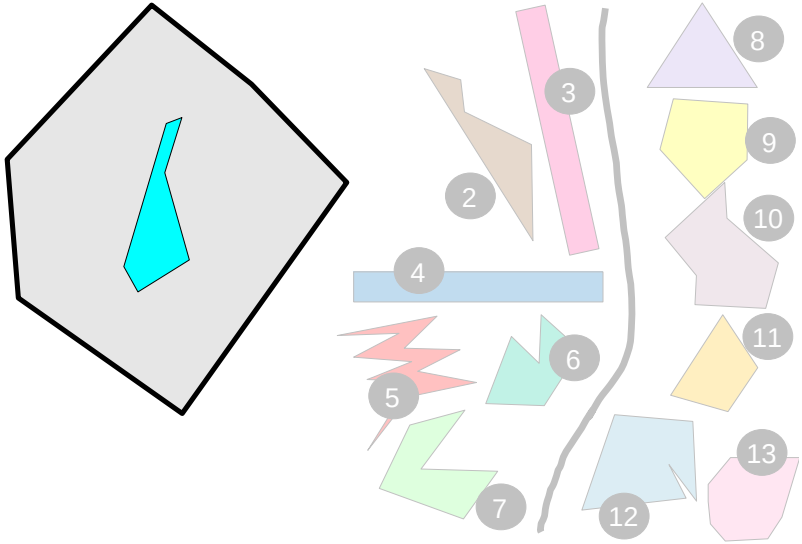

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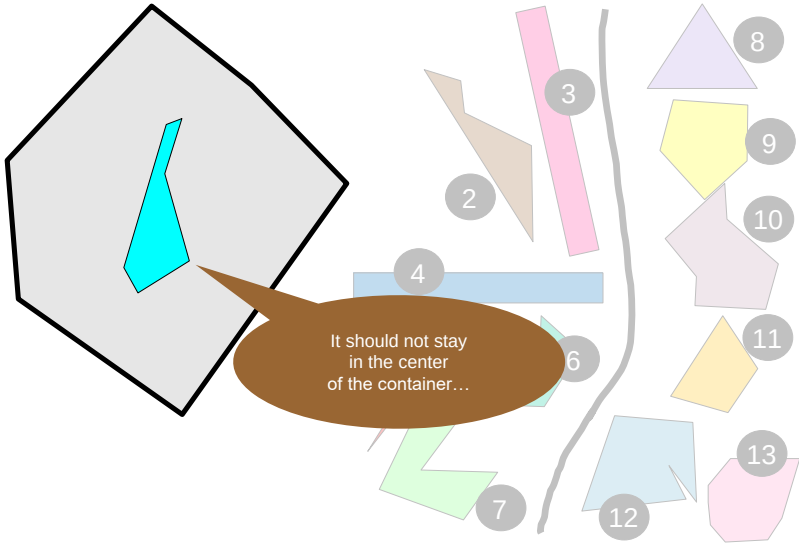

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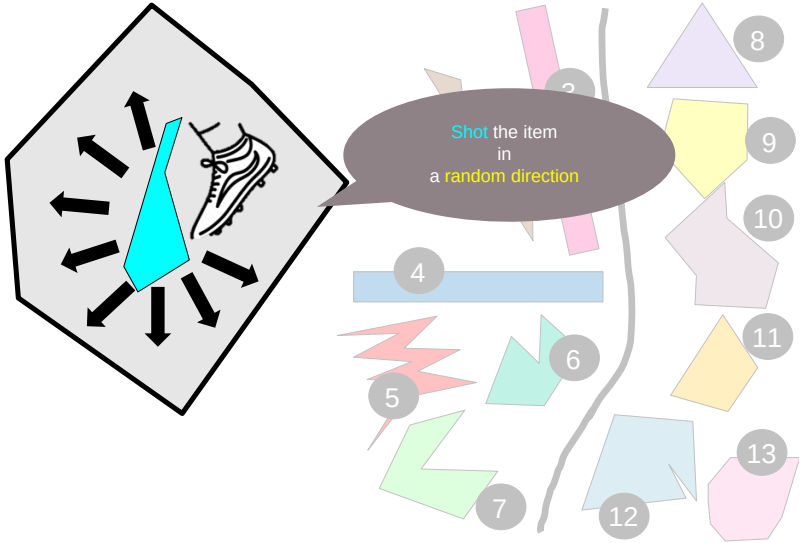

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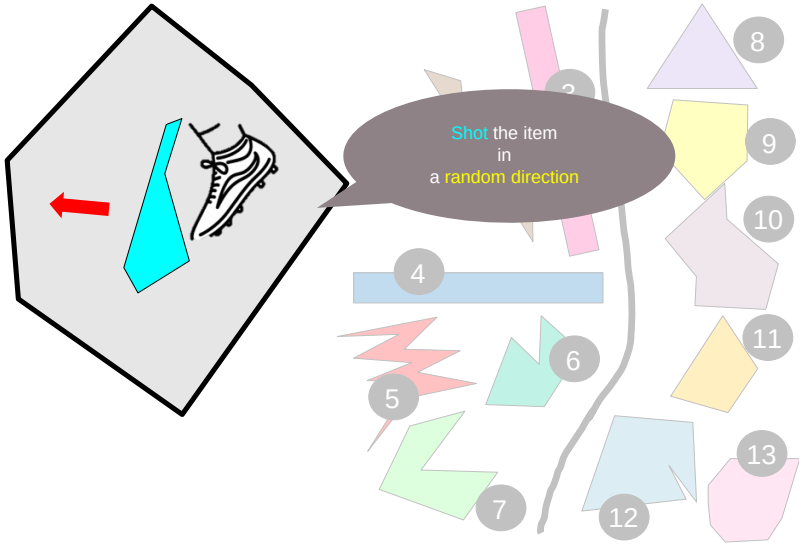

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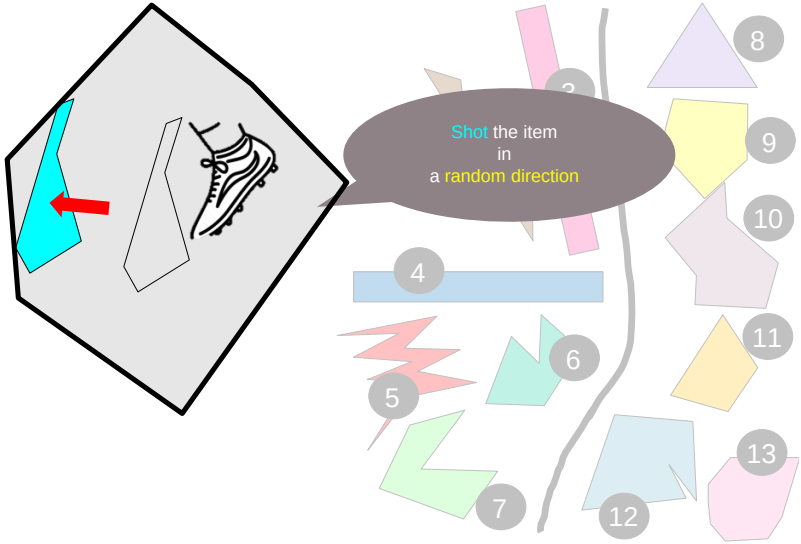

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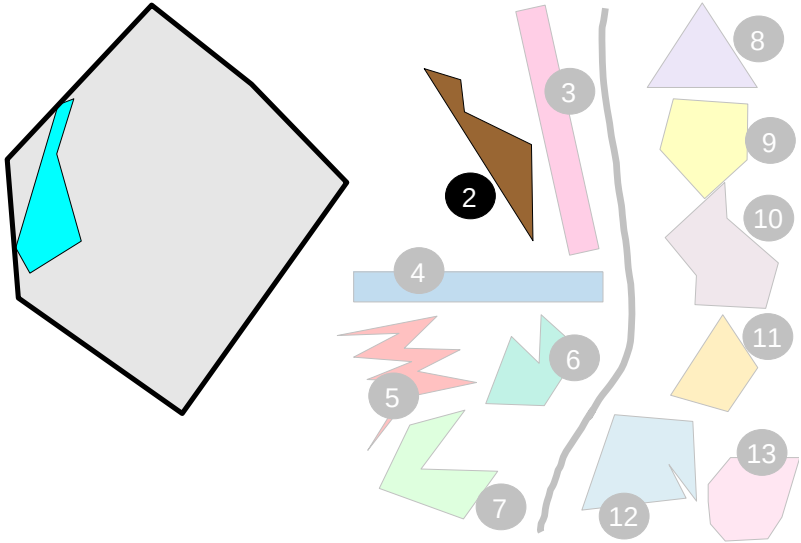

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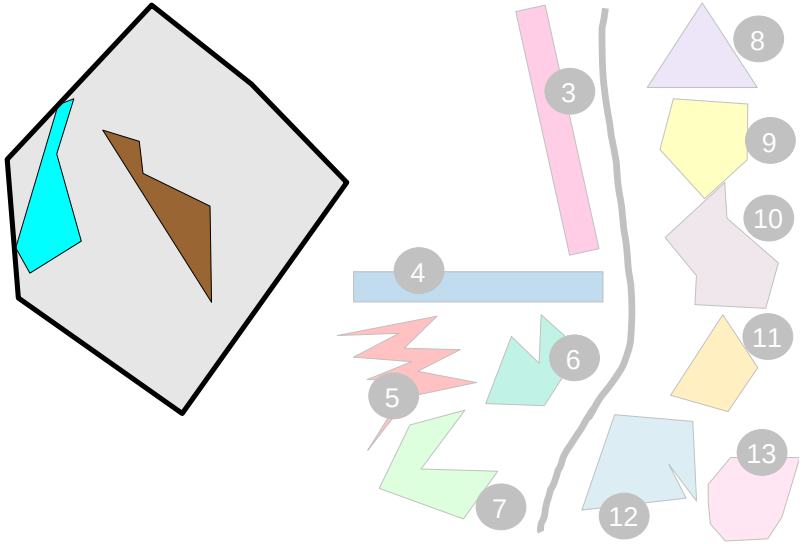

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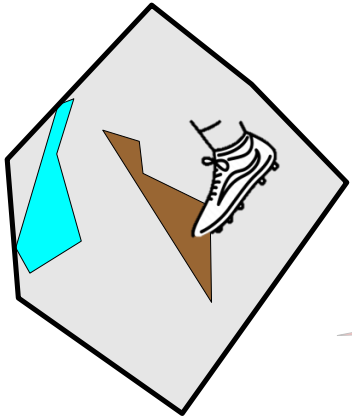

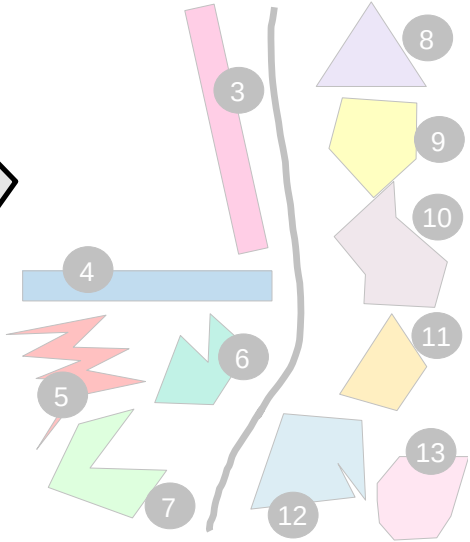

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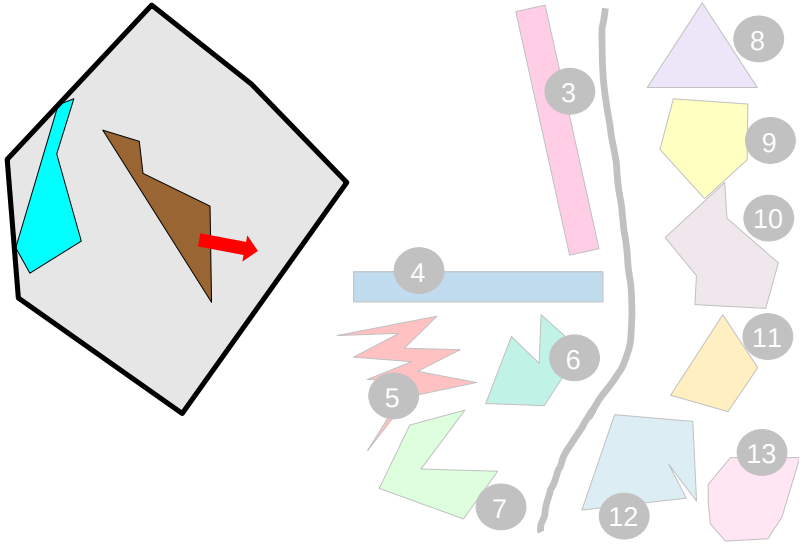

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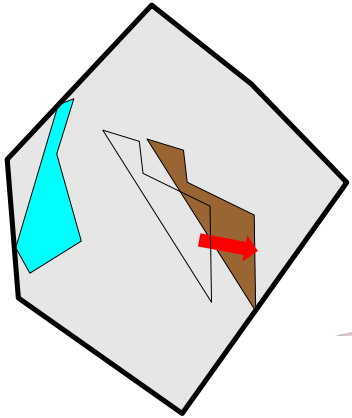

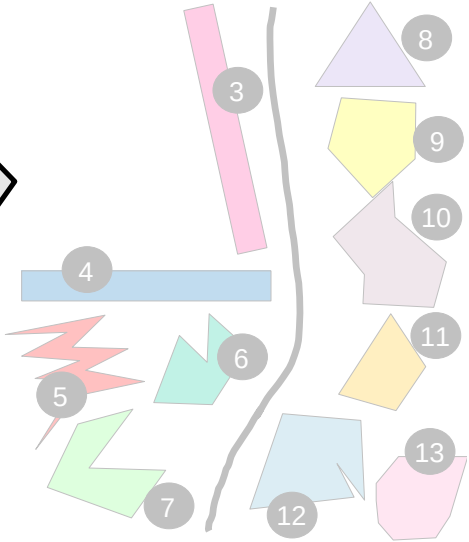

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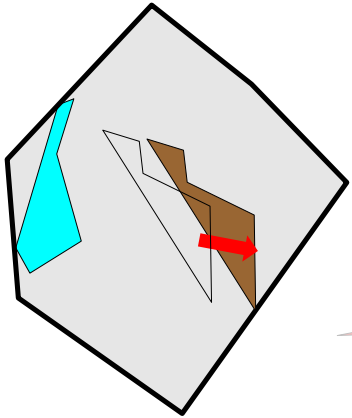

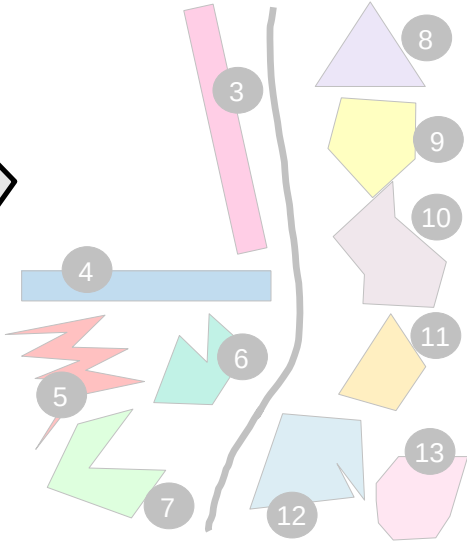

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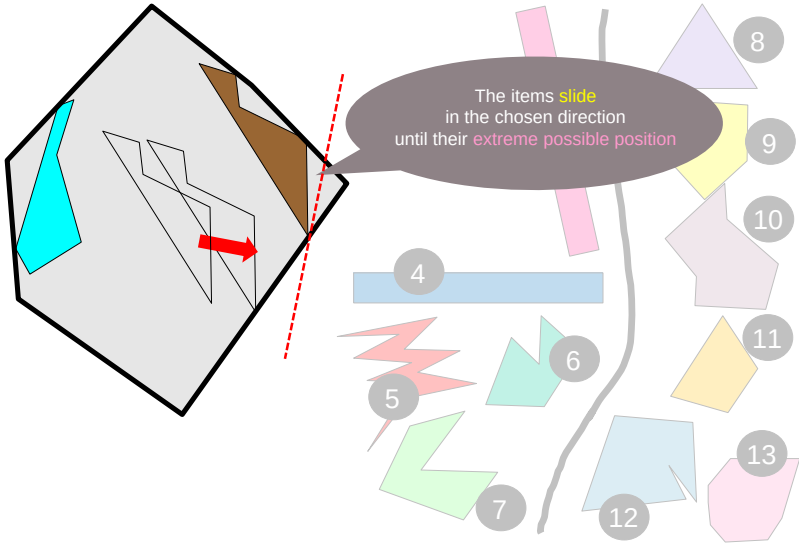

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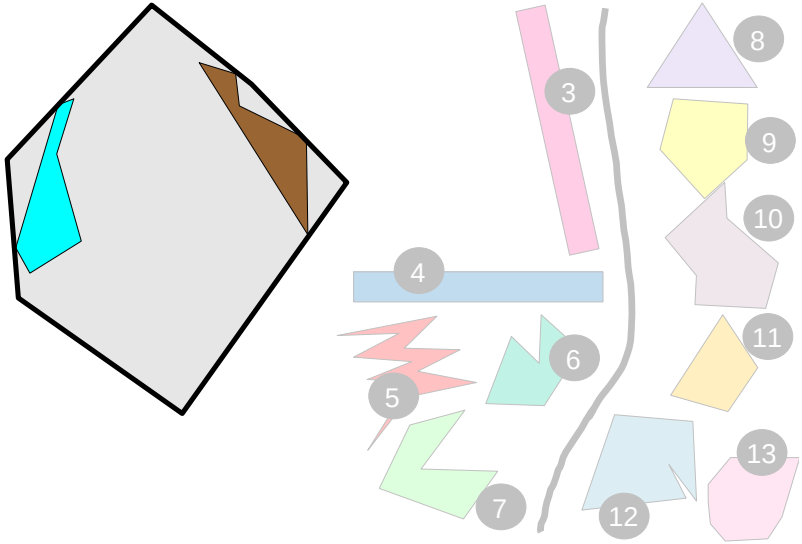

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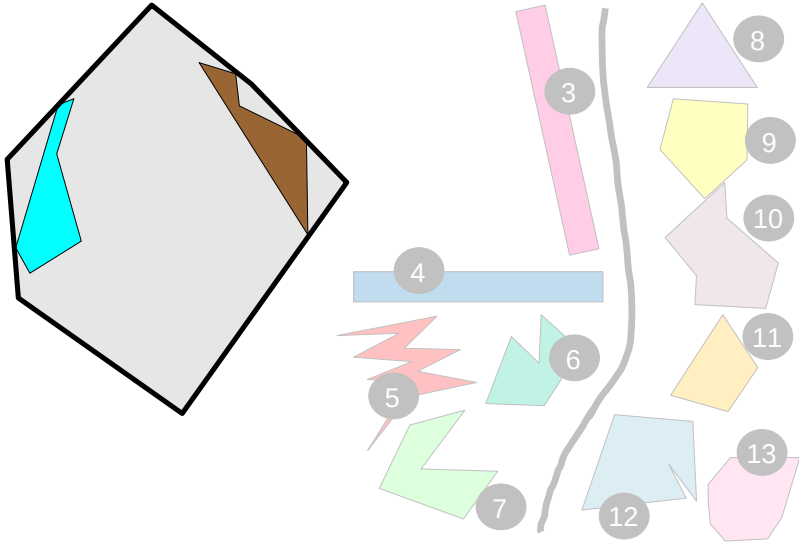

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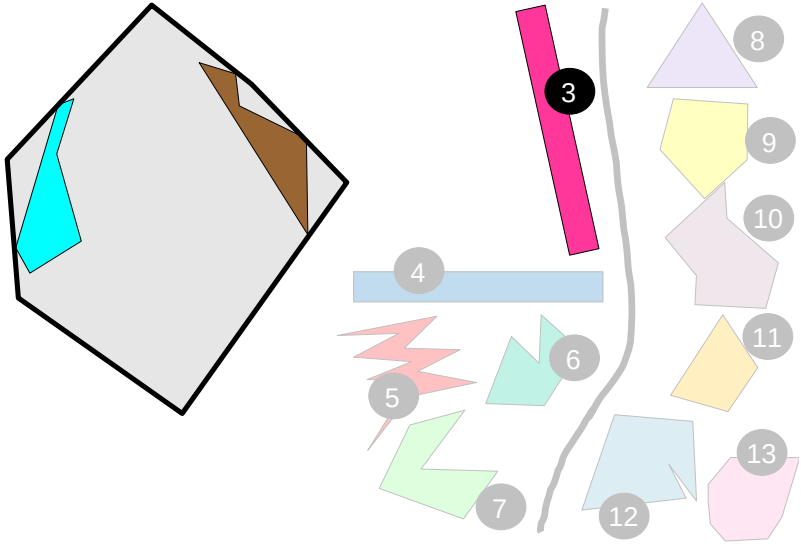

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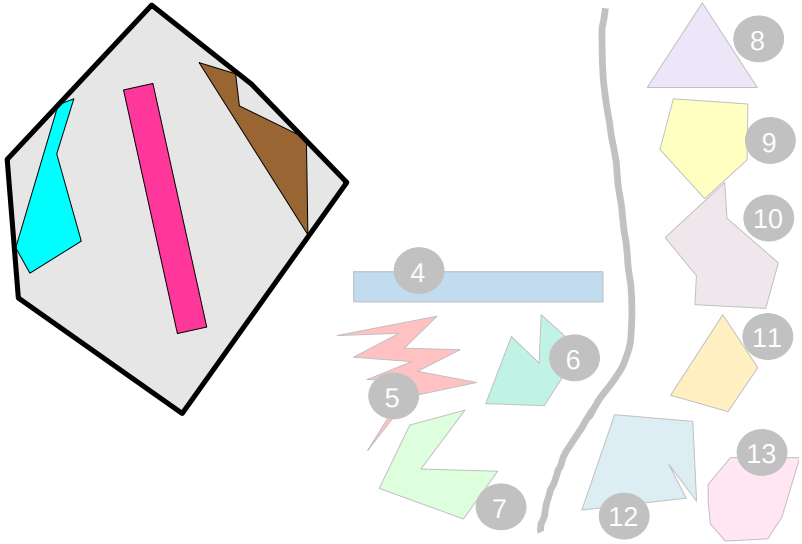

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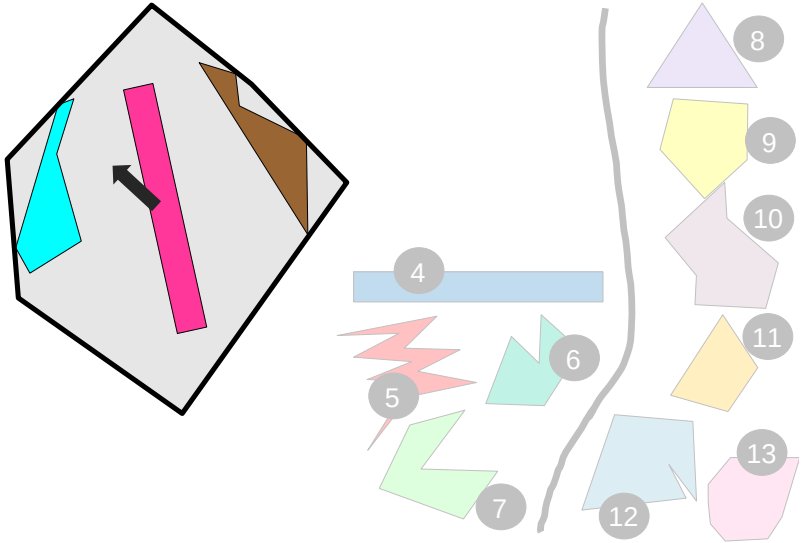

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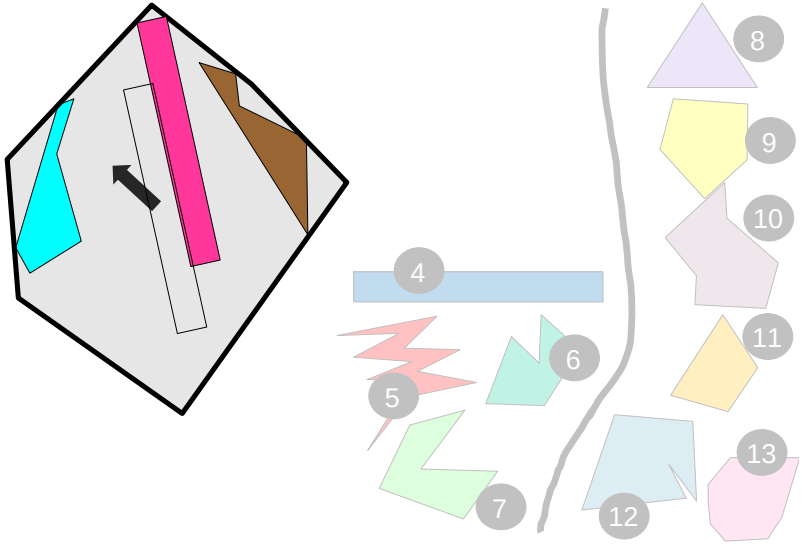

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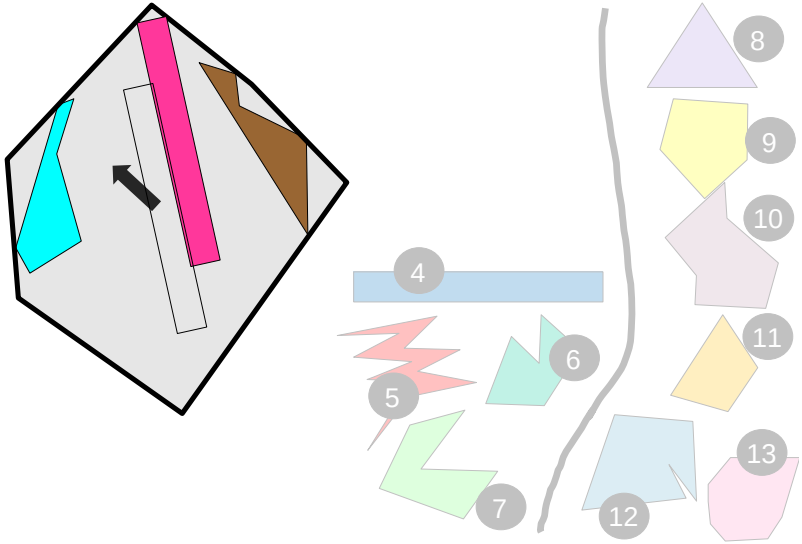

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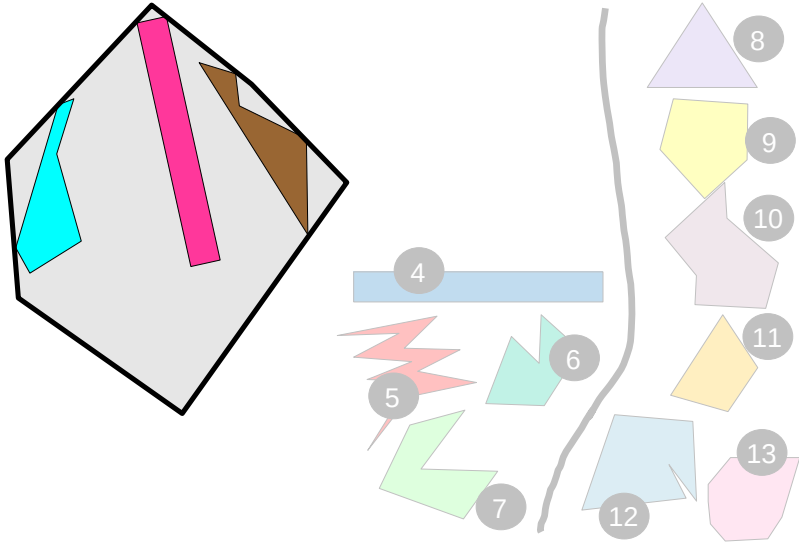

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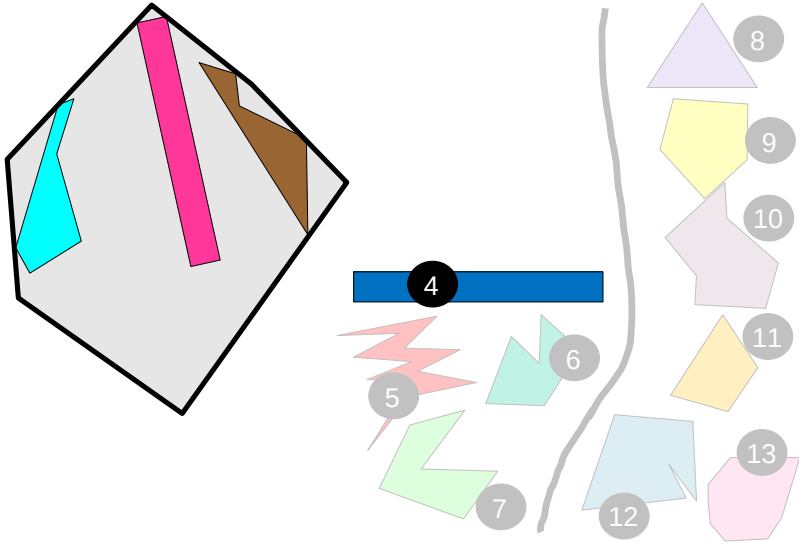

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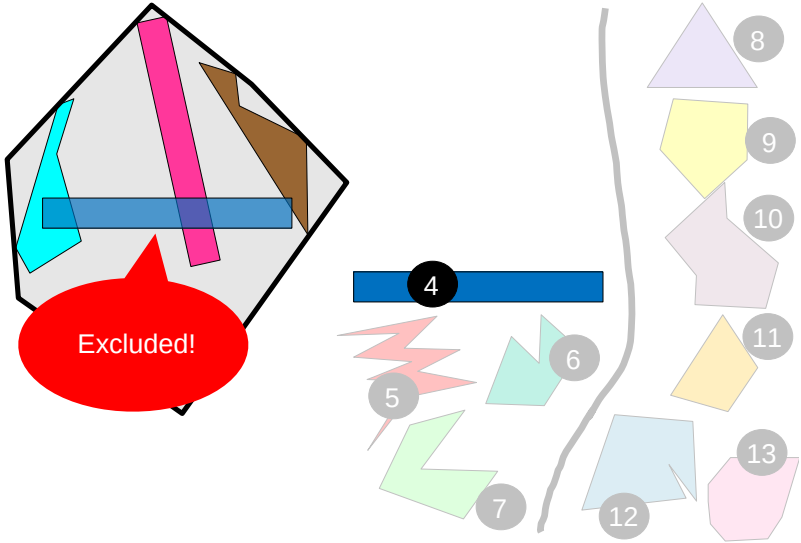

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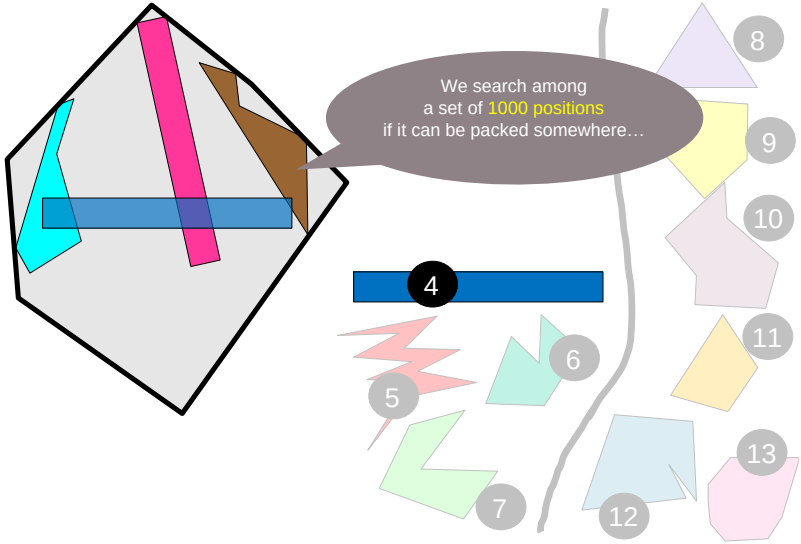

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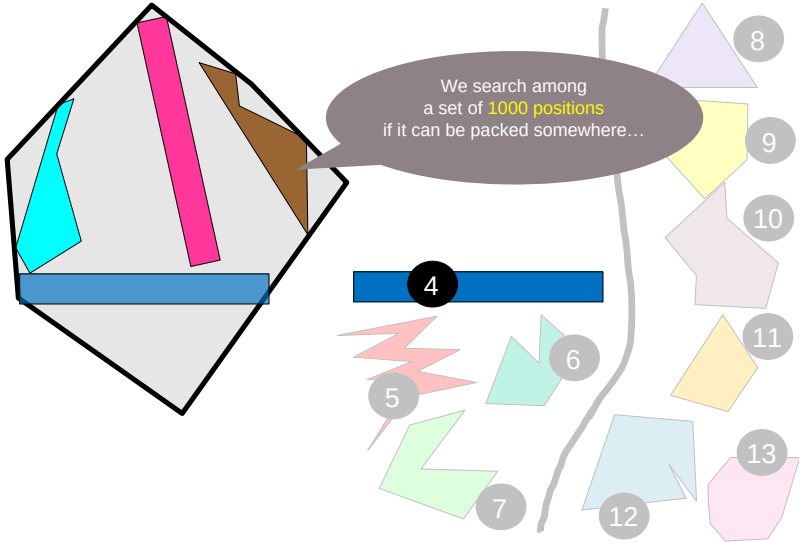

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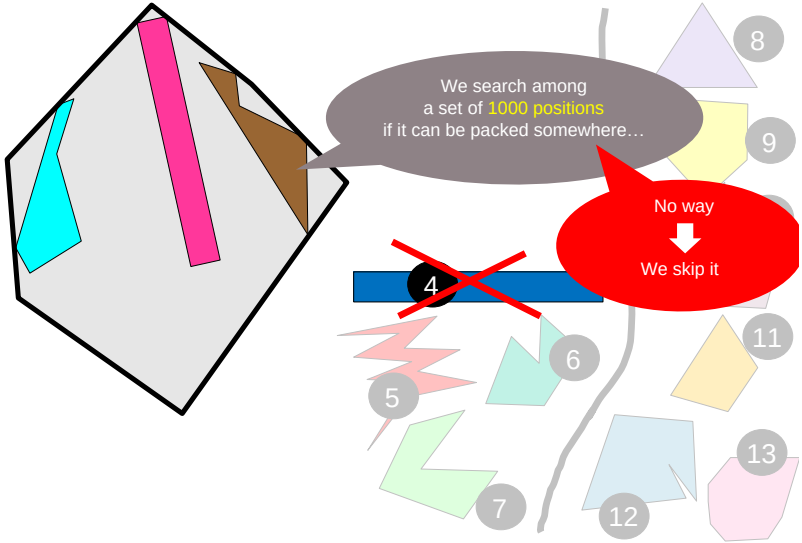

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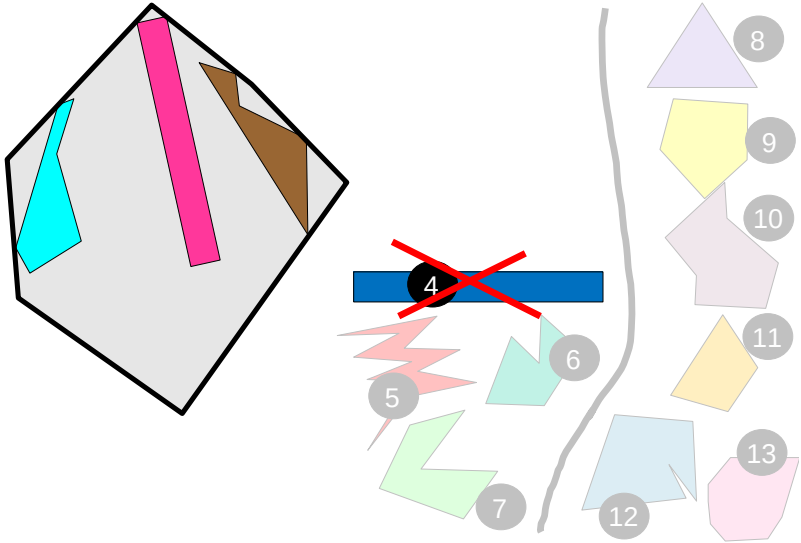

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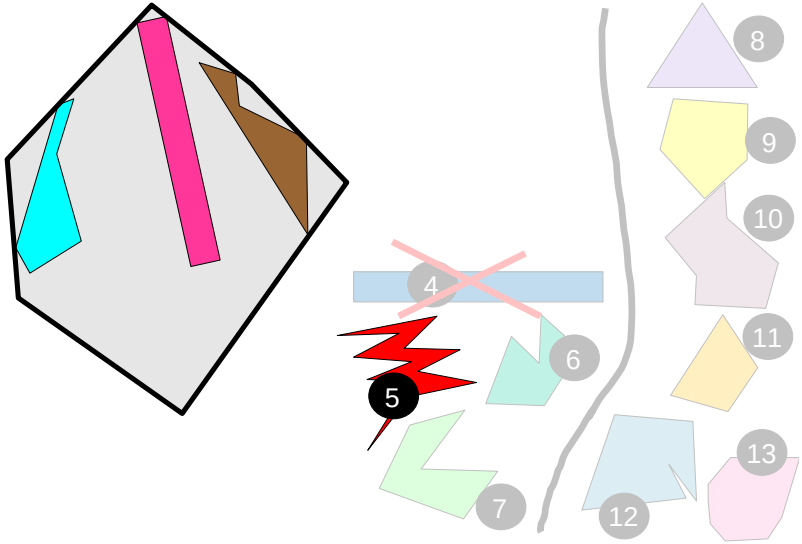

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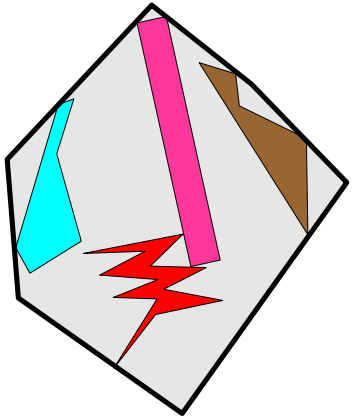

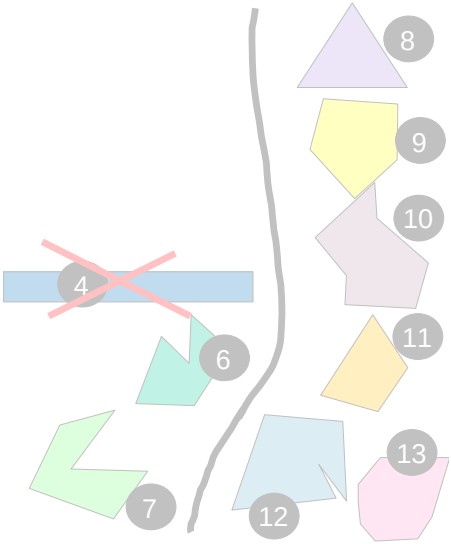

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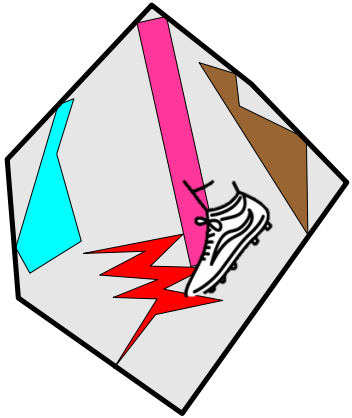

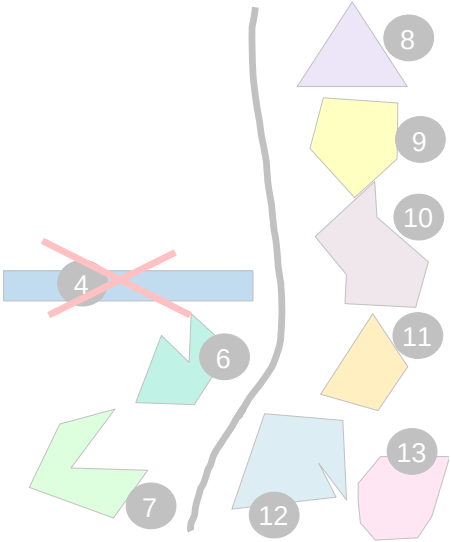

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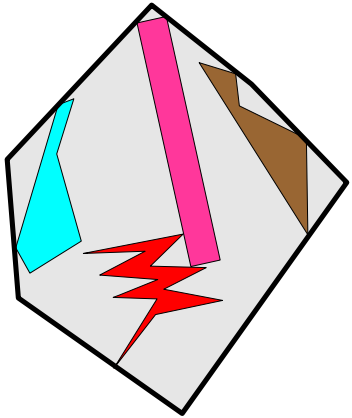

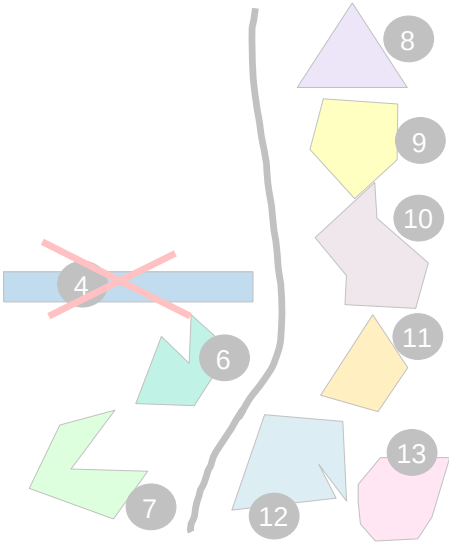

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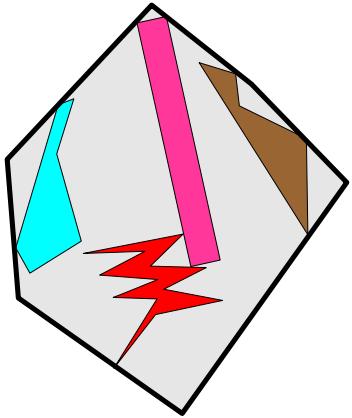

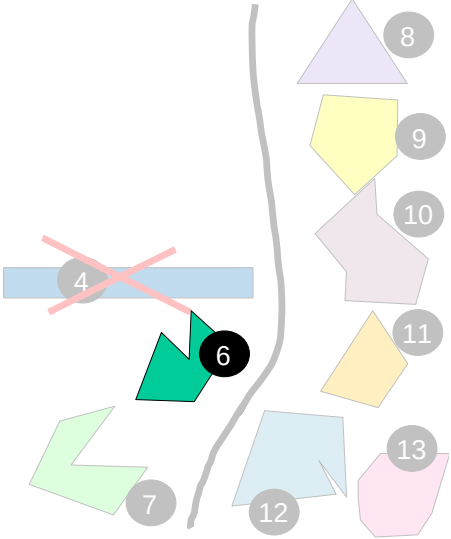

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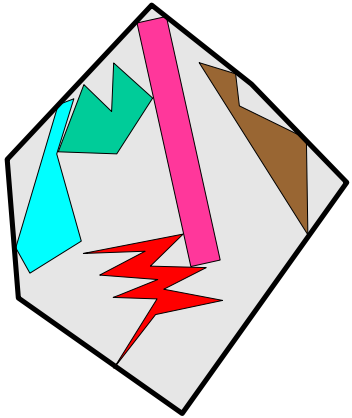

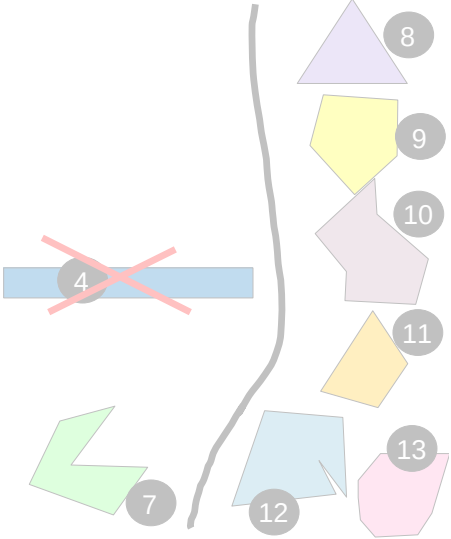

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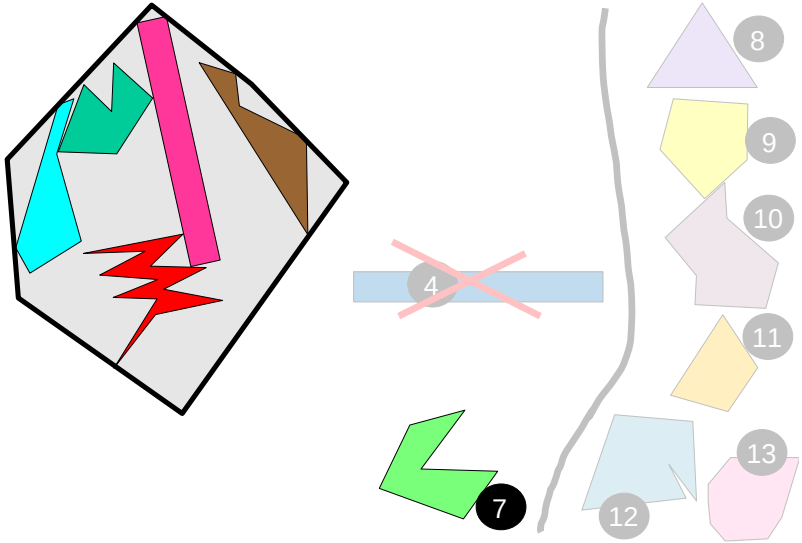

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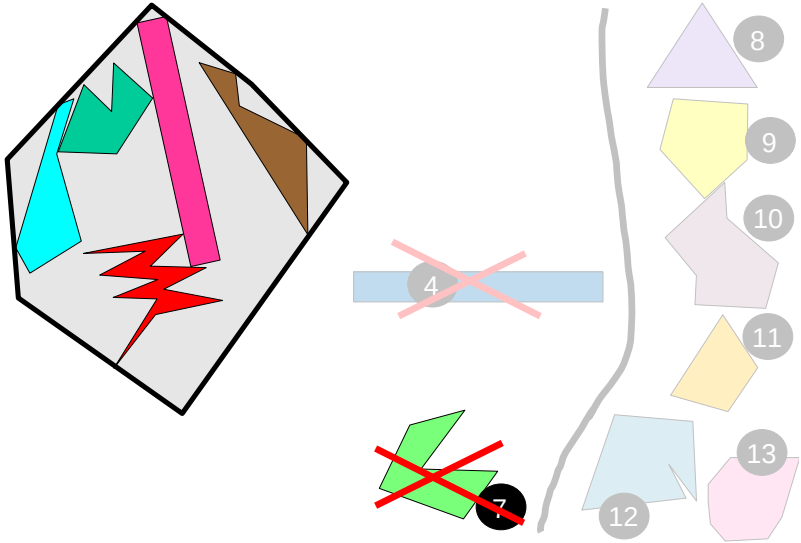

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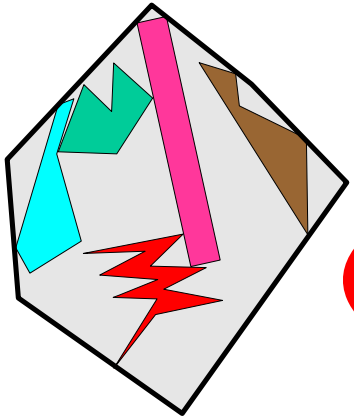
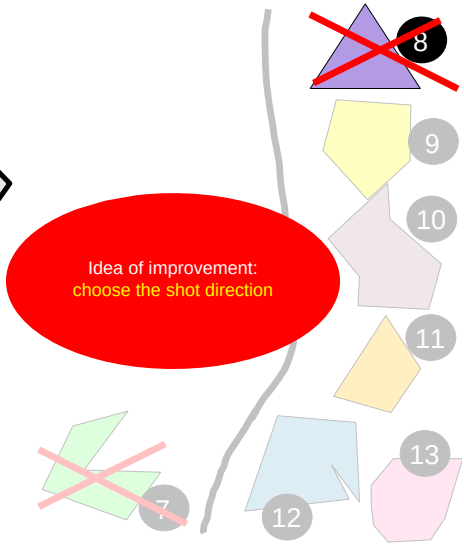
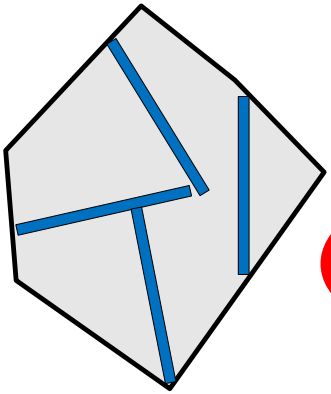




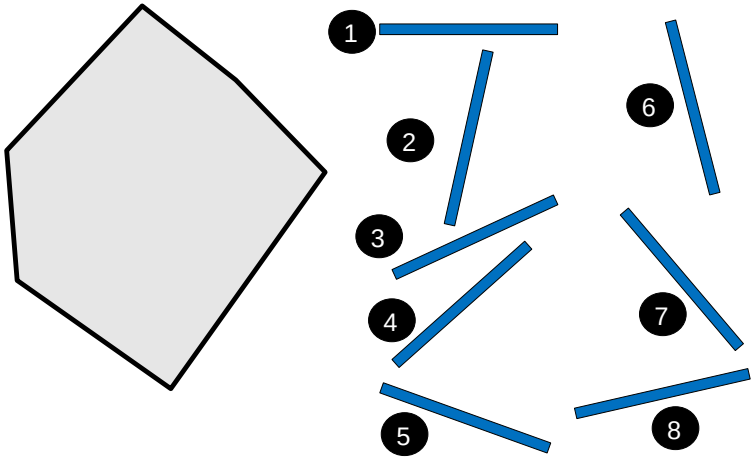
Choosing a Better Direction



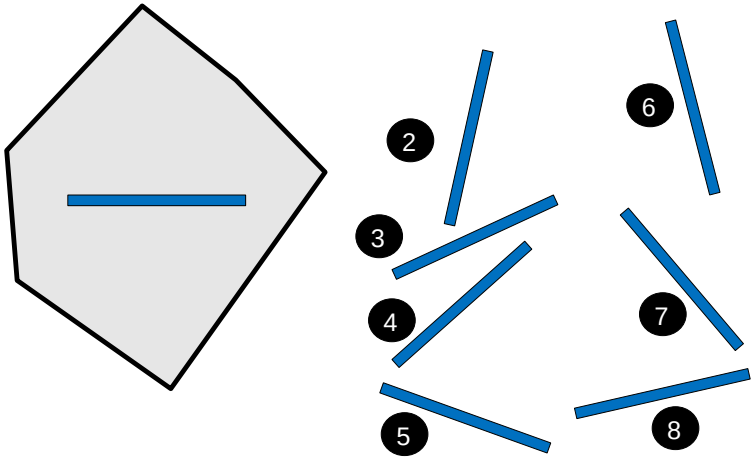
For long items:
Random is BAD

Idea of improvement:
choose the shot direction

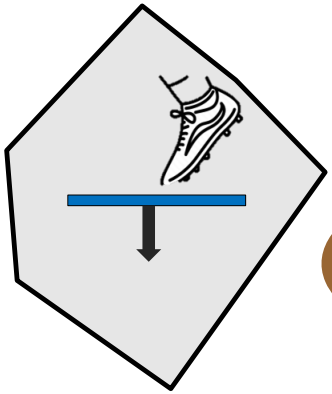
Choosing a Better Direction

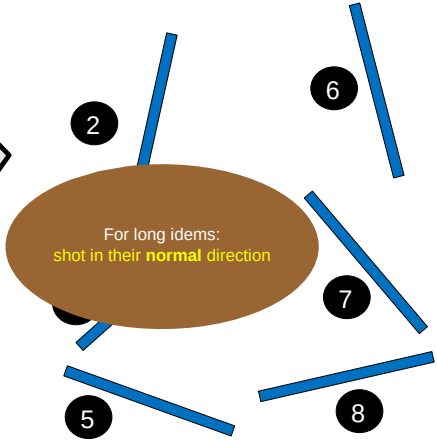


Choosing a Better Direction

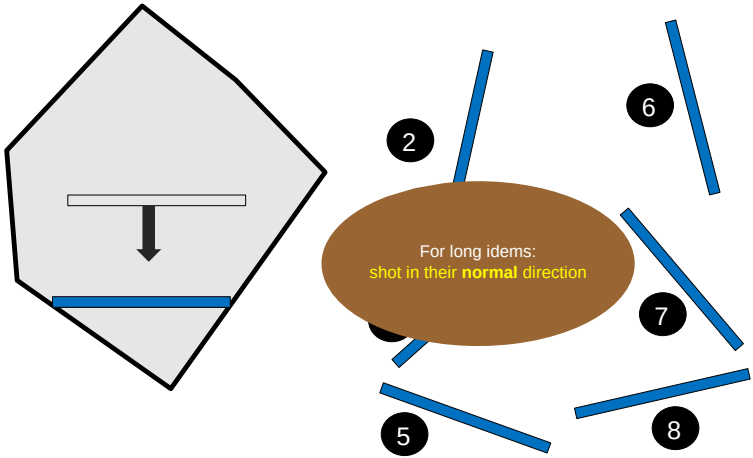


Choosing a Better Direction

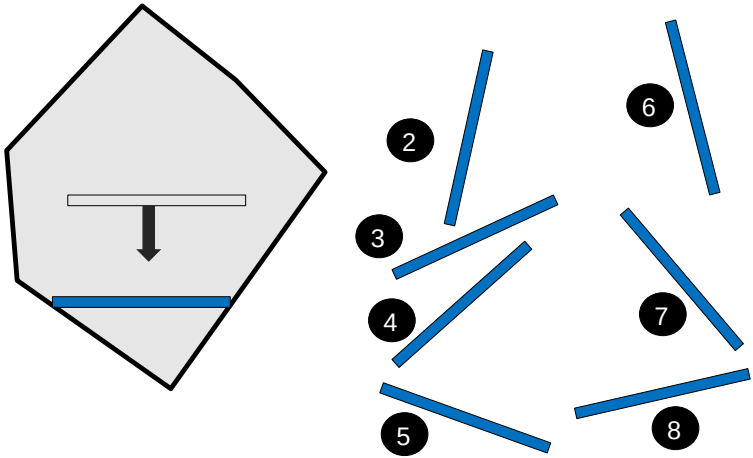




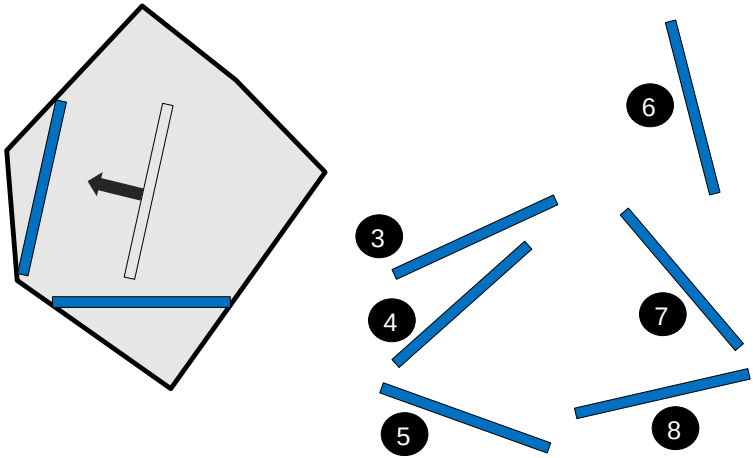
Choosing a Better Direction



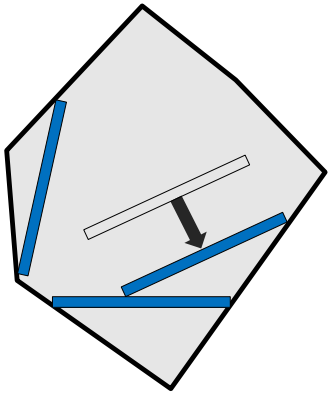
Choosing a Better Direction

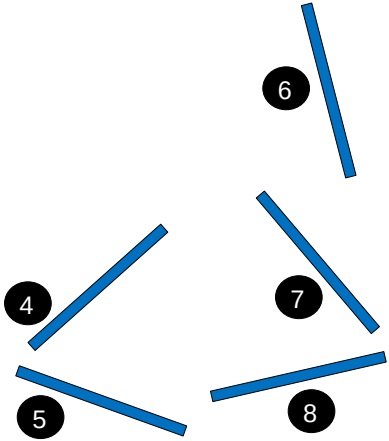


Choosing a Better Direction

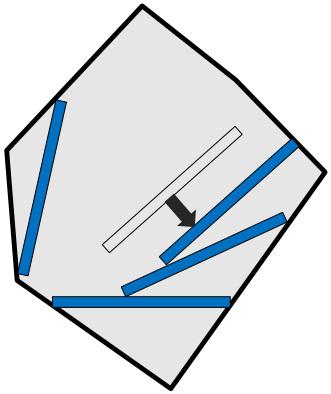


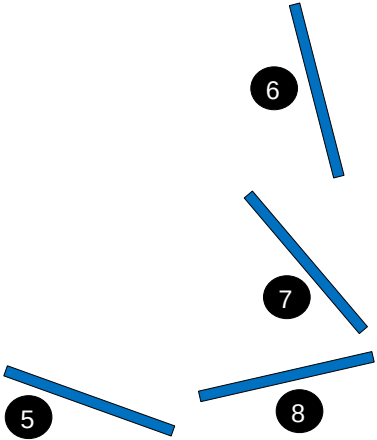
Choosing a Better Direction



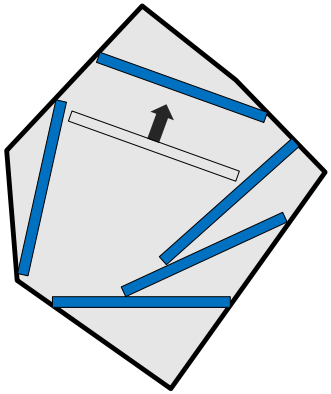


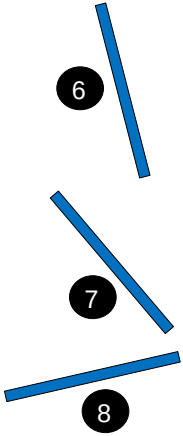
Choosing a Better Direction



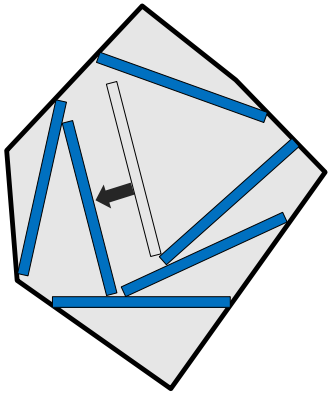


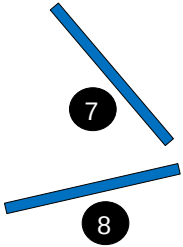
Choosing a Better Direction



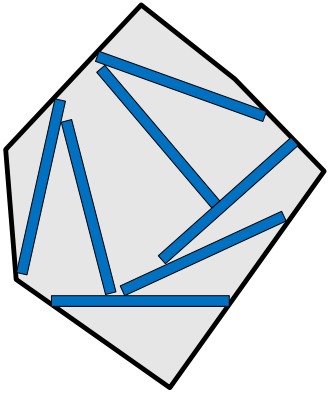


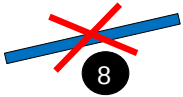
Choosing a Better Direction



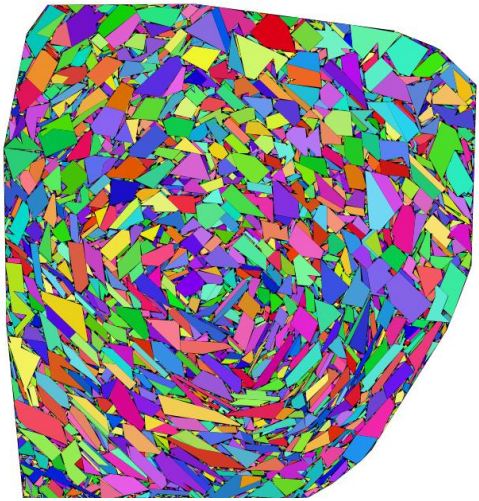


Choosing a Better Direction



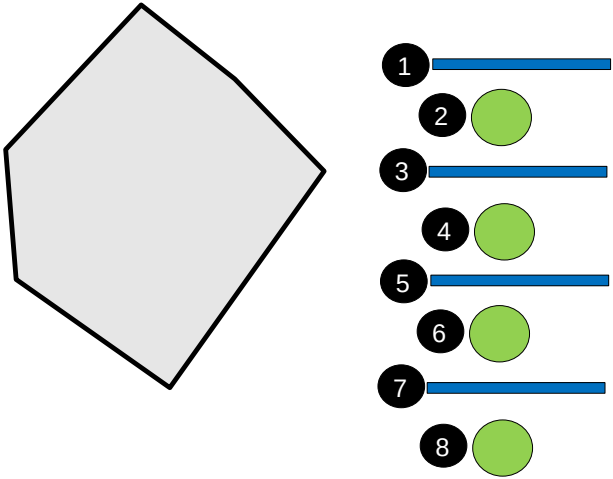


Choosing a Better Direction

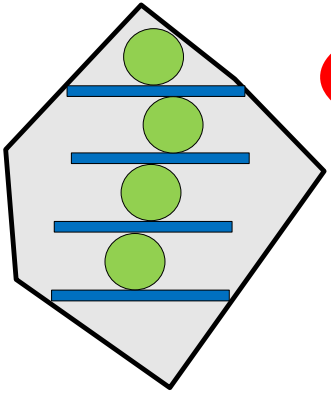


Choosing a Better Direction

Choosing a Better Direction

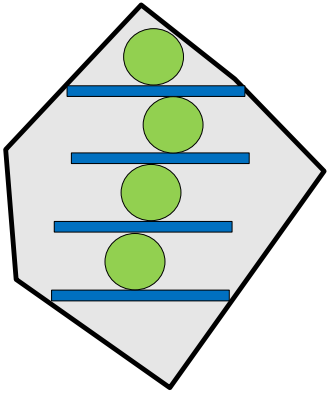


Choosing a Better Direction

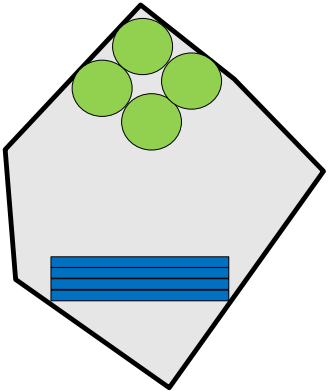


Not efficient

Choosing a Better Direction

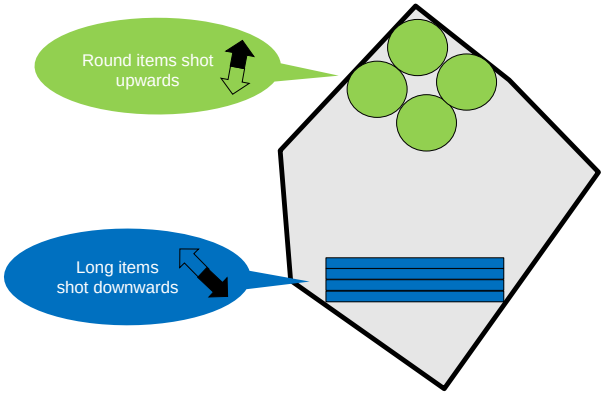


Not efficient

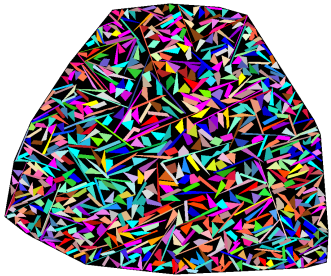


Much better

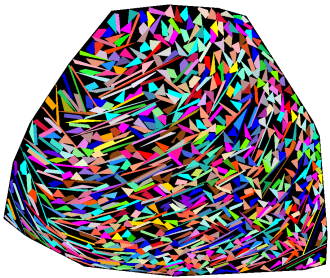
Choosing a Better Direction



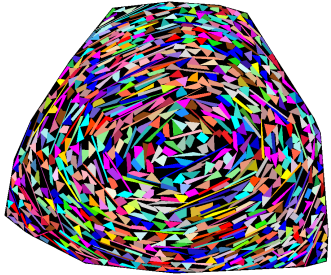
Different Packing Strategies



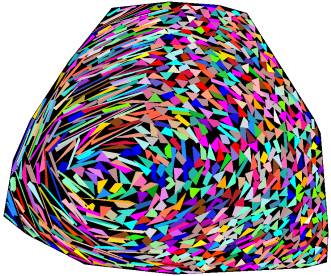
Score= 0,82



Score= 0,85

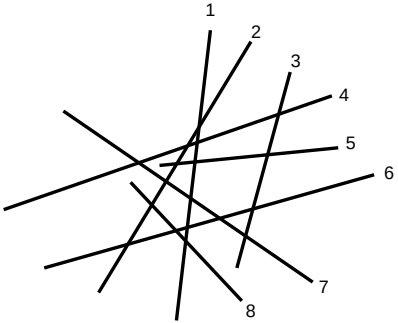


Score= 0,89



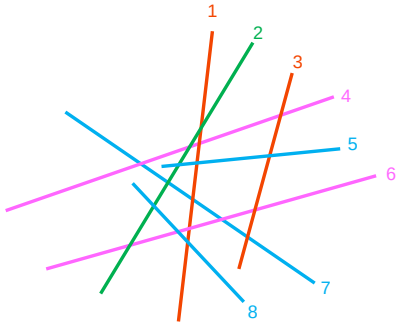
Score= 0,92

Coloring



Input: n segments in the plane

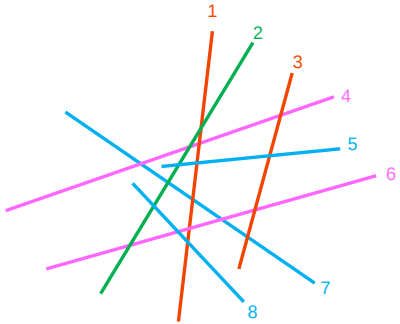
Coloring



Input: n segments in the plane

Output: segments coloring with a minimum number of colors so that two crossing segments don't have the same color…

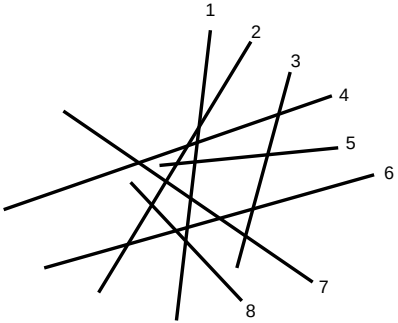
Coloring

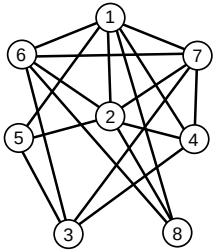


Input: n segments in the plane

Output: segments coloring with a minimum number of colors so that two crossing segments don't have the same color…

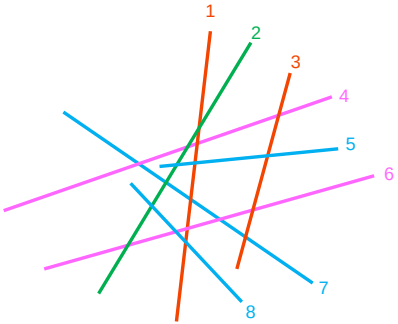
Coloring

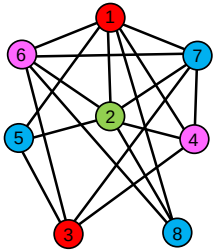




Conflict graph

Coloring

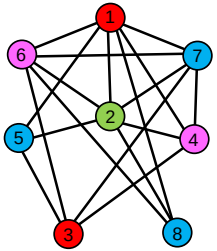




Any graph coloring algorithm can be used…
or algorithms using geometry

Coloring

Many graph coloring heurisitics…



DSATUR [Brélaz, 1979]

RLF [Leighton, 1979]

Tabu search [Hertz and de Werra, 1987]

Hybrid evolutionary algorithms

Population-based hybrid algorithms

Reduce and solve algorithms

Modified cuckoo algorithm

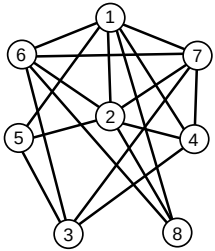
Boolean satisfiability formulation

Coloring

Most simple heuristic

Initialization: order the vertices

Repeat: take the first uncolored vertex of the list
color it with the first possible color
(create a new color if necessary)



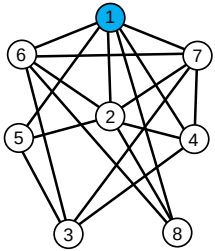
Coloring

Color 1

Most simple heuristic

Initialization: order the vertices

Repeat: take the first uncolored vertex of the list
color it with the first possible color
(create a new color if necessary)



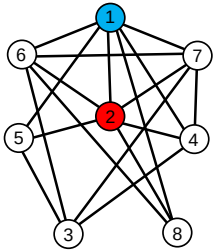
Coloring

Color 1 Color 2

Most simple heuristic

Initialization: order the vertices

Repeat: take the first uncolored vertex of the list
color it with the first possible color
(create a new color if necessary)



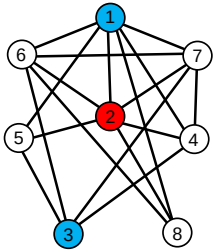
Coloring

Color 1 Color 2

Most simple heuristic

Initialization: order the vertices

Repeat: take the first uncolored vertex of the list
color it with the first possible color
(create a new color if necessary)



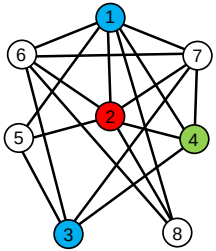
Coloring

Color 1 | Color 2 | Color 3

Most simple heuristic

Initialization: order the vertices

Repeat: take the first uncolored vertex of the list
color it with the first possible color
(create a new color if necessary)



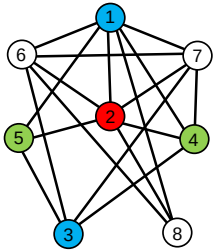
Coloring

Color 1 | Color 2 | Color 3

Most simple heuristic

Initialization: order the vertices

Repeat: take the first uncolored vertex of the list
color it with the first possible color
(create a new color if necessary)



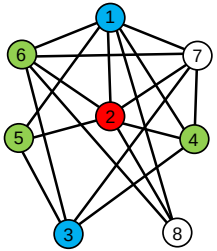
Coloring

Color 1 | Color 2 | Color 3

Most simple heuristic

Initialization: order the vertices

Repeat: take the first uncolored vertex of the list
color it with the first possible color
(create a new color if necessary)



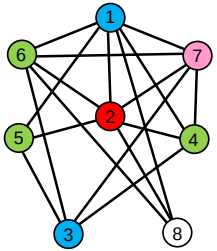
Coloring

Color 1 | Color 2 | Color 3 | Color 4

Most simple heuristic

Initialization: order the vertices

Repeat: take the first uncolored vertex of the list
color it with the first possible color
(create a new color if necessary)



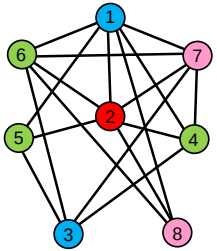
Coloring

Color 1 | Color 2 | Color 3 | Color 4

Most simple heuristic

Initialization: order the vertices

Repeat: take the first uncolored vertex of the list
color it with the first possible color
(create a new color if necessary)



Coloring

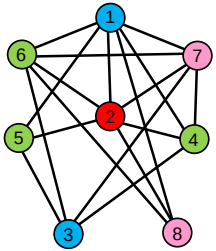
Color 1 | Color 2 | Color 3 | Color 4

Most simple heuristic

Initialization: order the vertices

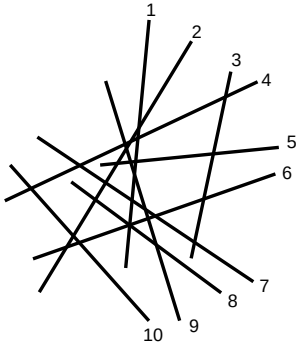
Repeat: take the first uncolored vertex of the list
color it with the first possible color
(create a new color if necessary)

Cool, but which order ?

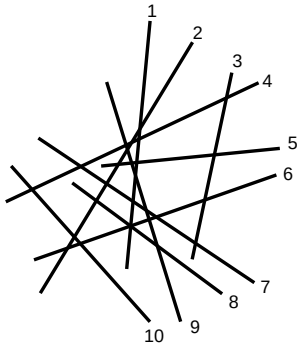


Coloring

Cool, but which order ?



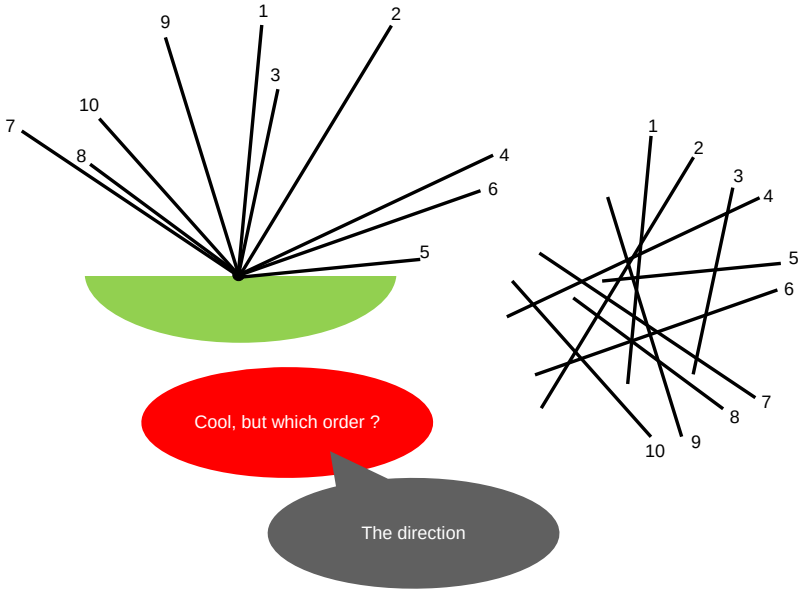
Coloring



Cool, but which order ?

The direction

Coloring


9
1
2
3
10
7
8
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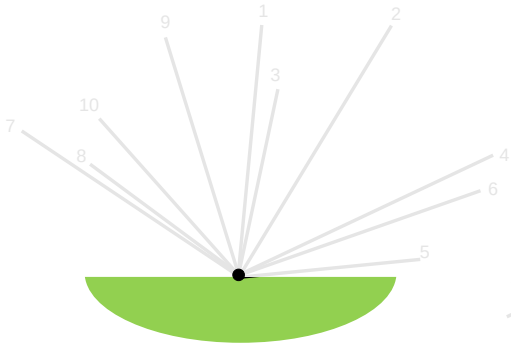
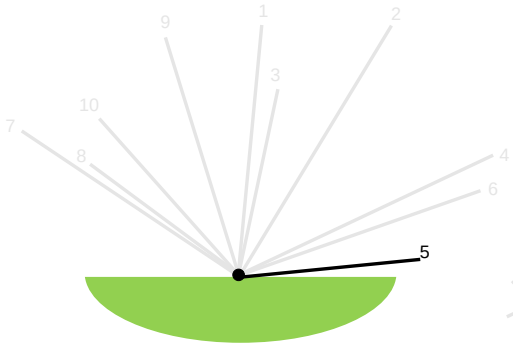
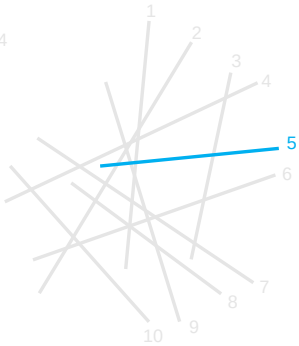
6
5
1
2
3
4
5
6
7
8
10
9
Cool, but which order ?
The direction

Coloring



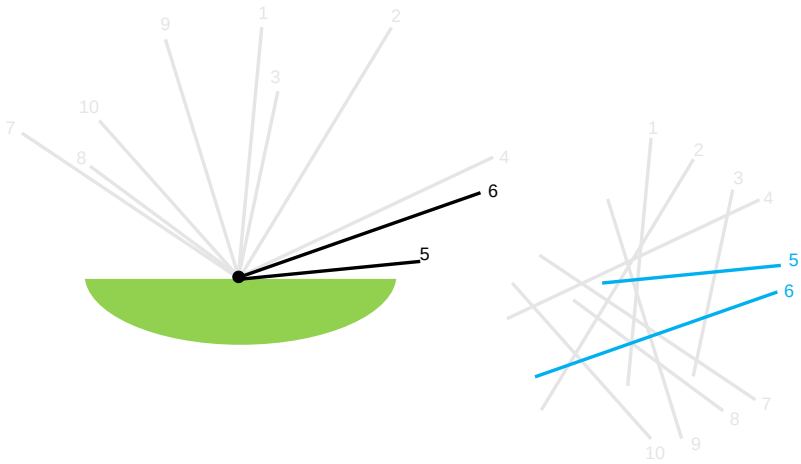
Coloring



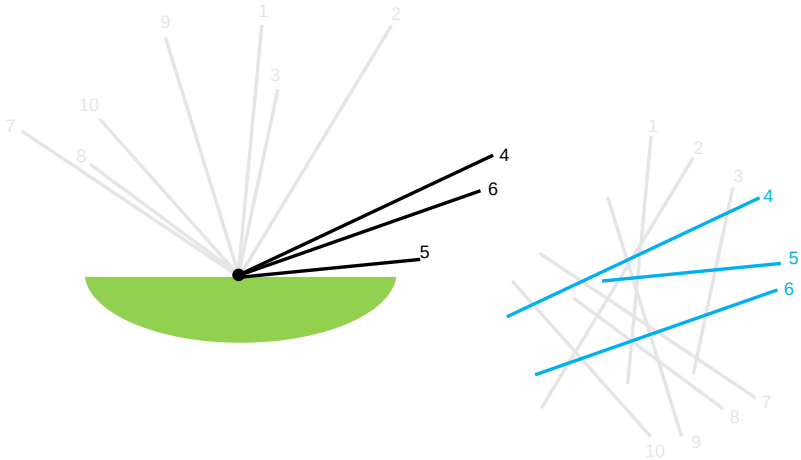


Color 1

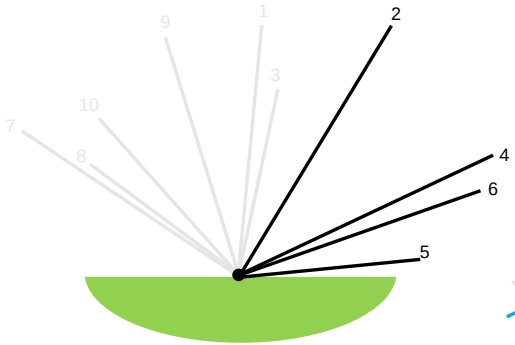
Coloring

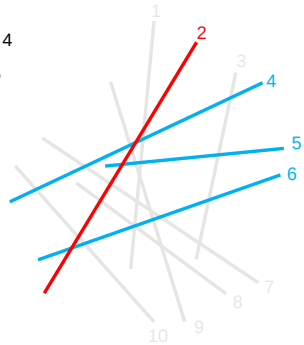


Coloring



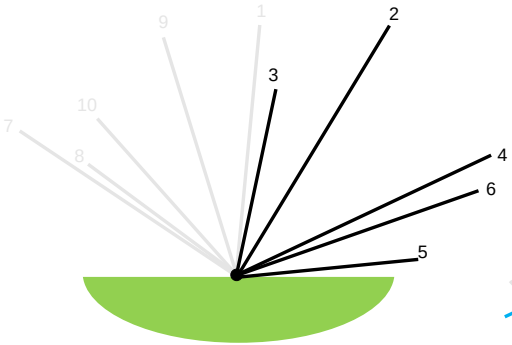
Coloring

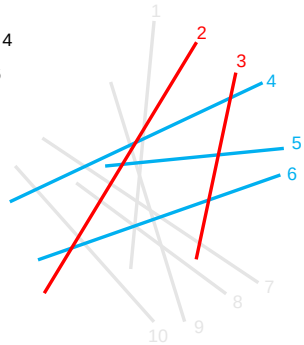




Color 1 Color 2

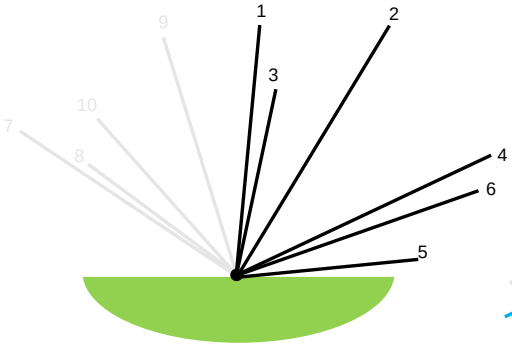
Coloring

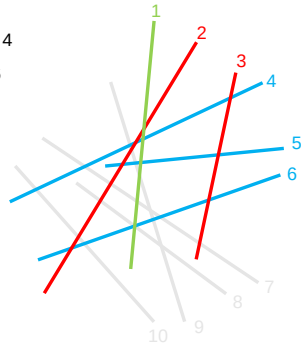




Color 1 Color 2

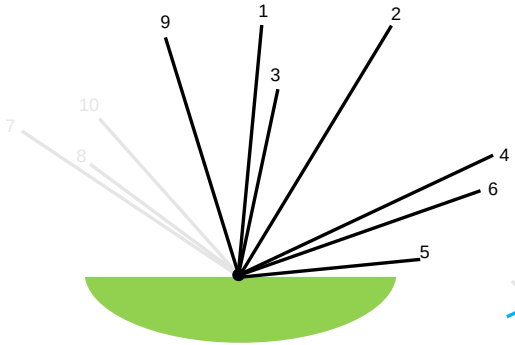
Coloring

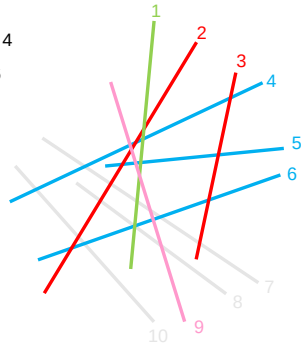




Color 1 | Color 2 | Color 3

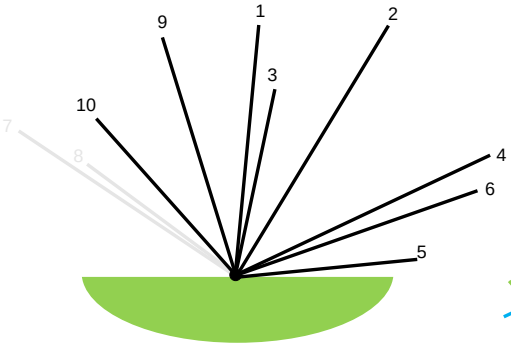
Coloring

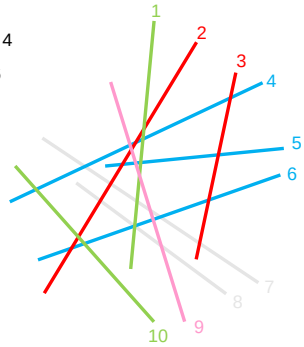




Color 1 | Color 2 | Color 3 | Color 4

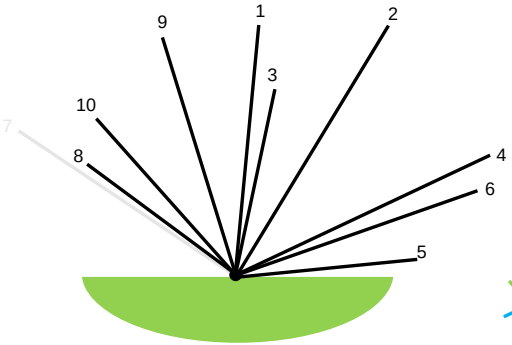
Coloring

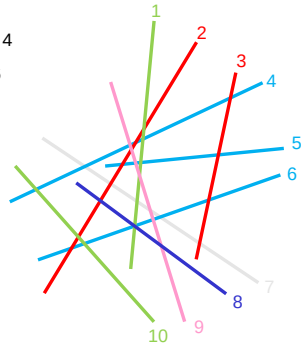




Color 1 | Color 2 | Color 3 | Color 4

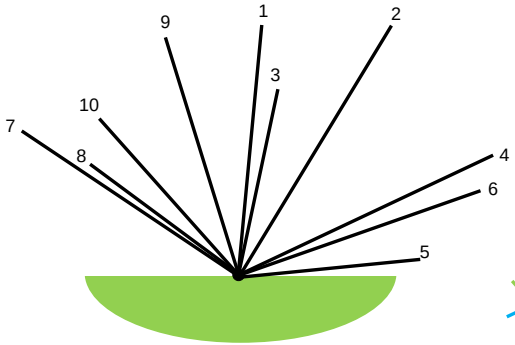
Coloring

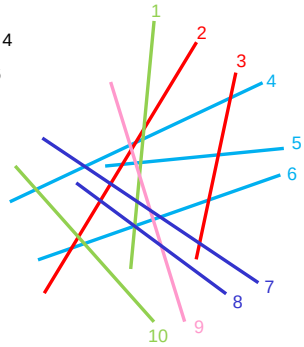




Color 1 | Color 2 | Color 3 | Color 4 | Color 5

Coloring

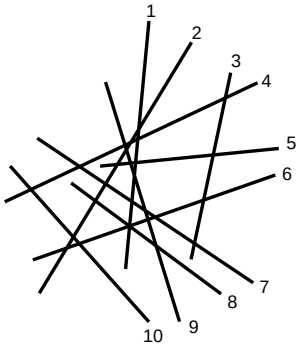




Color 1 | Color 2 | Color 3 | Color 4 | Color 5

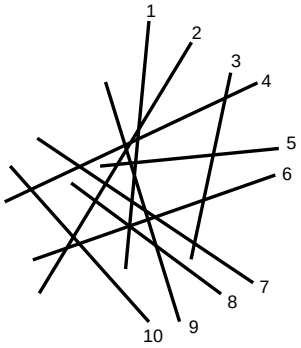
DSatHull Coloring

Another greedy heuristic
(variant of DSatur)



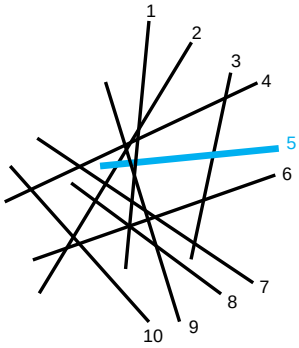
DSatHull Coloring

DSatur



DSatHull Coloring

DSatur

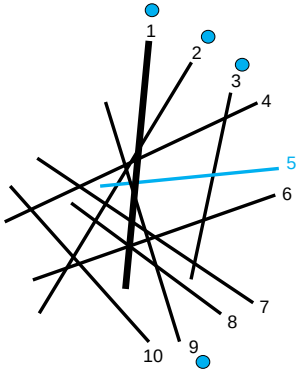


Color 1

DSatHull Coloring

DSatur

Color a segment that crosses the largest number of colors

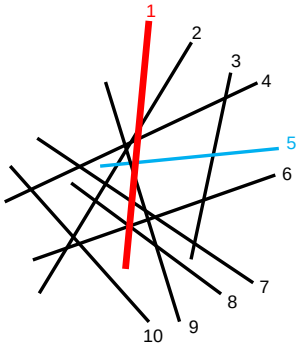


Color 1

DSatHull Coloring

DSatur

Color a segment that crosses the largest number of colors



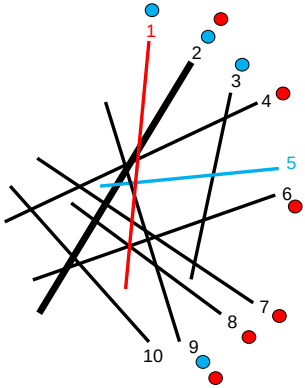
Color 1

Color 2

DSatHull Coloring

DSatur

Color a segment that crosses the largest number of colors

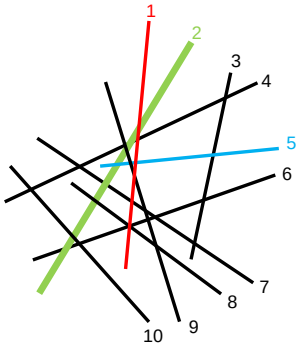


Color 1 Color 2

DSatHull Coloring

DSatur

Color a segment that crosses the largest number of colors

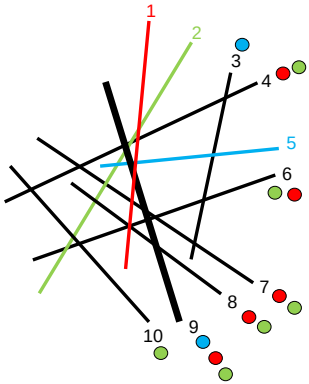


Color 1 | Color 2 | Color 3

DSatHull Coloring

DSatur

Color a segment that crosses the largest number of colors

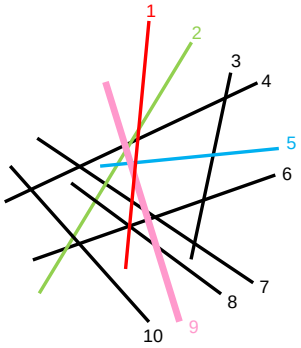


Color 1 | Color 2 | Color 3

DSatHull Coloring

DSatur

Color a segment that crosses the largest number of colors

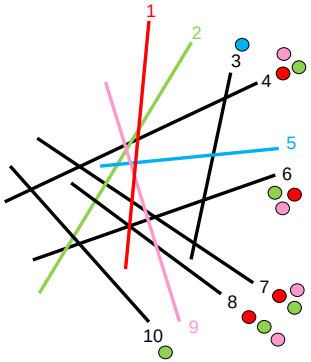


Color 1 | Color 2 | Color 3 | Color 4

DSatHull Coloring

DSatur

Color a segment that crosses the largest number of colors



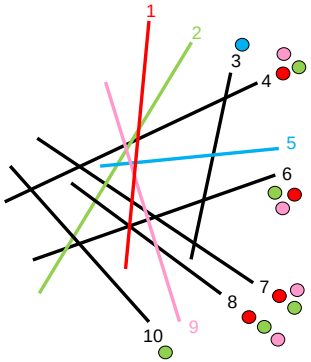
Color 1 | Color 2 | Color 3 | Color 4

DSatHull Coloring

DSatur

Color a segment that crosses the largest number of colors

And we continue…

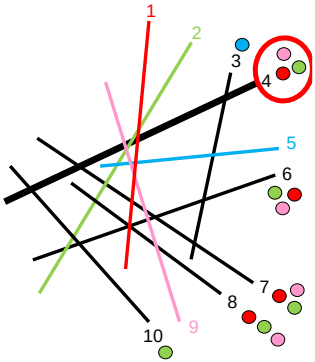


Color 1 | Color 2 | Color 3 | Color 4

DSatHull Coloring

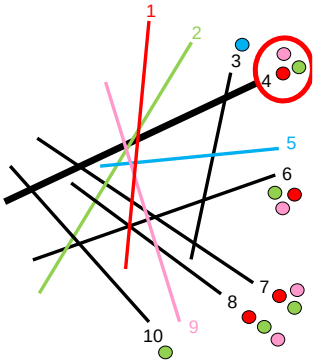
DSatur

Color a segment that crosses the largest number of colors



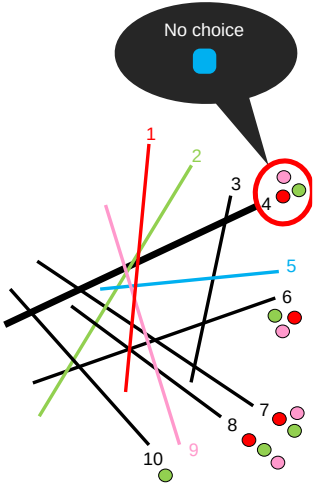
Color 1 | Color 2 | Color 3 | Color 4

DSatHull Coloring



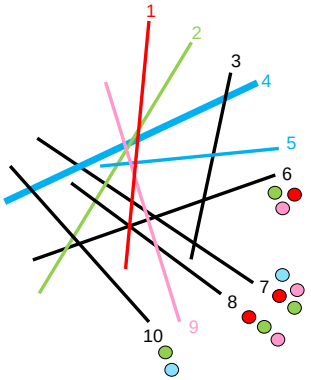
Color 1 | Color 2 | Color 3 | Color 4

DSatHull Coloring



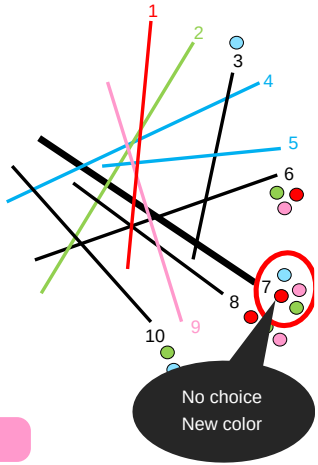
Color 1 | Color 2 | Color 3 | Color 4

DSatHull Coloring



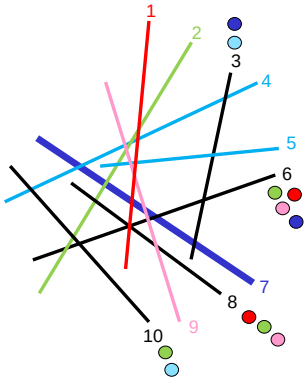
Color 1 | Color 2 | Color 3 | Color 4

DSatHull Coloring



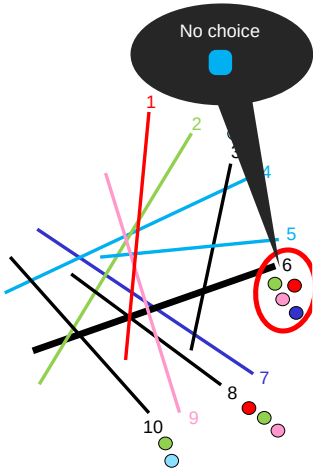
Color 1 | Color 2 | Color 3 | Color 4

DSatHull Coloring



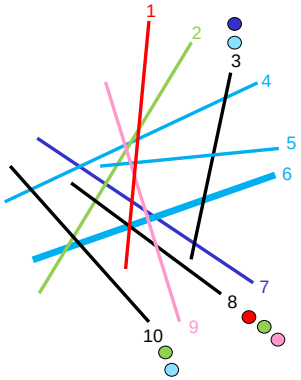
Color 1 | Color 2 | Color 3 | Color 4 | Color 5

DSatHull Coloring



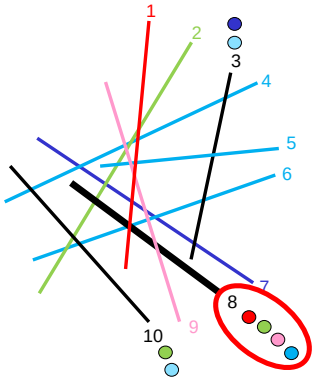
Color 1 | Color 2 | Color 3 | Color 4 | Color 5

DSatHull Coloring



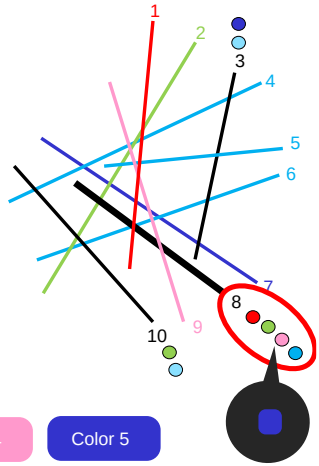
Color 1 | Color 2 | Color 3 | Color 4 | Color 5

DSatHull Coloring

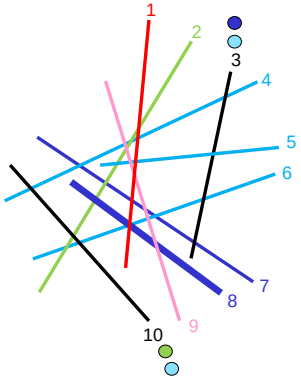


Color 1 | Color 2 | Color 3 | Color 4 | Color 5

DSatHull Coloring

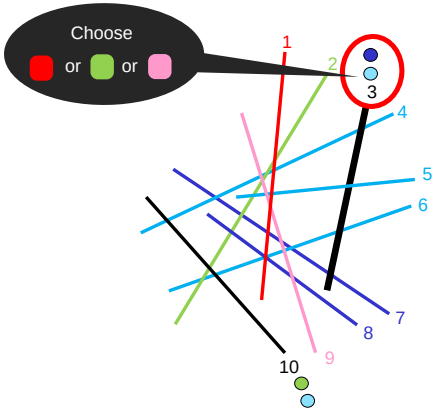


DSatHull Coloring



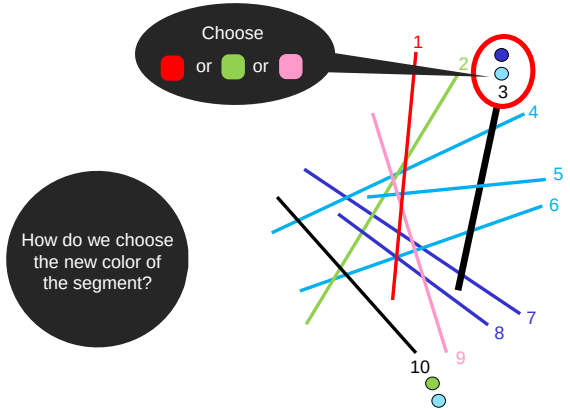
Color 1 | Color 2 | Color 3 | Color 4 | Color 5

DSatHull Coloring

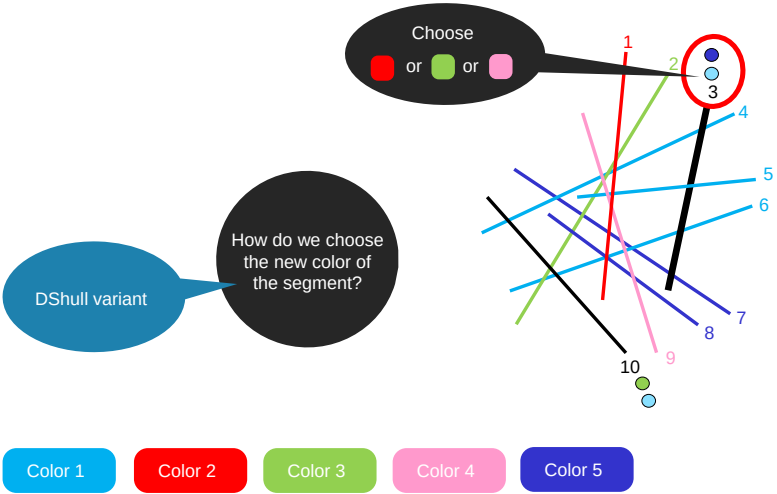


Color 1 | Color 2 | Color 3 | Color 4 | Color 5

DSatHull Coloring



DSatHull Coloring



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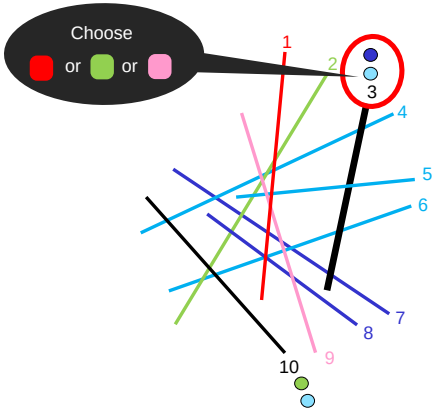

Color 1 | Color 2 | Color 3 | Color 4 | Color 5

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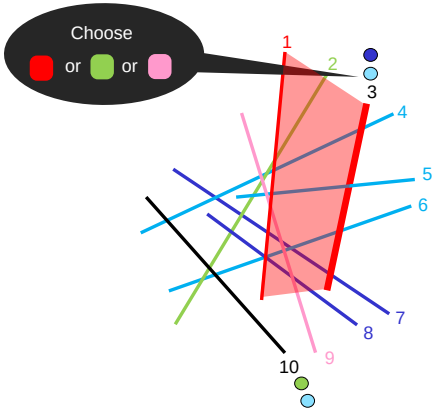

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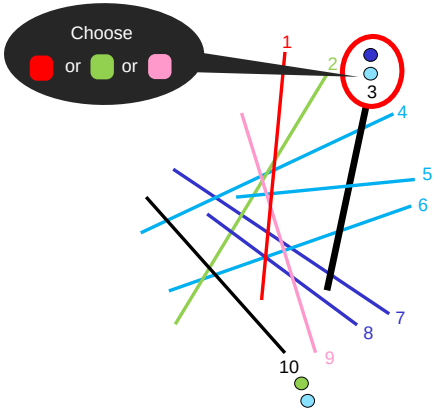

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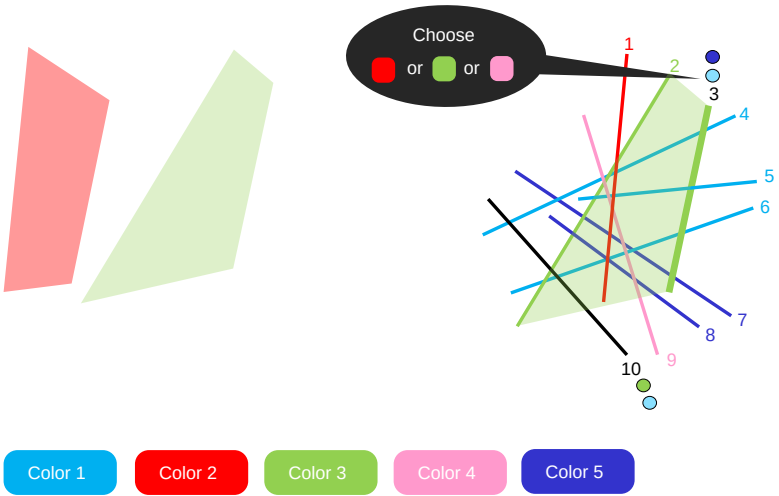

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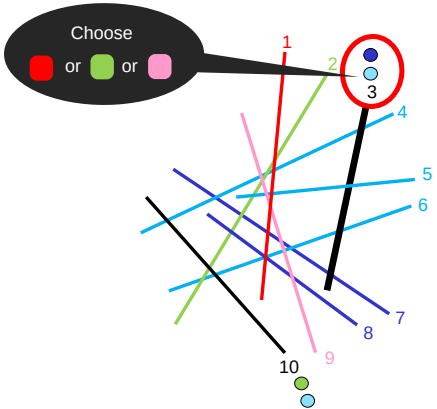

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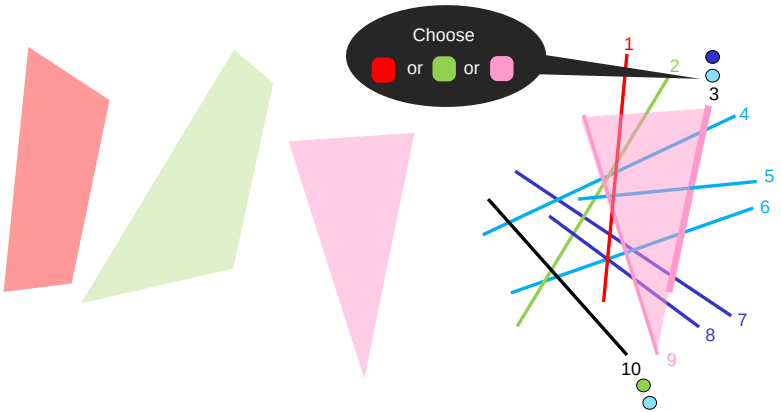

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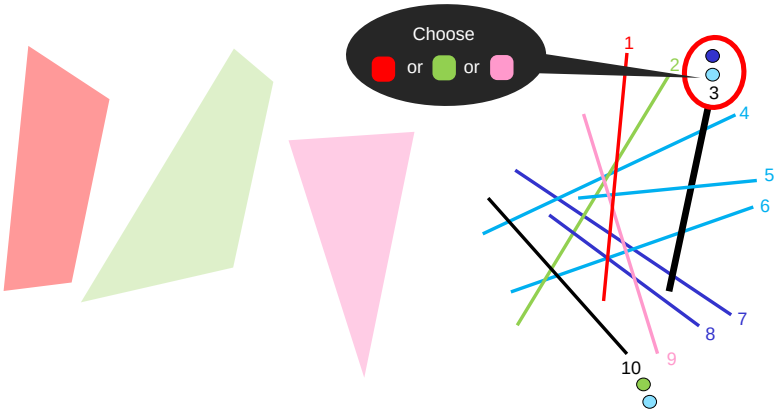

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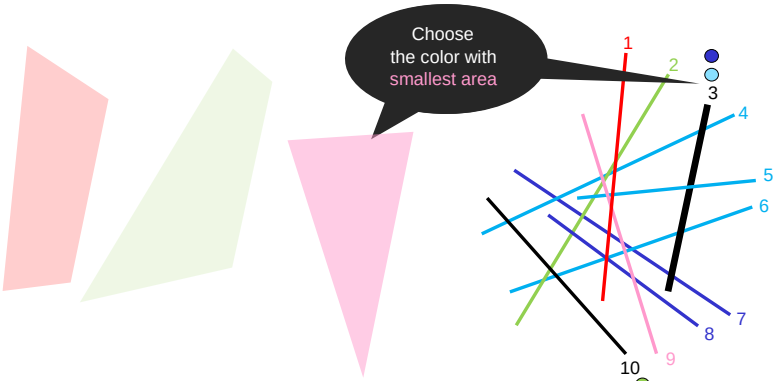

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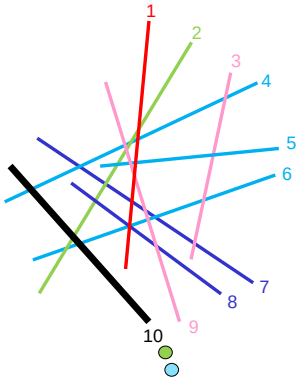

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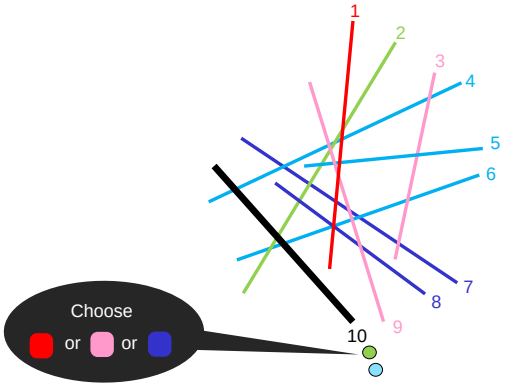

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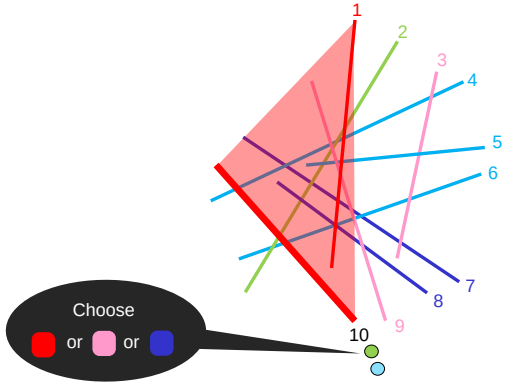

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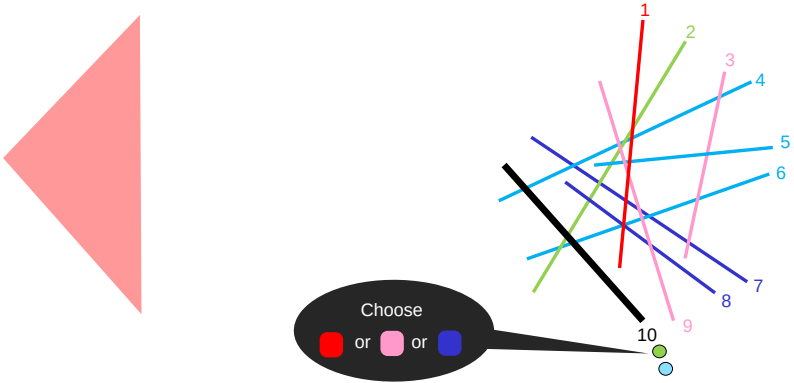

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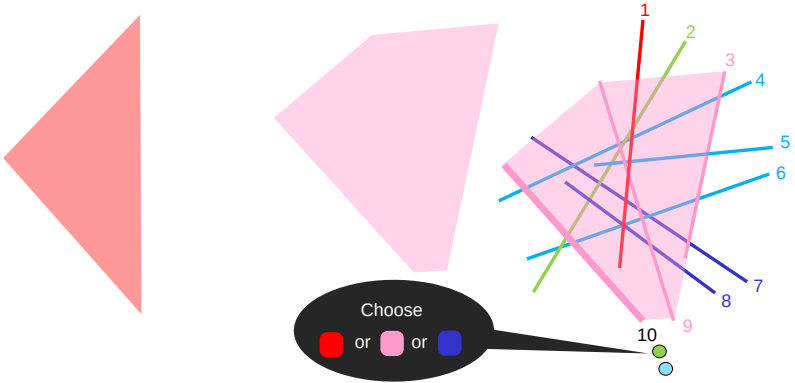

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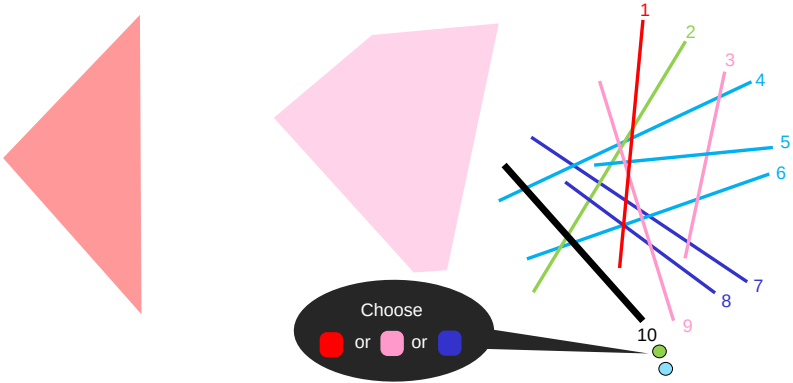

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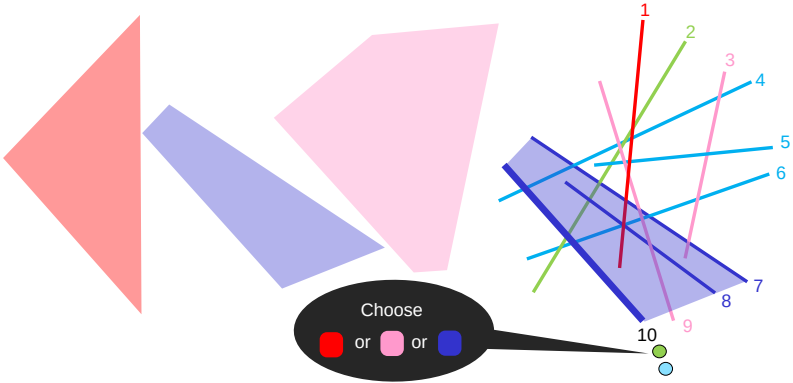

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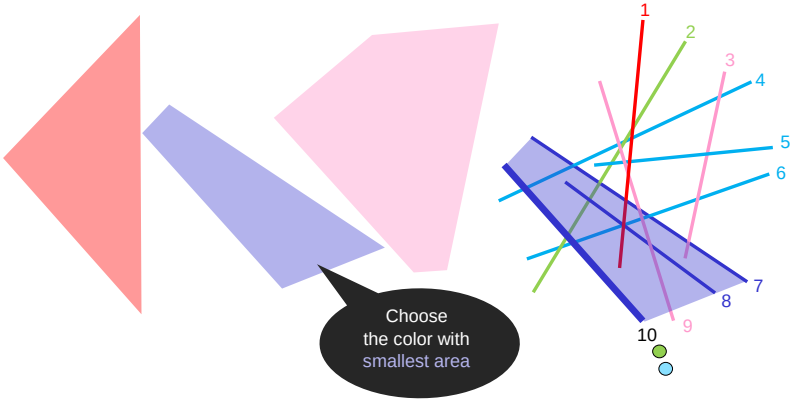

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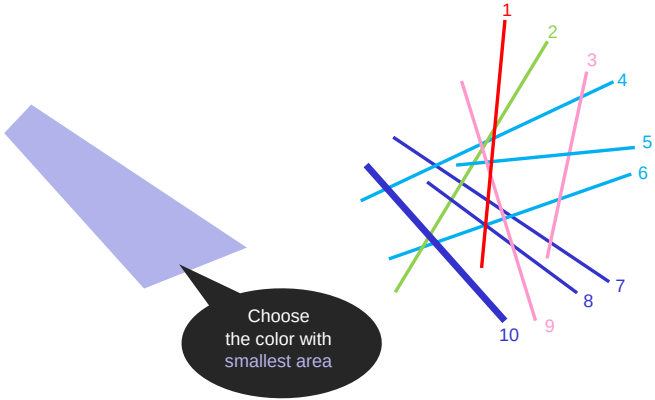

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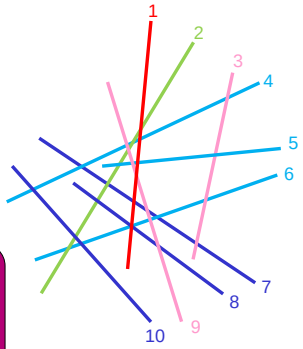
Greedy algorithms

Fast and simple (easy and fast to code)

Can be randomized to provide many solutions



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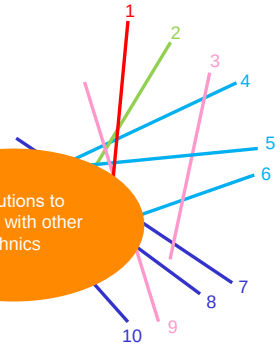
… solutions to optimize with other technics



Integer Programming (IP)

Integer Programming (IP)

Integer programming (IP):

- Variables: Take integer values
- Constraints: A set of linear inequalities
- Objective: A linear function to maximize or minimize

Solvers:

- Commercial solvers are quite efficient (CPLEX, Gurobi...)
- Some open source alternatives (GLPK, COIN-OR...)
- Optimal solutions or small gap are guaranteed
- Cannot solve huge problems

Packing as Independent Set

- Goal: Reduce a geometric problem to a combinatorial problem
- Create some random translations of each item inside the container
- Each translated item is a vertex
- A clique for translations of the same item
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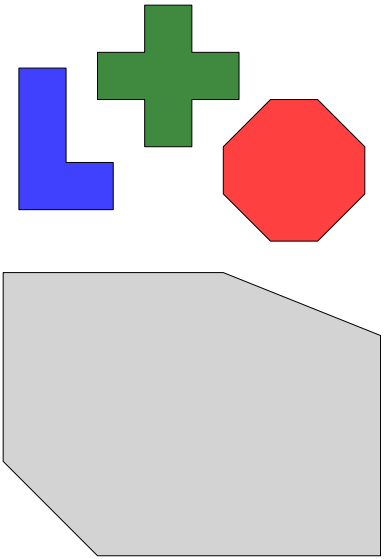
- Solve maximum independent set
- Use the corresponding packing



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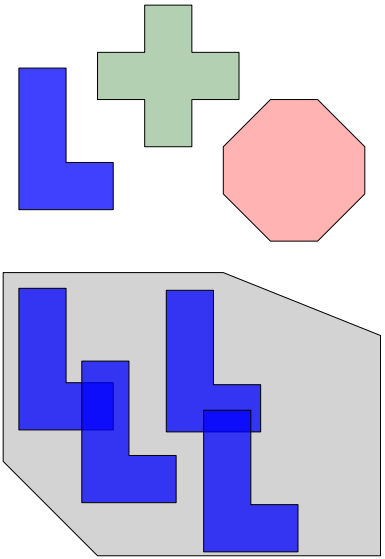

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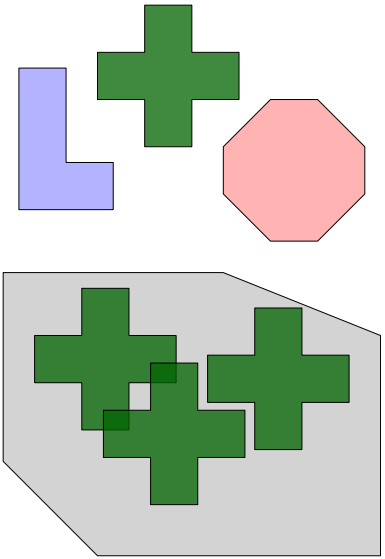

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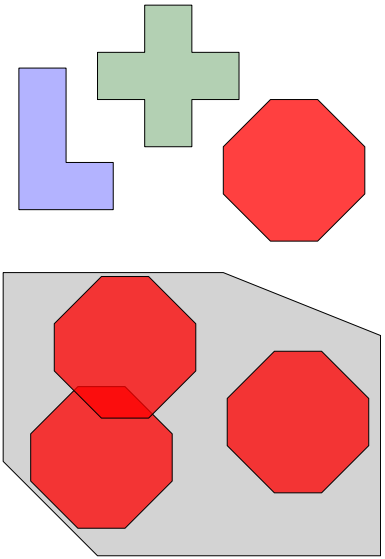

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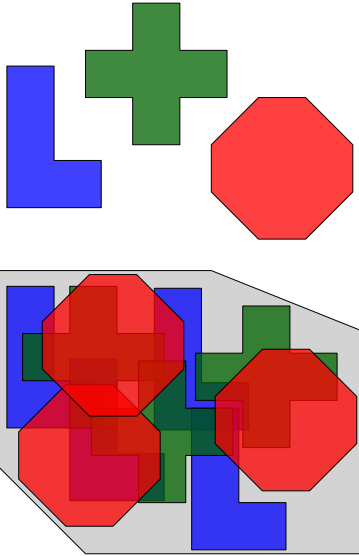

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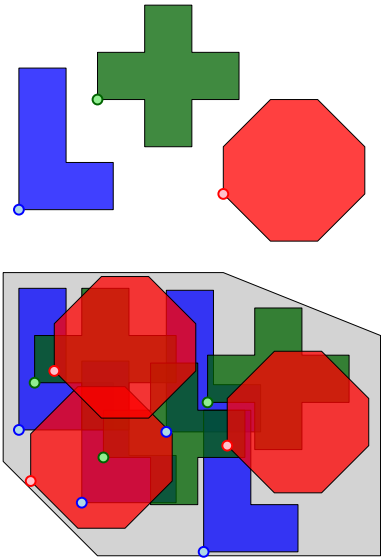

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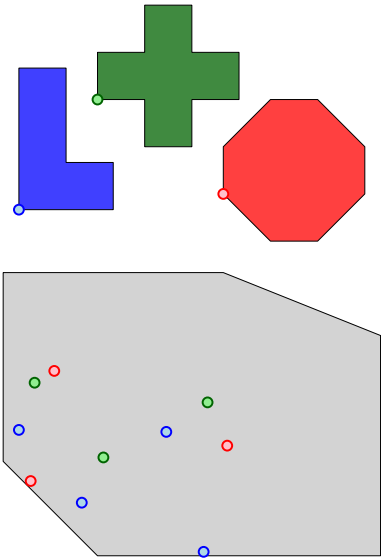

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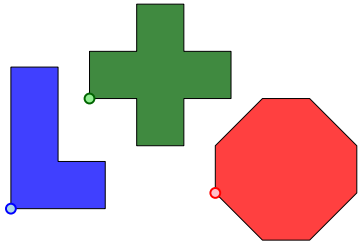

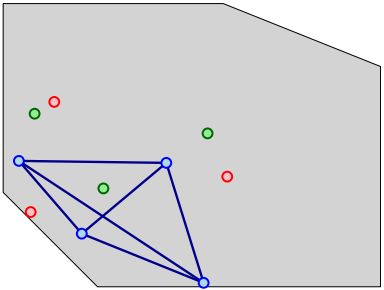

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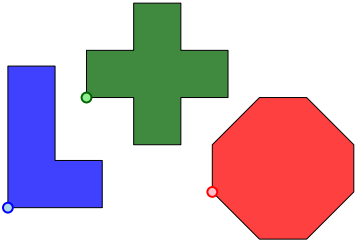

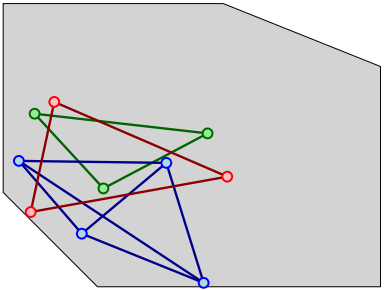

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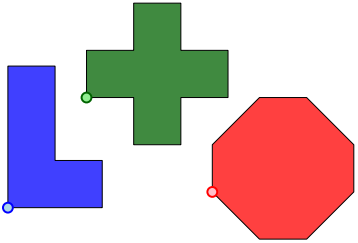

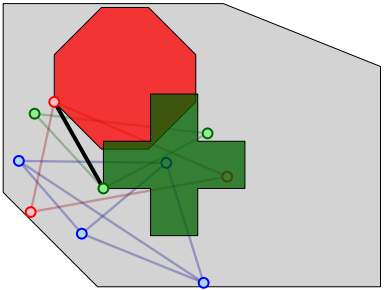

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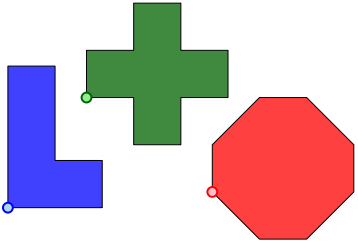

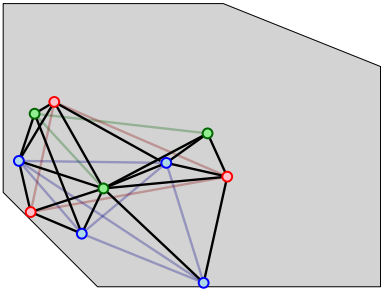

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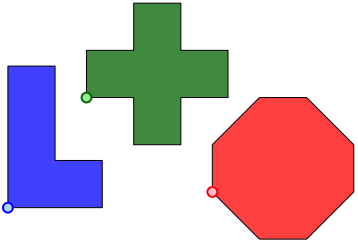

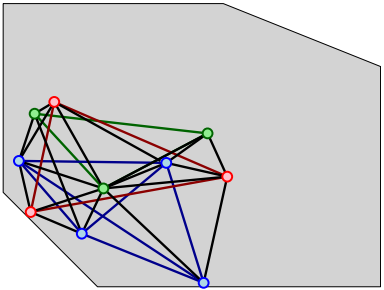

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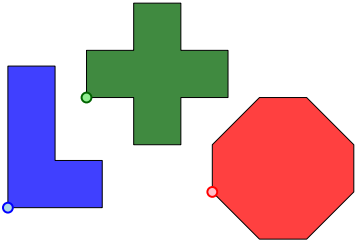

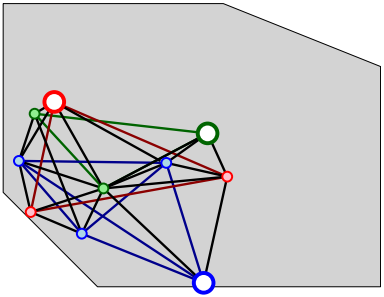

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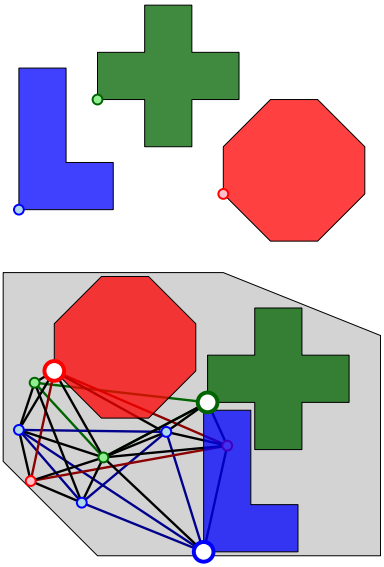


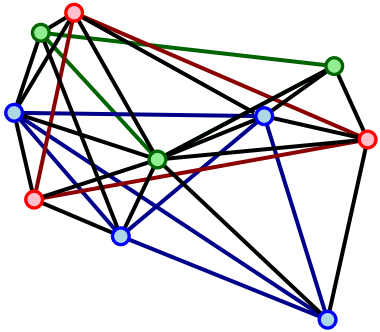
Maximum Independent Set as Linear Programming

- Variables: vertices
  0: not in the solution
  1: in the solution
- Objective: Maximize sum of variables
- Constraints: edge uv becomes $u + v \leq 1$
- Better constraint for clique C

$$\sum_{v \in C} v \leq 1$$

- And for k copies of the same item

$$\sum_{v \in C} v \leq k$$



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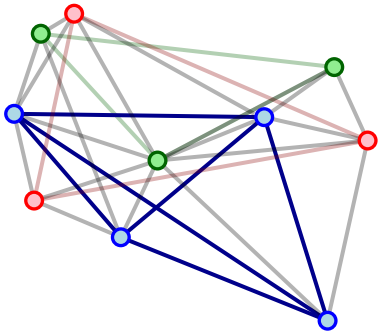
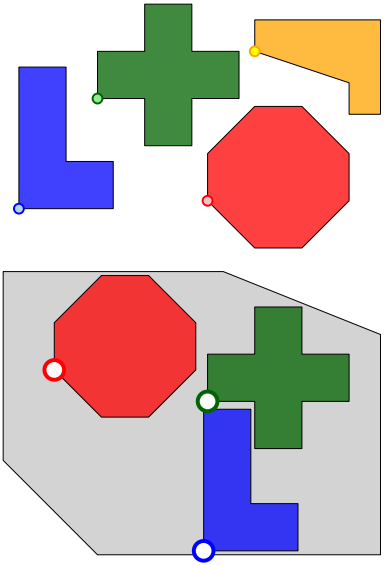


IP Limitation

- CPLEX can only handle ~ 1000 vertices
- Not enough copies of each item
- Solution: We repeat with some translations near the previous solution
- Also random translations of every item
- If we are lucky, we pack more items



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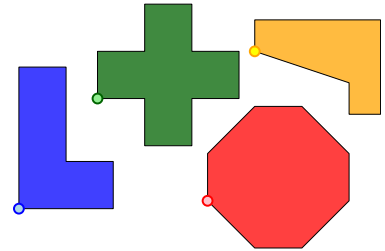

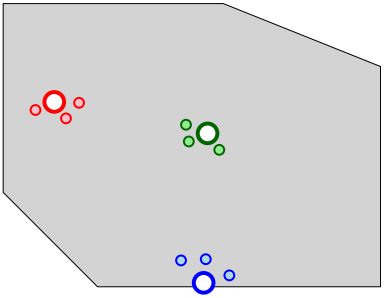

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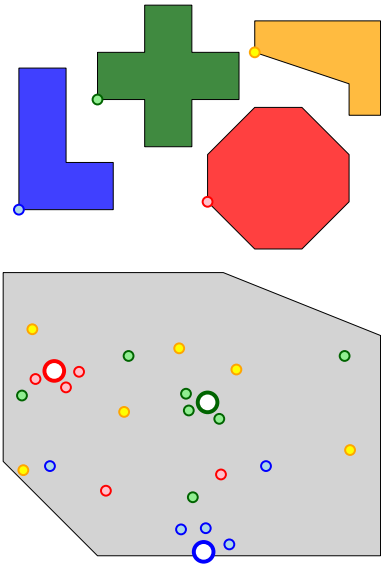

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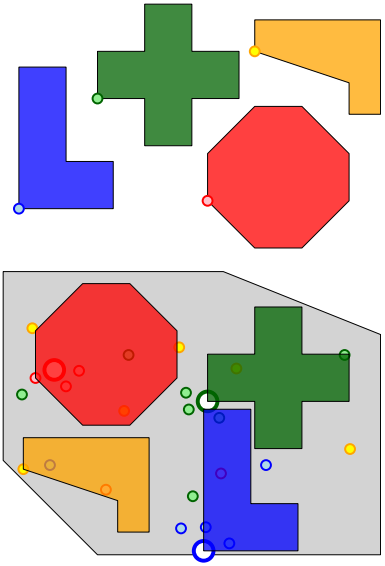


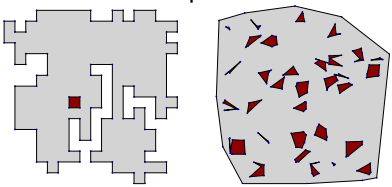
Convex Covering in Two Steps

- We want to cover a polygon (with holes) using convex polygons
- Goal: Reduce a geometric problem to a combinatorial problem

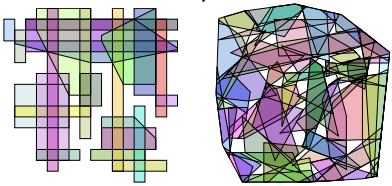
1. Create a collection of many large convex polygons
2. Find a small subset of the collection

- We show how to solve step 2

Input



Output



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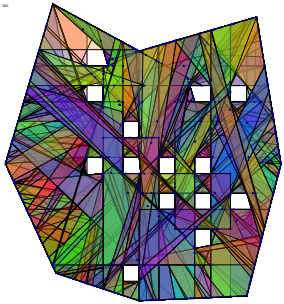

382 convex polygons

Convex Covering in Two Steps

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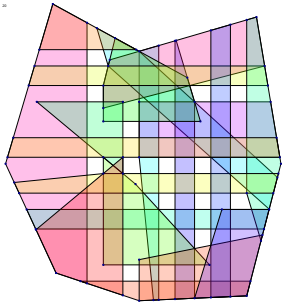
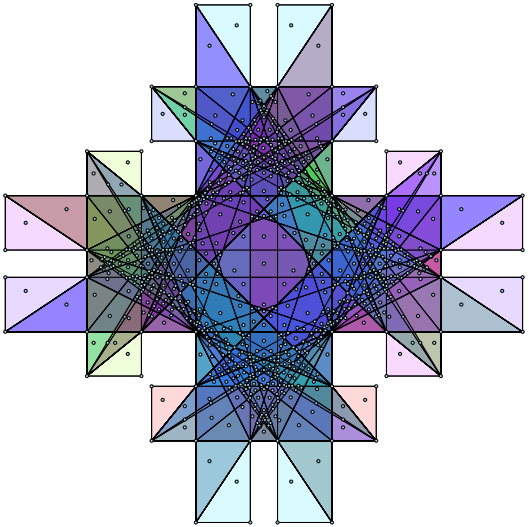


20 convex polygons

Convex Covering as Set Cover

$\mathcal{C}$: Convex polygons from phase 1
P: Instance polygon with holes

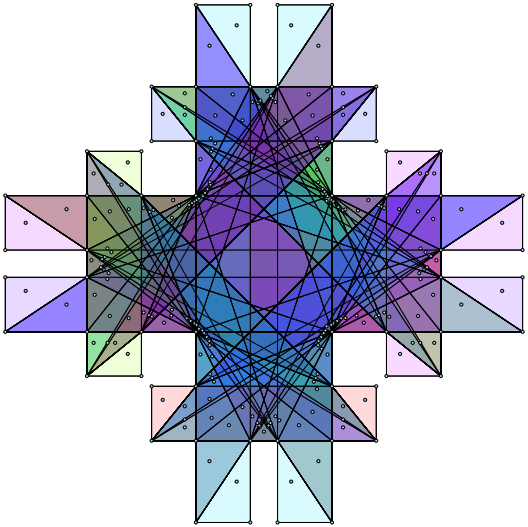
- $(\mathcal{C}, P)$ define a set system
- P has infinitely many points
- First attempt: reduce P to a quadratic number of witnesses, one point per arrangement cell
- Too many witnesses!
- Building the arrangement is slow!



1009 witnesses for 82 convex polygons

Vertex Witnesses

- Solution: only place witnesses near vertices of P
- Does not guarantee that P is covered
- Two possible fixes:
  - Add a witnesses inside each uncovered area and repeat (generally better, but slower)
  - Cover the uncovered area using some quick heuristic (faster and sometimes better)



200 witnesses for 82 convex polygons

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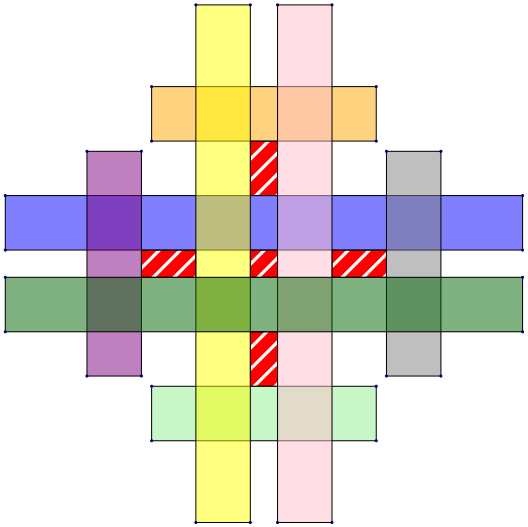

5 uncovered regions 8 convex polygons

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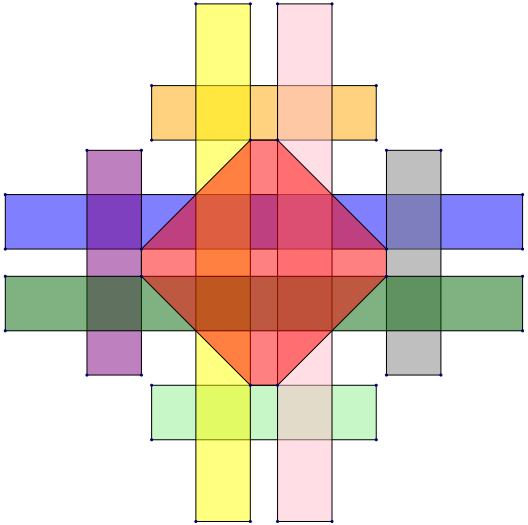

0 uncovered regions 9 convex polygons

Solving Combinatorial Set Cover with IP

Use integer programming (IP):

- Each set becomes a binary variable
  0: Not in the solution
  1: In the solution
- Each witness becomes a constraint:
  the sum of the sets that contain it must be at least 1
- Objective: minimize the sum of all variables

- Very fast for small to medium instances
- Solutions often guaranteed optimal
- On some large instances: slow and very bad solutions

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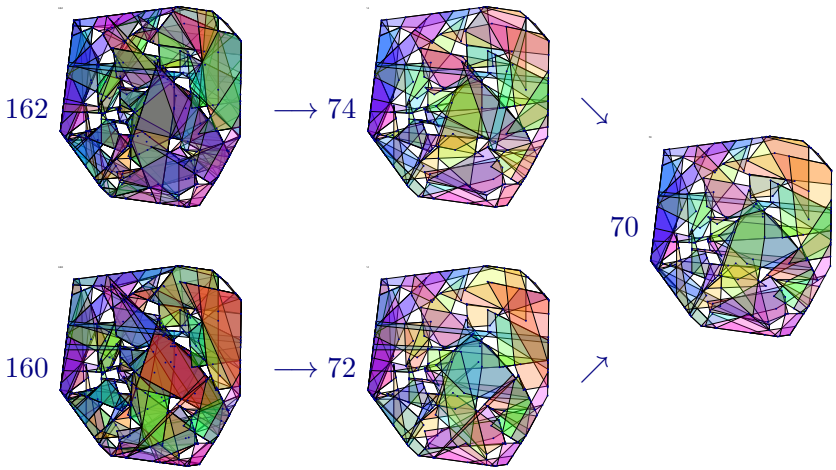
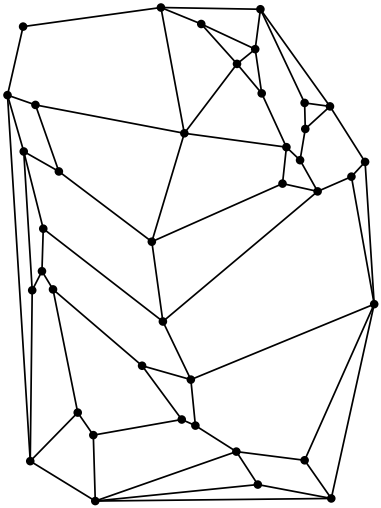
Improving Convex Cover with IP

We use multiple good solutions to get a better one



Improving Convex Partition with IP

- Convex partition can easily be modelled as IP
- CPLEX can only solve small instances
- We improve a solution using CPLEX:
  1. Select some adjacent cells
  2. Remove selected cells
  3. Solve the polygon in the selected area
- The initial solution is not very relevant, we may start with a triangulation



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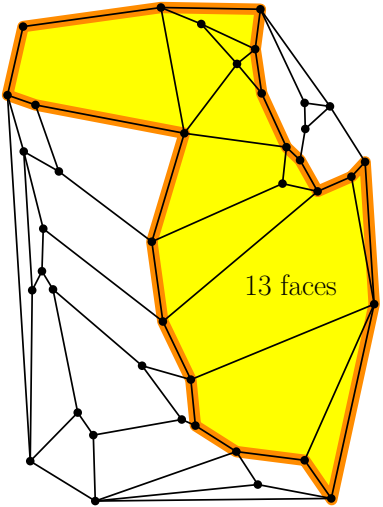

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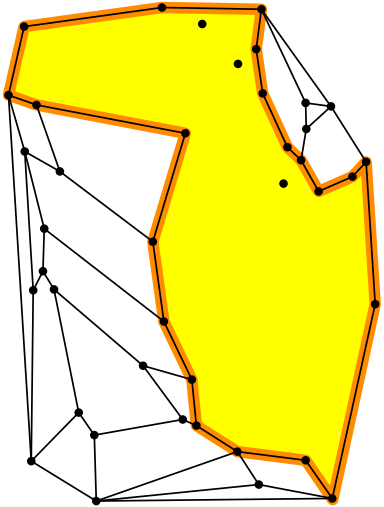

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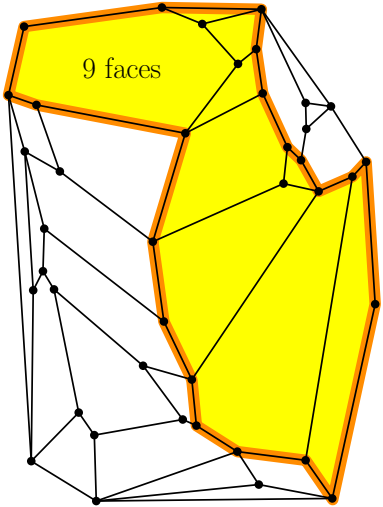

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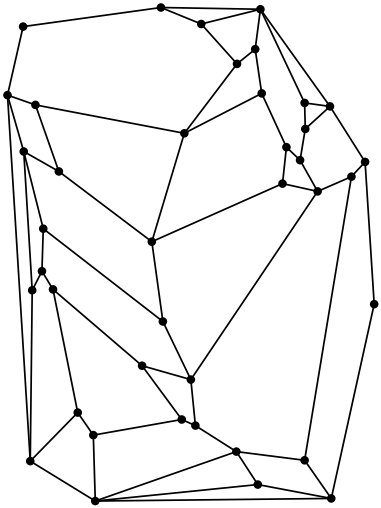
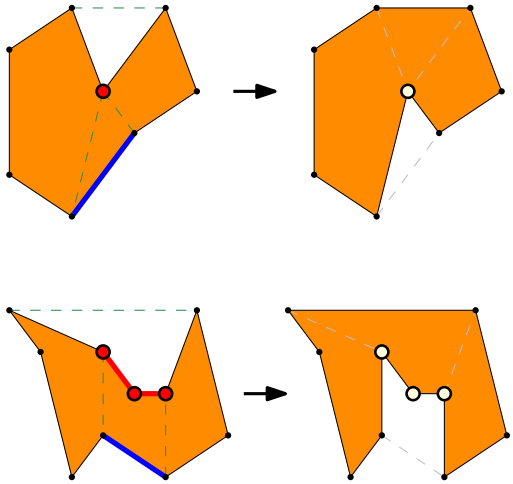
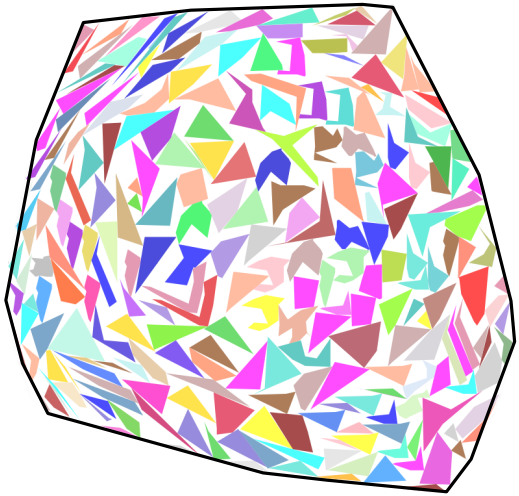


Improving Maximum Area Polygon with Local Search

- We try to improve many solutions
- Sometimes a worse solution may improve better
- We move one or more vertices in the polygon order in order to increase the area
- For this problem, changes are small

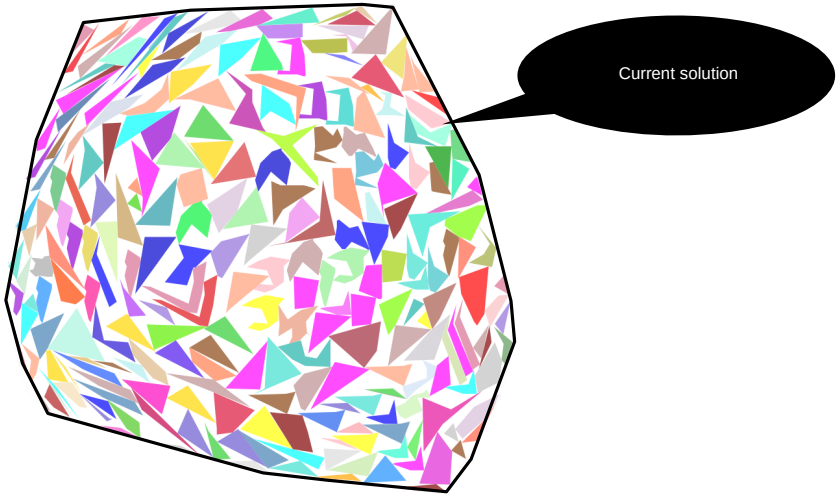


Packing Local Improvement



Local Search for Packing

Packing Local Improvement



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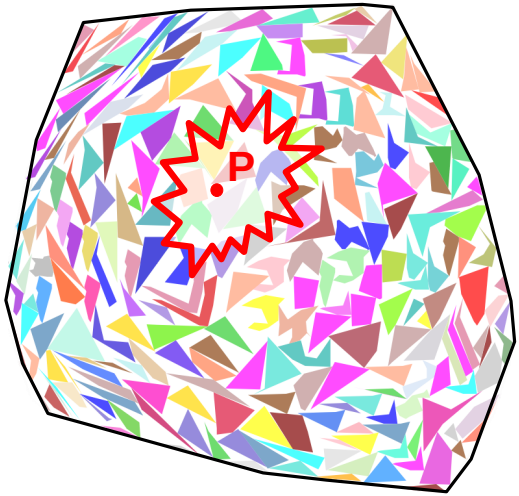
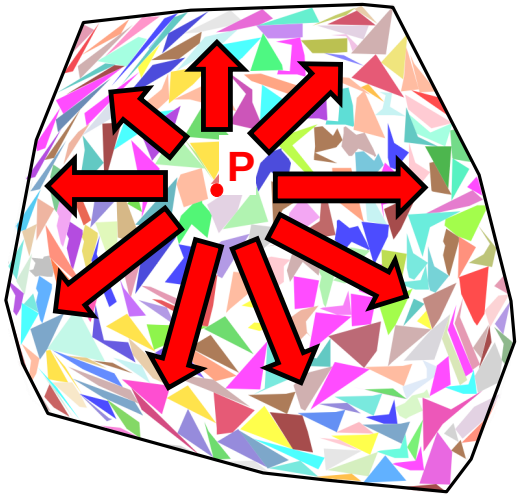


1) Choose **a random point P** in the solution

Packing Local Improvement



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2) **Push** every packed item away from **P** (from the farthest to the closest)

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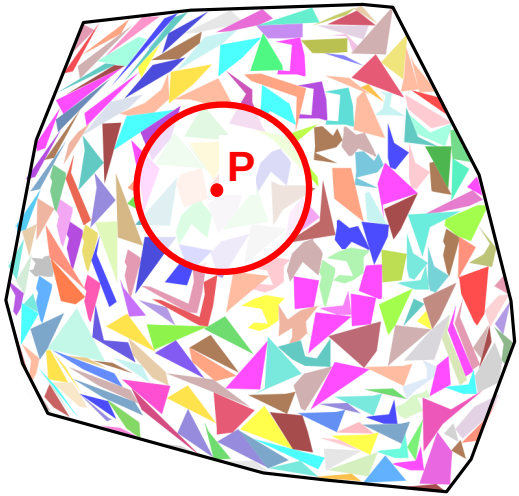
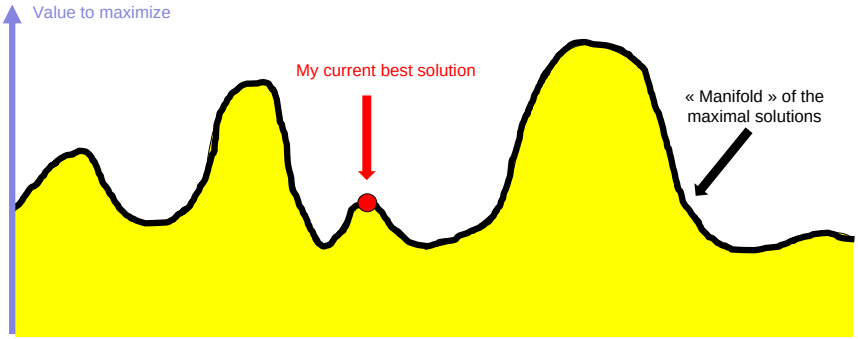


1) Choose **a random point P** in the solution

2) **Push** every packed item away from **P** (from the farthest to the closest)

3) **Try to pack** a new item around **P** (if it crosses some packed items but there could be a benefit, we pack the new item)

More than Local Search ?



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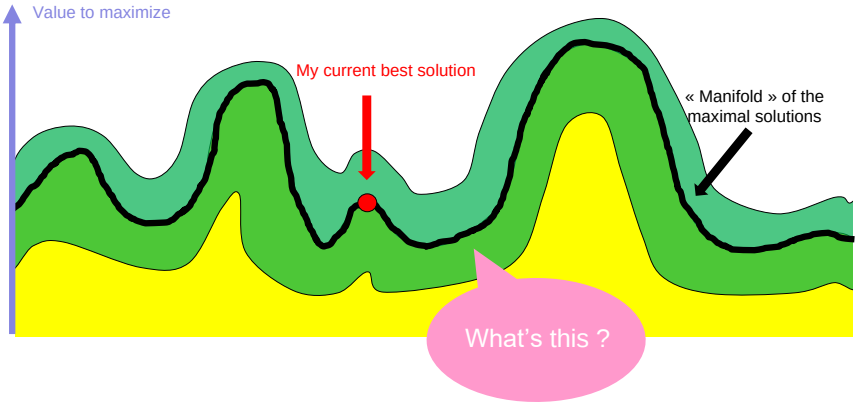

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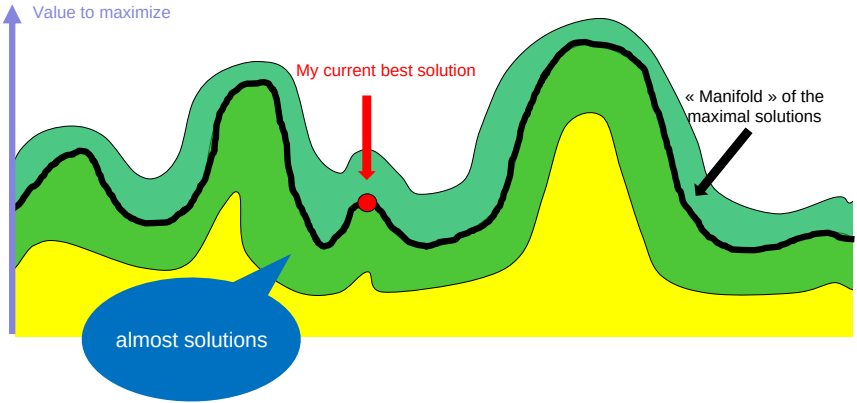

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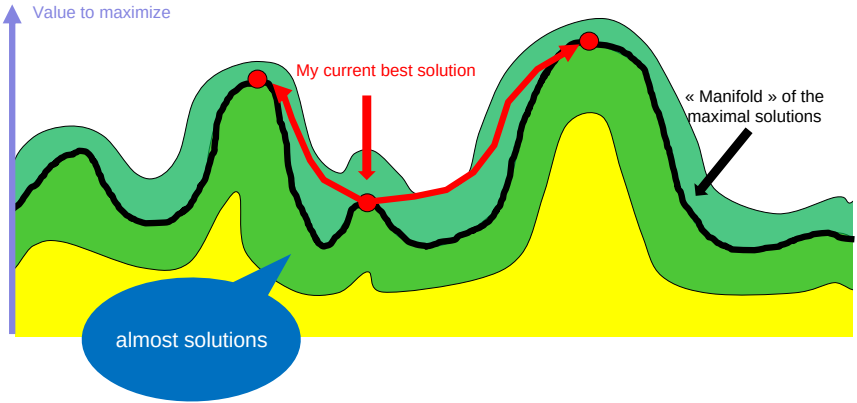
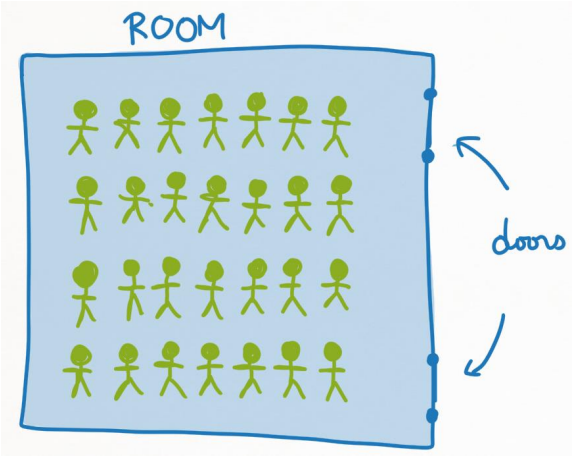
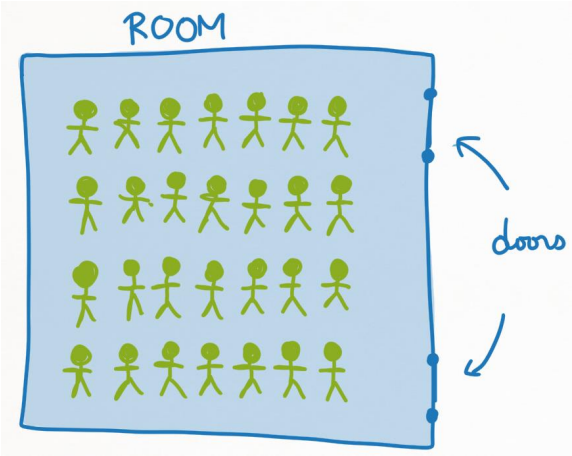


Conflict



28 agents in a room: find the best consensus of a complex combinatorial problem…

Conflict



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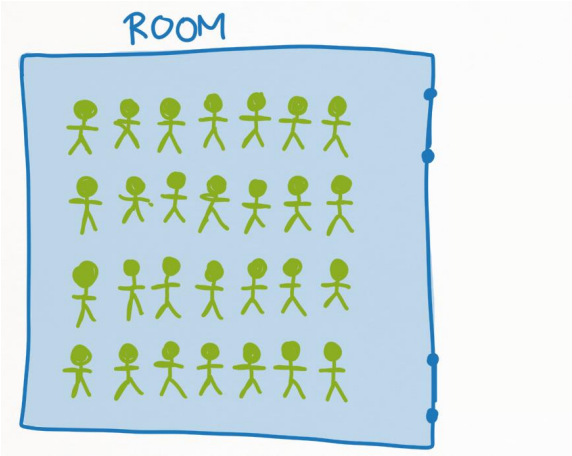

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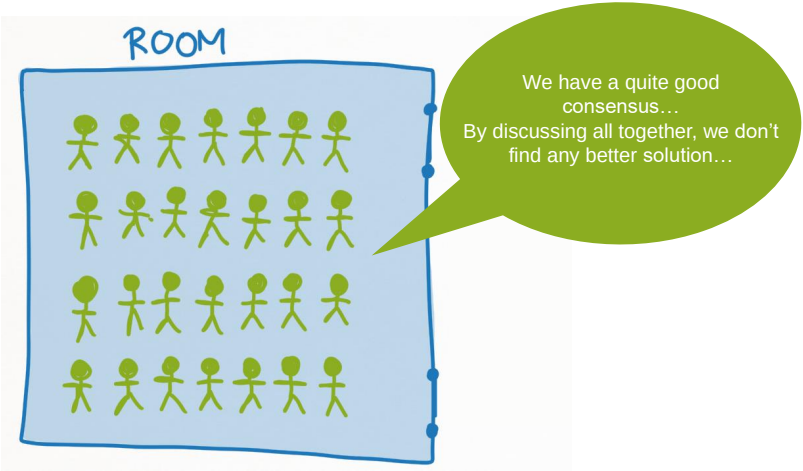

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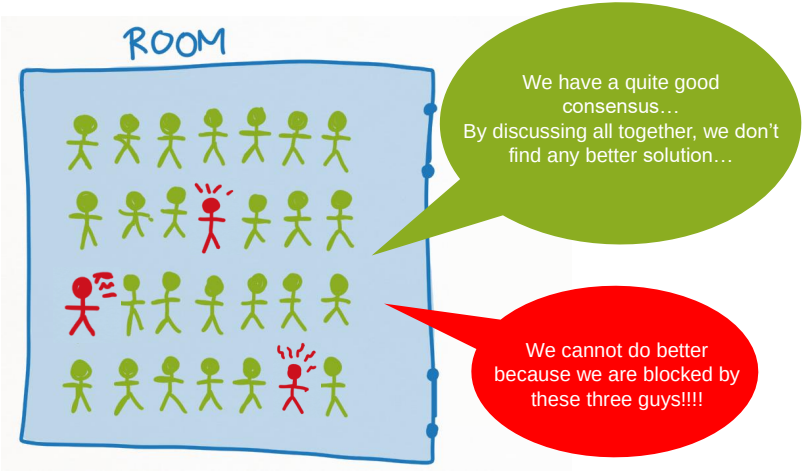
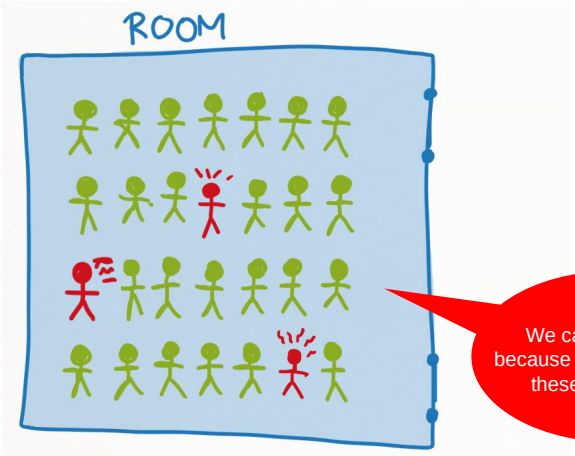

ROOM
We have a quite good consensus…
By discussing all together, we don't find any better solution…
We cannot do better because we are blocked by these three guys!!!!

Conflict



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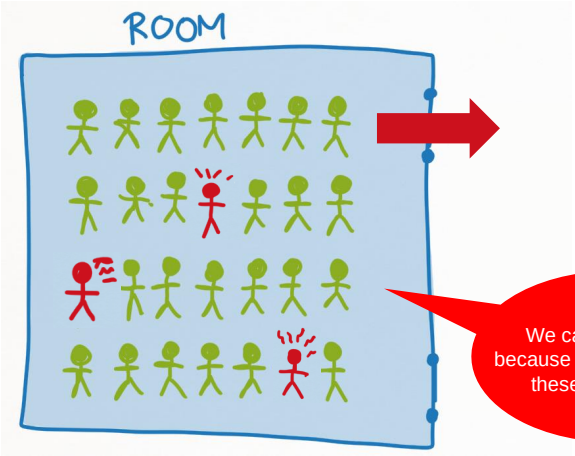

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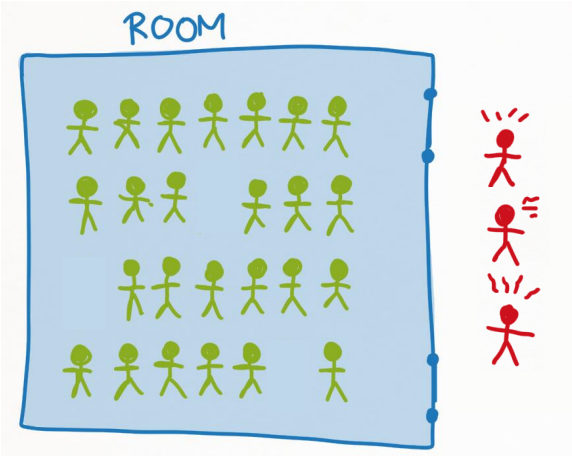

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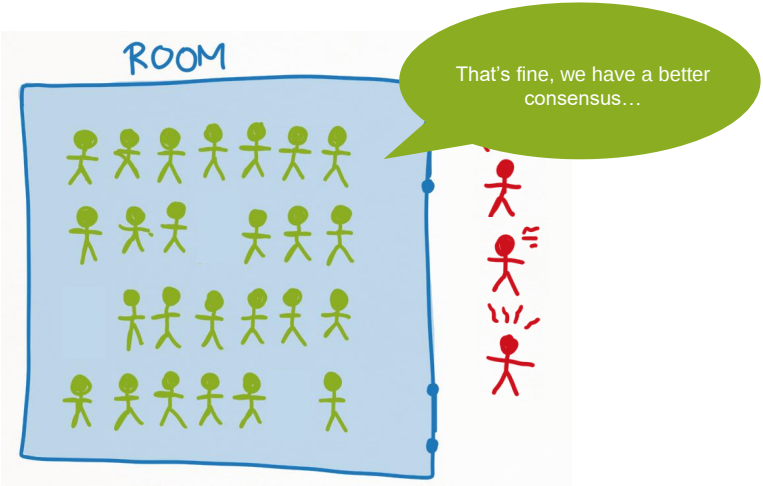

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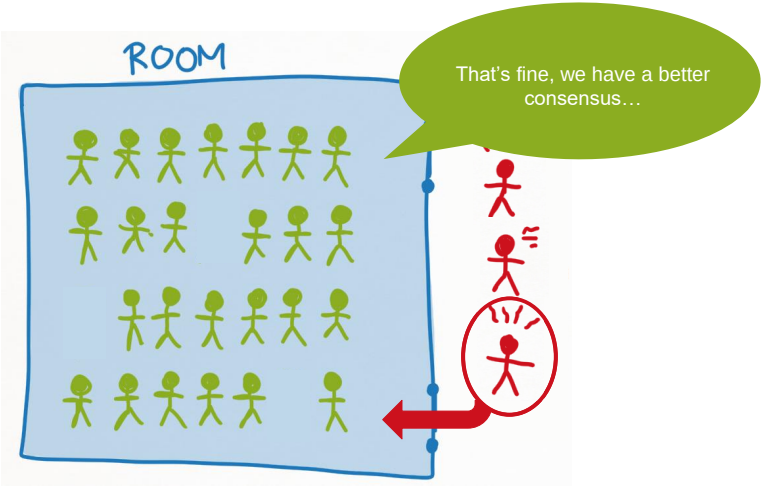

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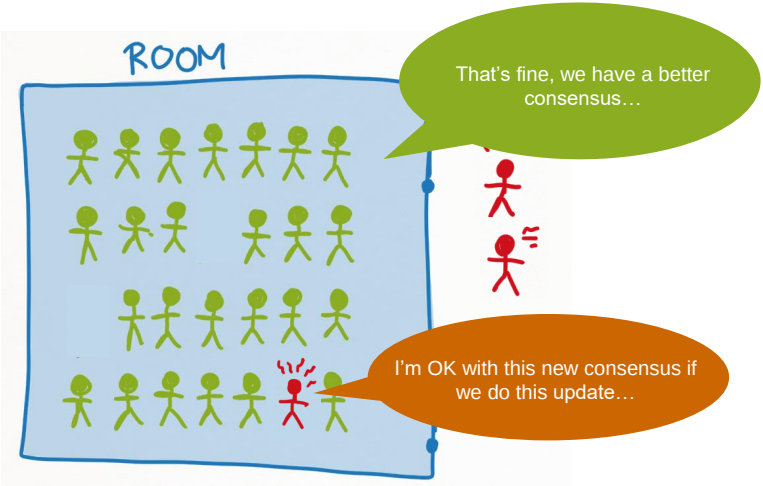
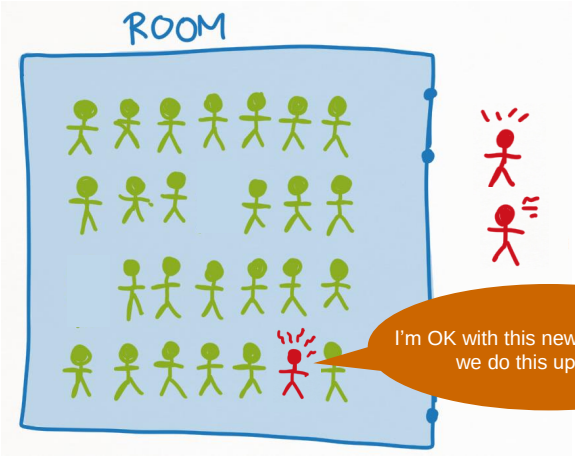

ROOM
That's fine, we have a better consensus…
I'm OK with this new consensus if we do this update…

Conflict



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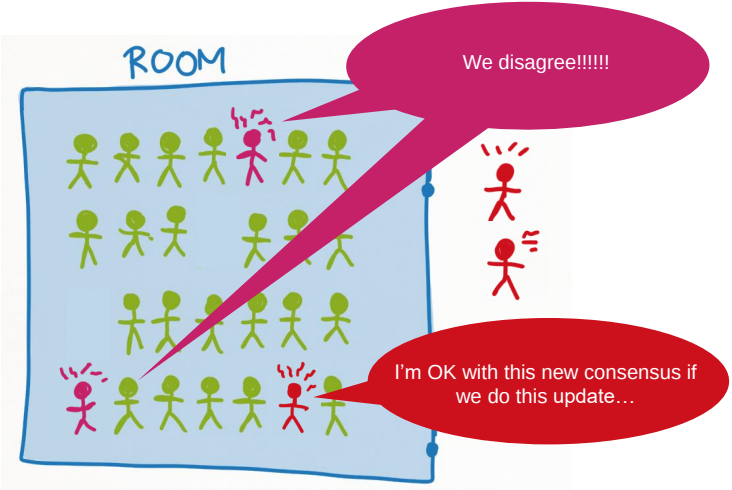

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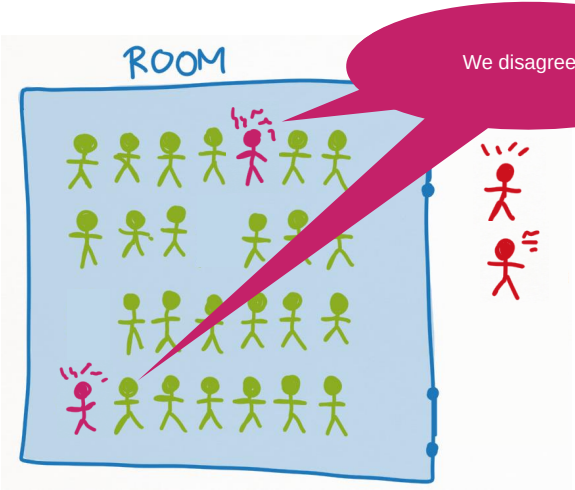

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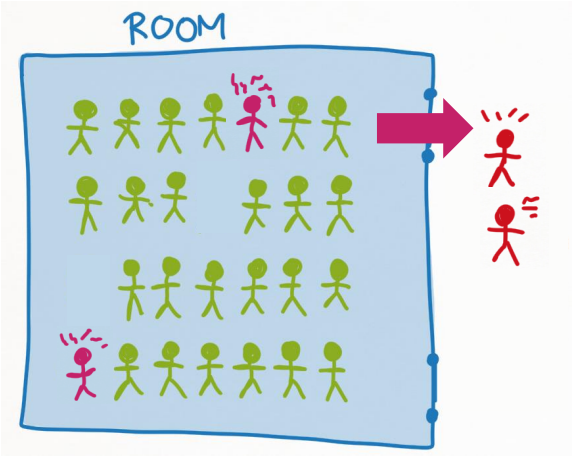

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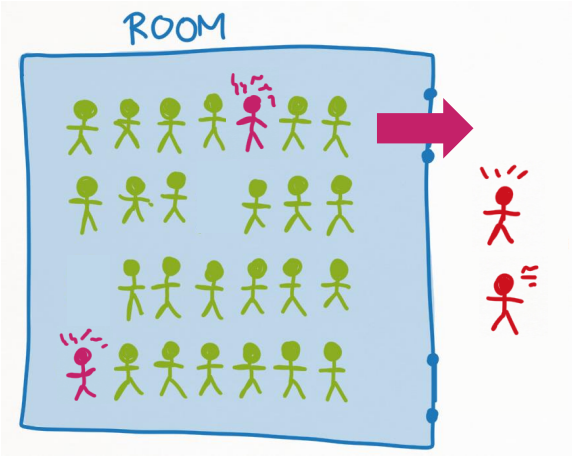

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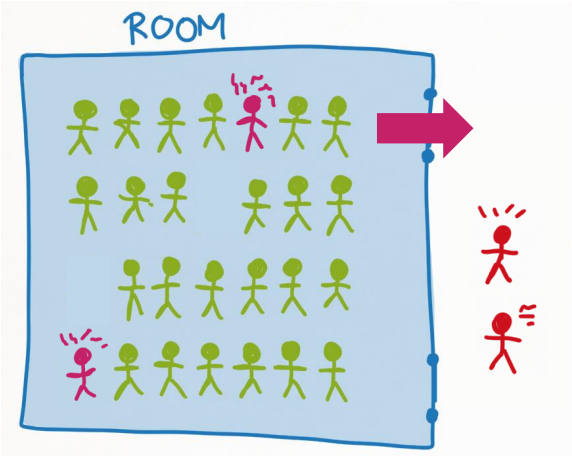

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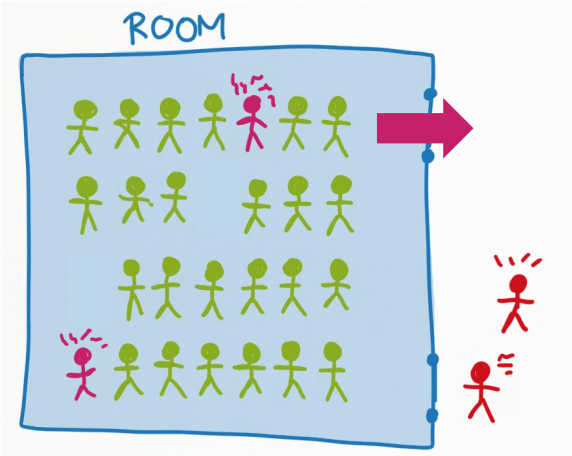

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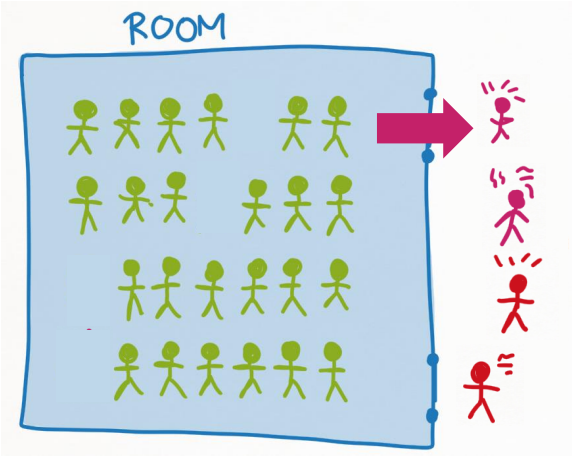

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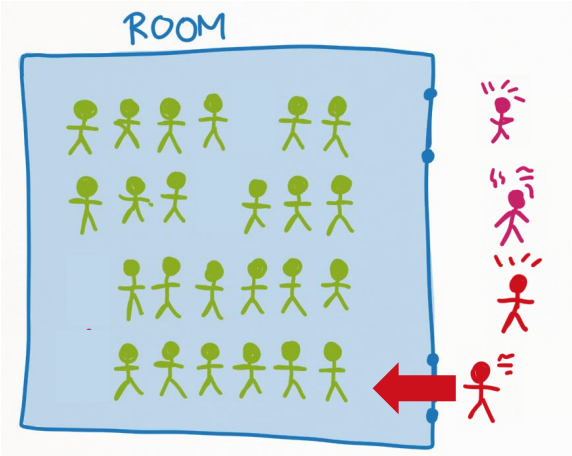

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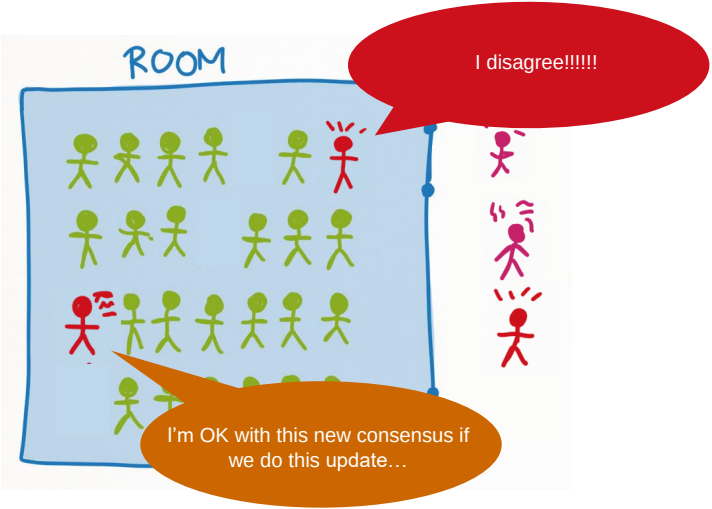

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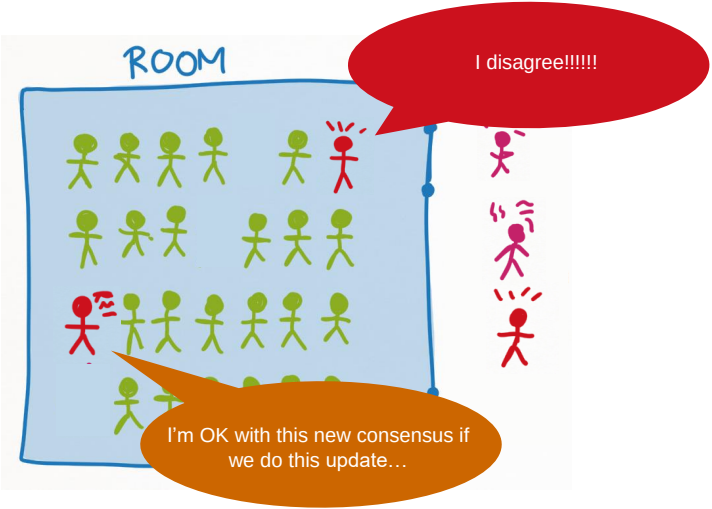

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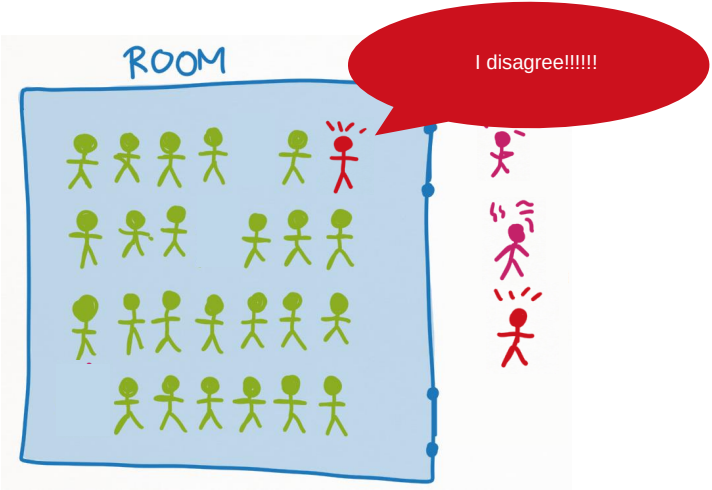

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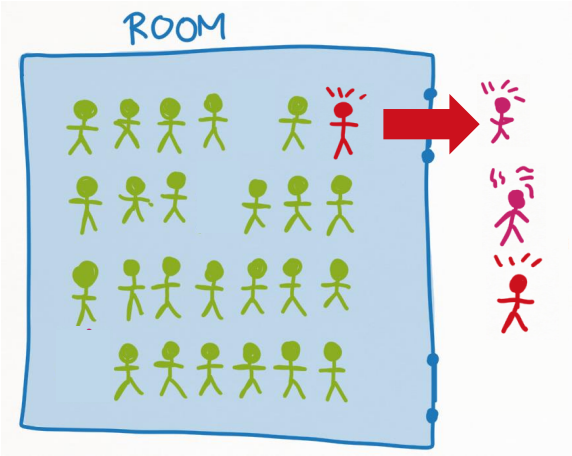
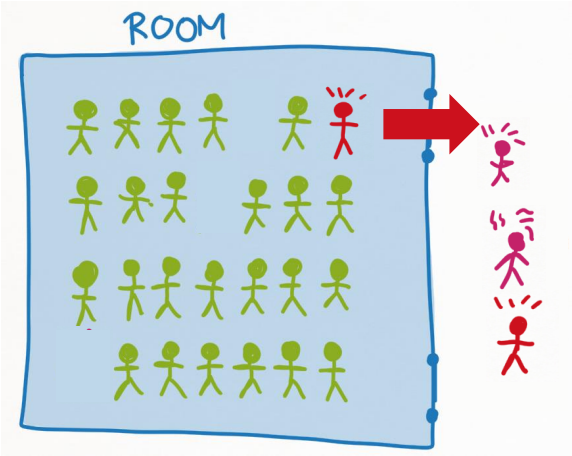
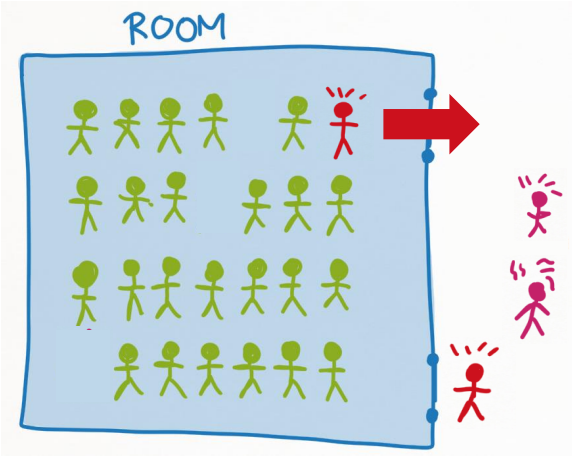


Conflict

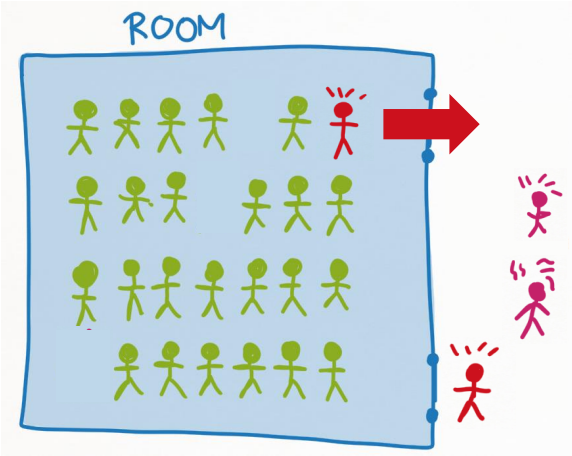


28 agents in a room: find the best consensus to a complex combinatorial problem…

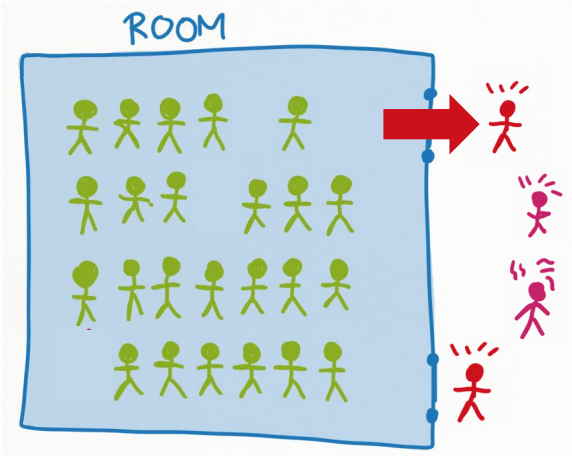
Conflict



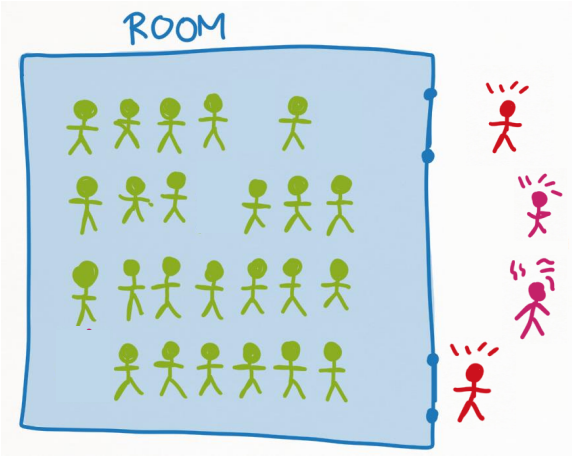
Conflict



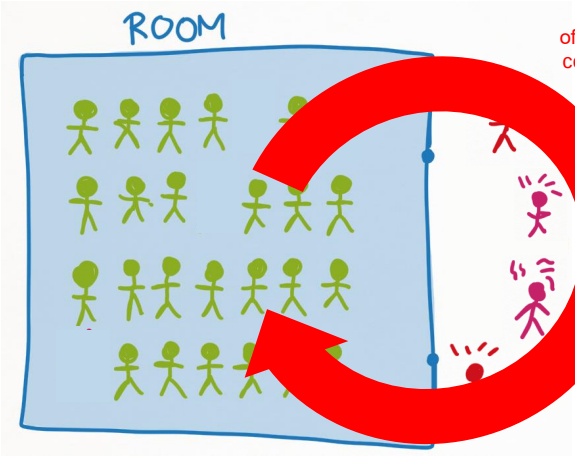
Conflict



Conflict

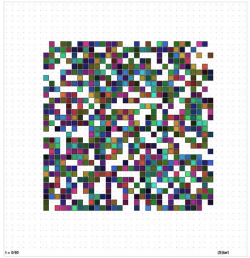


Conflict



Repeat thousands of times… until the new consensus is accepted by every agent….

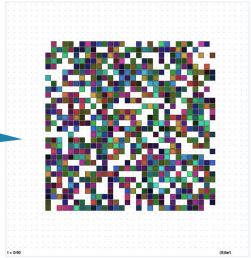
Conflict



What can we do to optimize a solution ?

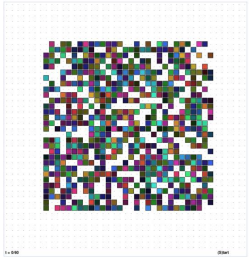
Conflict

Solution of makespan 61



Conflict

Solution of makespan 61

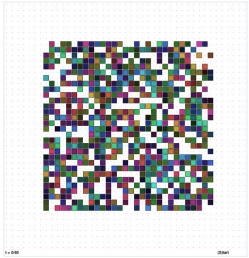


Remove the robots arriving at their target at t=61

All the other robots arrive at t≤60

Conflict

Solution of makespan 61

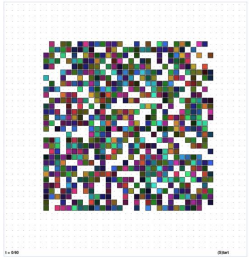


Remove the robots arriving at their target at t=61

- Put the removed robots in a queue Q

Conflict

Solution of makespan 61



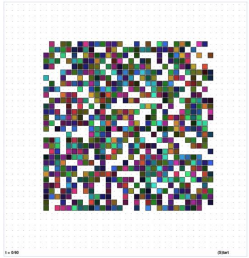
Remove the robots arriving at their target at t=61

- Put the removed robots in a queue Q

- Pick the first robot r_0 of the queue
  search for a path P arriving at time t=60
  with a minimum number of conflicts with other robots

Conflict

Solution of makespan 61



Remove the robots arriving at their target at t=61

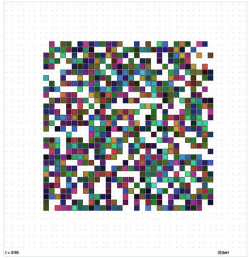
- Put the removed robots in a queue Q

- Pick the first robot r_0 of the queue
  search for a path P arriving at time t=60
  with a minimum number of conflicts with other robots

Add the path of the first robot r_0 in the current solution…
Remove the robots in conflict with this path…

Conflict

Solution of makespan 61



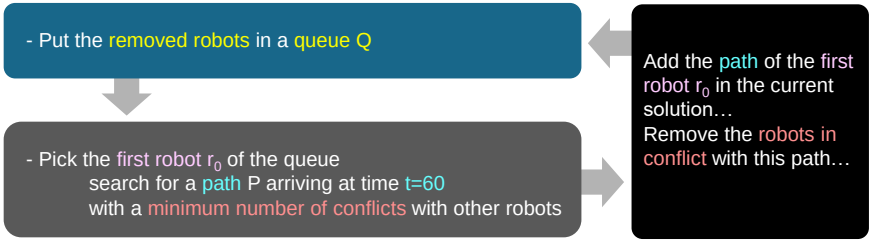
Remove the robots arriving at their target at t=61

- Put the removed robots in a queue Q

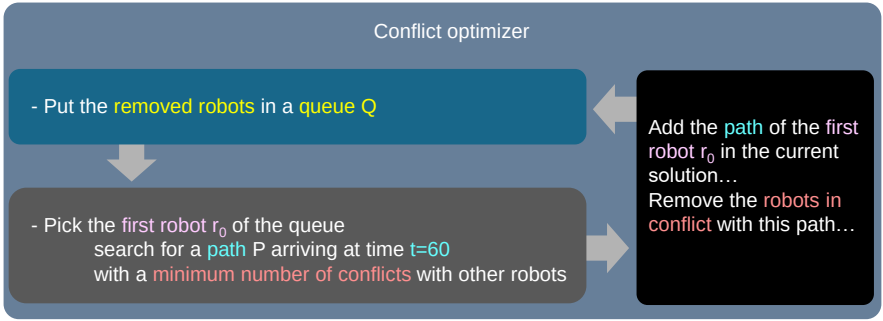
- Pick the first robot r_0 of the queue
  search for a path P arriving at time t=60
  with a minimum number of conflicts with other robots

Add the path of the first robot r_0 in the current solution…
Remove the robots in conflict with this path…

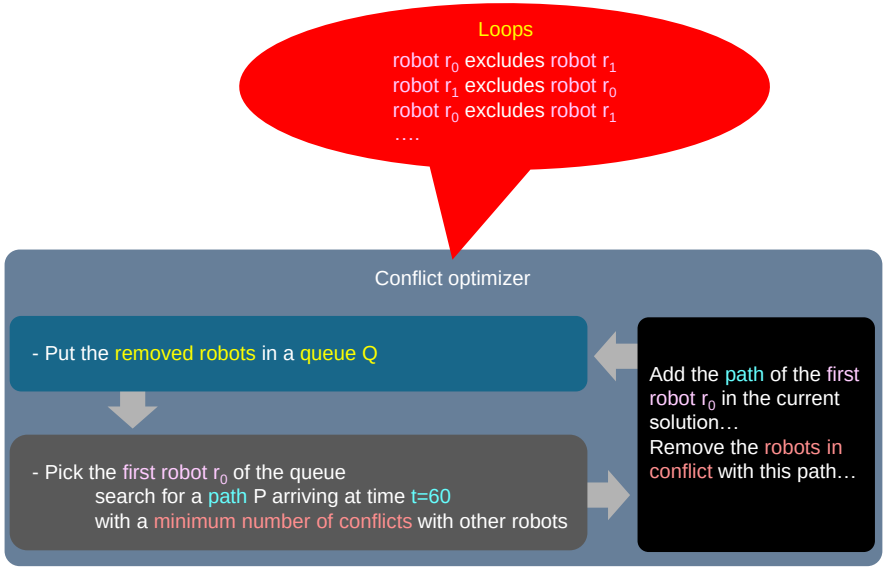
Conflict


- Put the removed robots in a queue Q
- Pick the first robot r_0 of the queue
search for a path P arriving at time t=60
with a minimum number of conflicts with other robots
Add the path of the first robot r_0 in the current solution…
Remove the robots in conflict with this path…

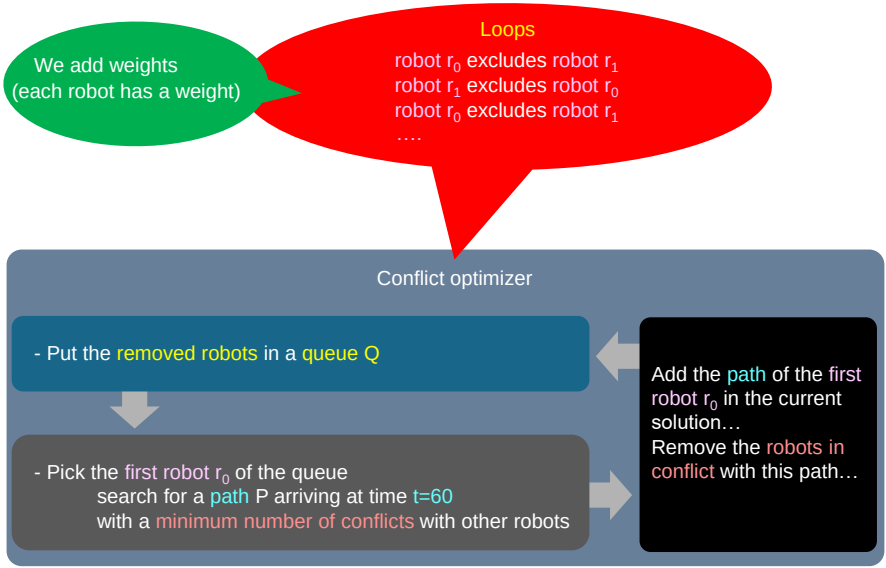
Conflict


Conflict optimizer
- Put the removed robots in a queue Q
- Pick the first robot r_0 of the queue
search for a path P arriving at time t=60
with a minimum number of conflicts with other robots
Add the path of the first robot r_0 in the current solution…
Remove the robots in conflict with this path…

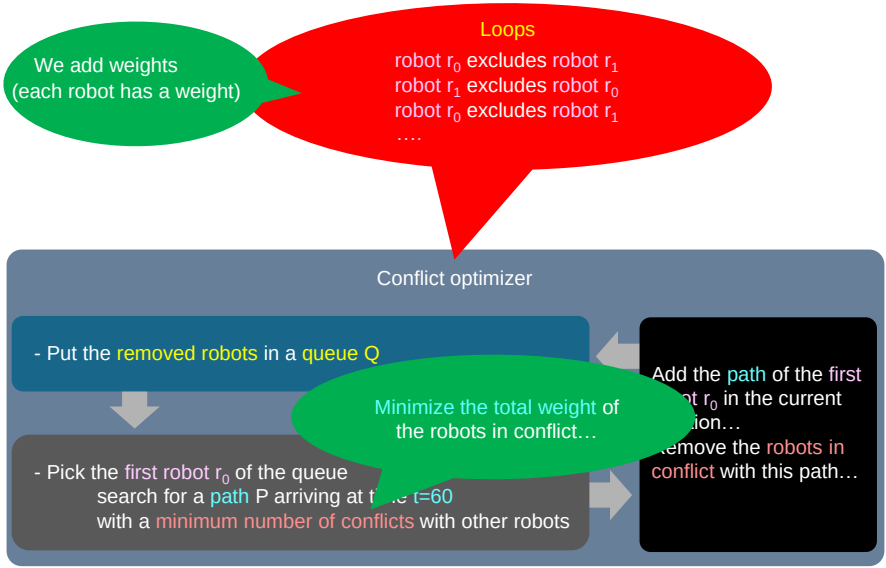
Conflict


Loops
robot r_0 excludes robot r_1
robot r_1 excludes robot r_0
robot r_0 excludes robot r_1
….
Conflict optimizer
- Put the removed robots in a queue Q
- Pick the first robot r_0 of the queue
search for a path P arriving at time t=60
with a minimum number of conflicts with other robots
Add the path of the first robot r_0 in the current solution…
Remove the robots in conflict with this path…

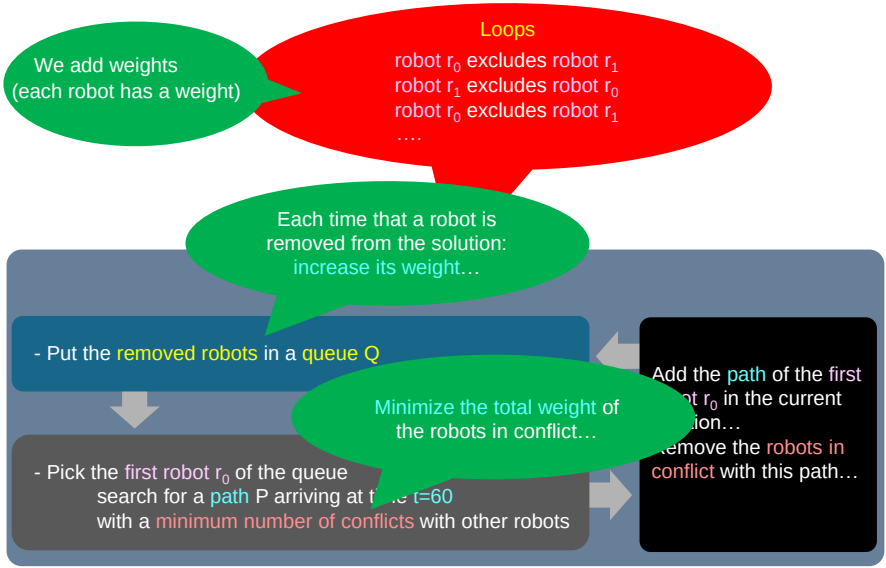
Conflict



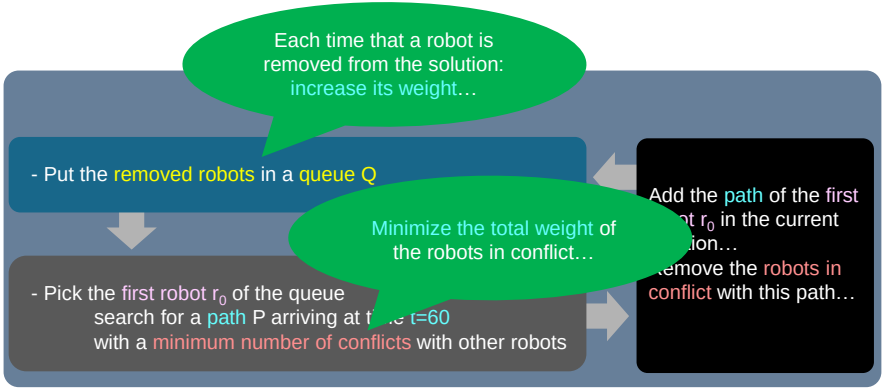
Conflict


We add weights
(each robot has a weight)
Loops
robot r_0 excludes robot r_1
robot r_1 excludes robot r_0
robot r_0 excludes robot r_1
….
Conflict optimizer
- Put the removed robots in a queue Q
Minimize the total weight of
the robots in conflict…
- Pick the first robot r_0 of the queue
search for a path P arriving at t=60
with a minimum number of conflicts with other robots
Add the path of the first
r_0 in the current
…ion…
…emove the robots in
conflict with this path…

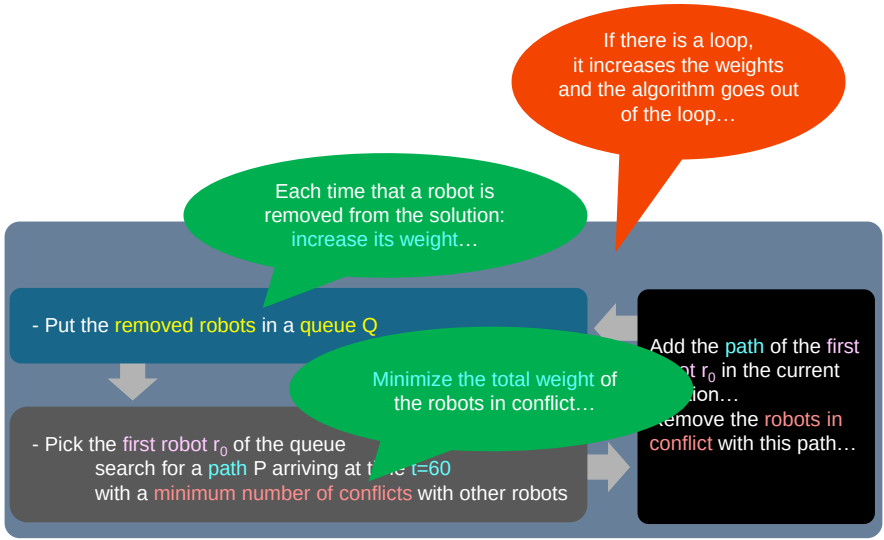
Conflict



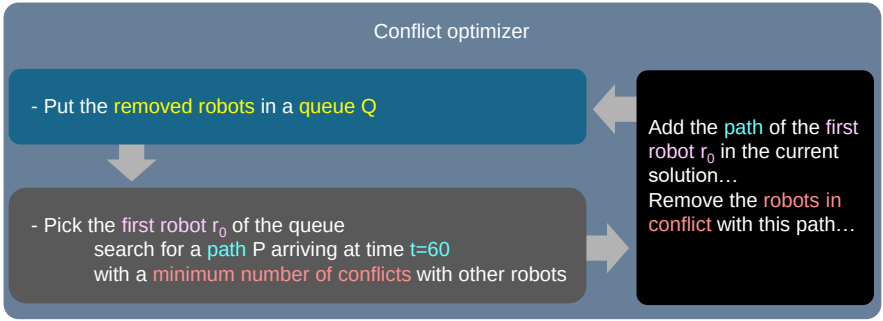
Conflict



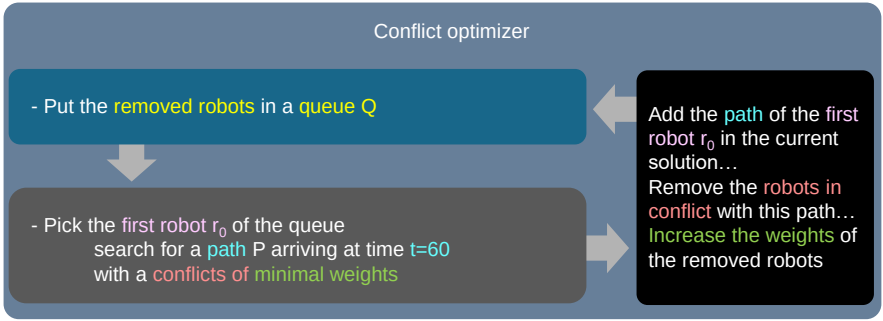
Conflict



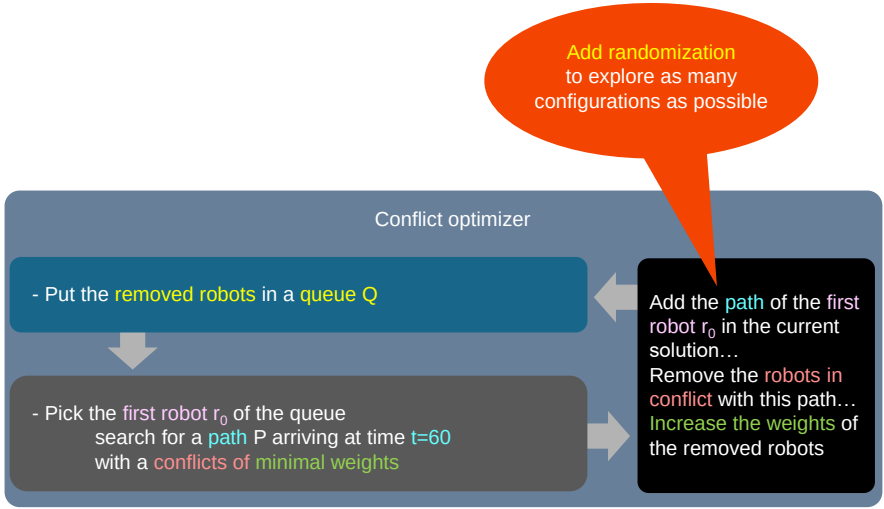
Conflict


Conflict optimizer
- Put the removed robots in a queue Q
- Pick the first robot r_0 of the queue
search for a path P arriving at time t=60
with a minimum number of conflicts with other robots
Add the path of the first robot r_0 in the current solution…
Remove the robots in conflict with this path…

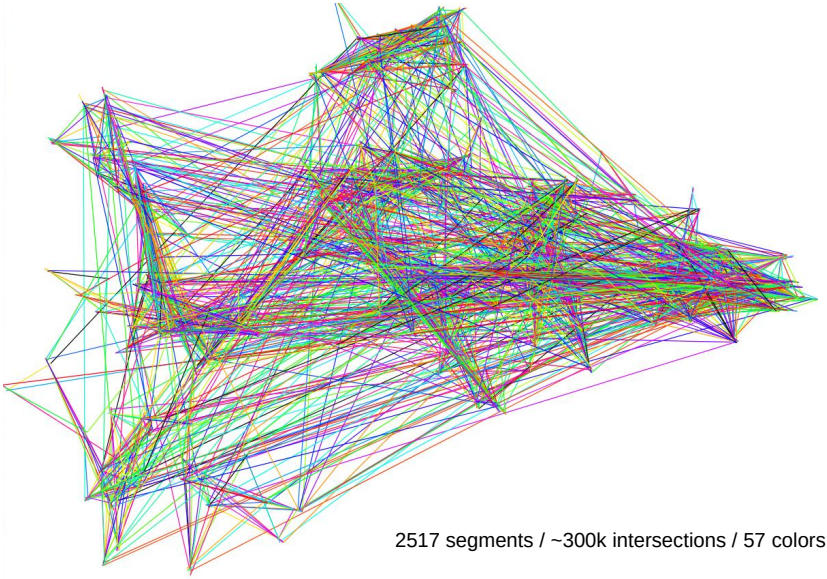
Conflict



Conflict

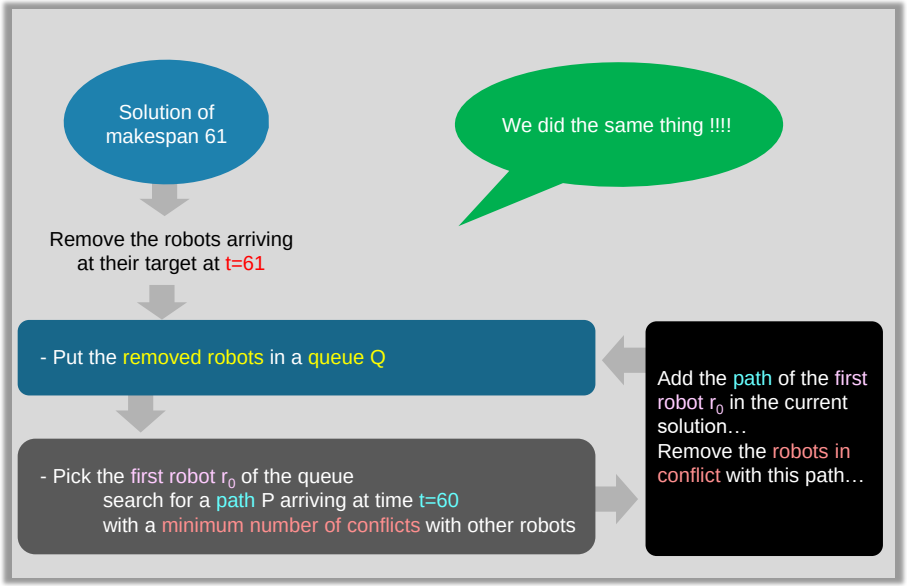

Add randomization
to explore as many
configurations as possible
Conflict optimizer
- Put the removed robots in a queue Q
- Pick the first robot r_0 of the queue
search for a path P arriving at time t=60
with a conflicts of minimal weights
Add the path of the first robot r_0 in the current solution…
Remove the robots in conflict with this path…
Increase the weights of the removed robots

Conflict



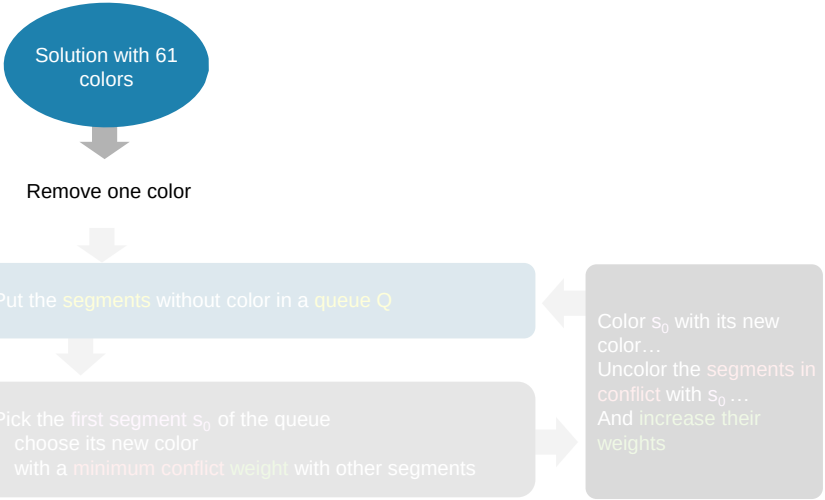
2517 segments / ~300k intersections / 57 colors

Conflict

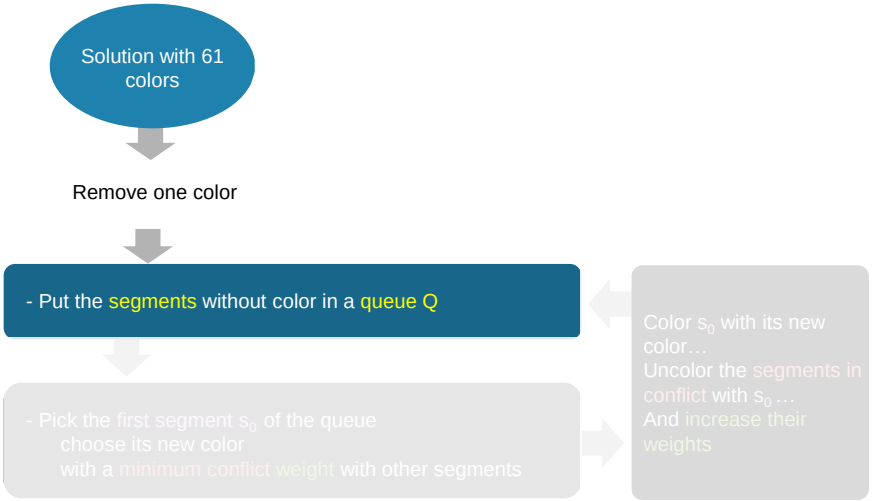

Solution of makespan 61
We did the same thing !!!!
Remove the robots arriving at their target at t=61
- Put the removed robots in a queue Q
- Pick the first robot r_0 of the queue
search for a path P arriving at time t=60
with a minimum number of conflicts with other robots
Add the path of the first robot r_0 in the current solution…
Remove the robots in conflict with this path…

Conflict

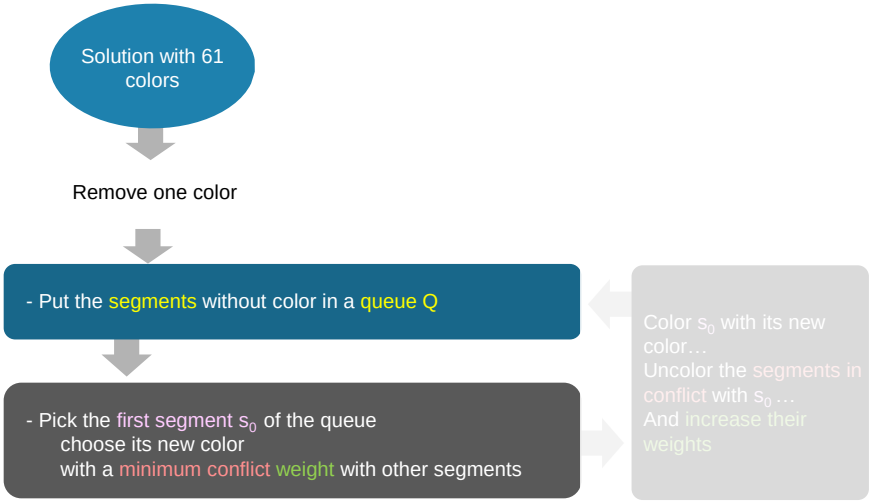
Conflict



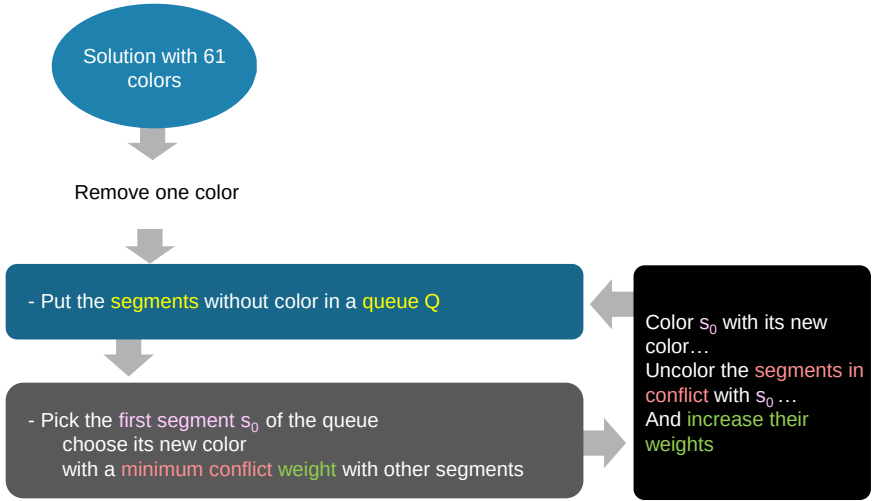
Conflict



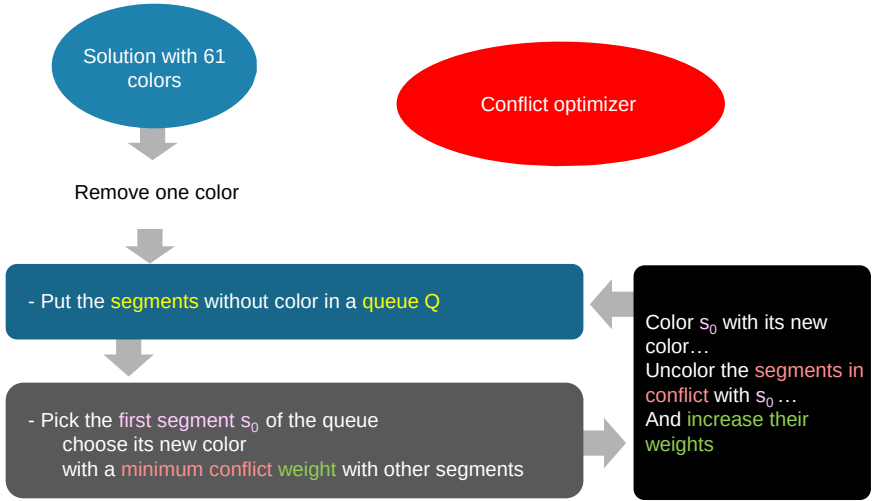
Conflict



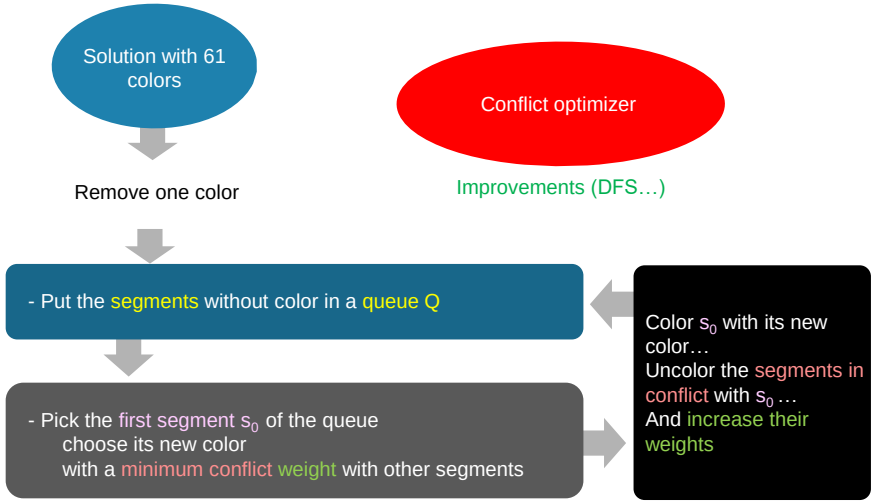
Conflict


Solution with 61 colors
Remove one color
- Put the segments without color in a queue Q
- Pick the first segment s_0 of the queue
choose its new color
with a minimum conflict weight with other segments
Color s_0 with its new color…
Uncolor the segments in conflict with s_0 …
And increase their weights

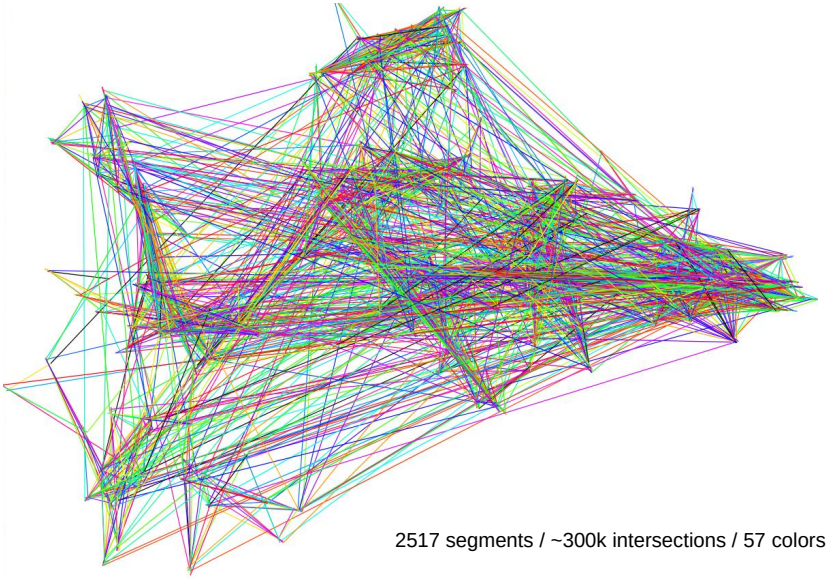
Conflict


Solution with 61 colors
Conflict optimizer
Remove one color
- Put the segments without color in a queue Q
- Pick the first segment s_0 of the queue
choose its new color
with a minimum conflict weight with other segments
Color s_0 with its new color…
Uncolor the segments in conflict with s_0 …
And increase their weights

Conflict



Conflict

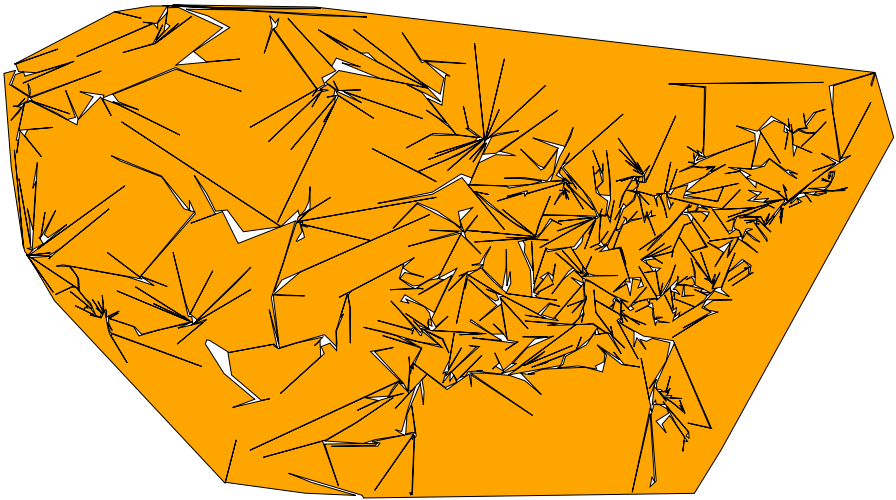


2517 segments / ~300k intersections / 57 colors

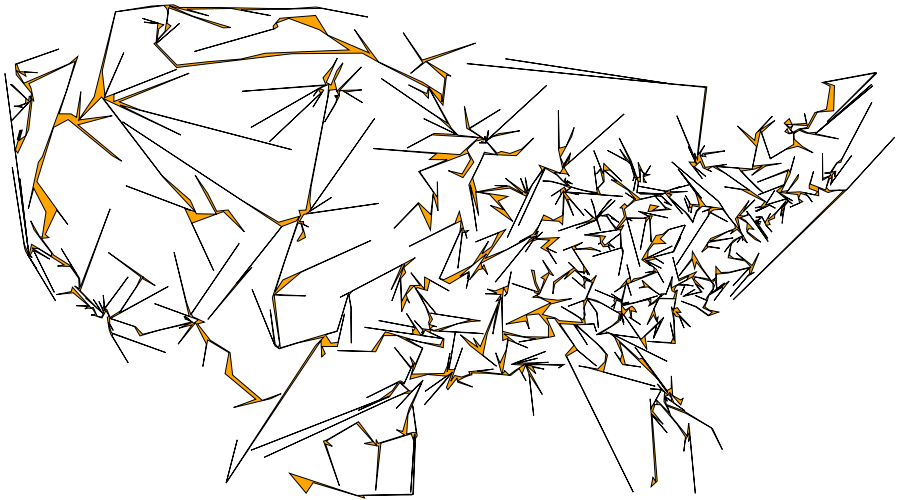
What did we get with these algorithms?

The (very hard) problem of going on vacation

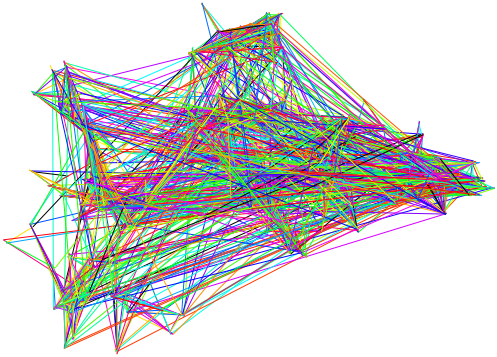
Maximum Area Polygon

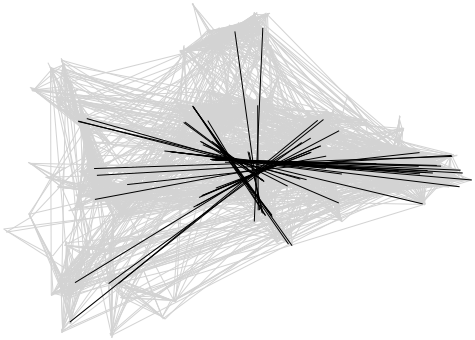


Minimum Area Polygon

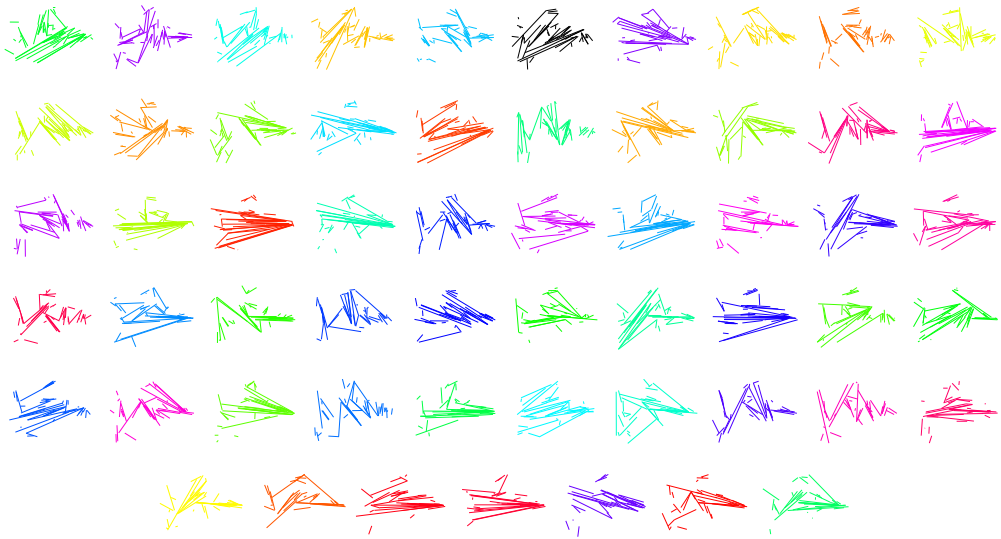


Segment Intersection Graph Coloring and Clique

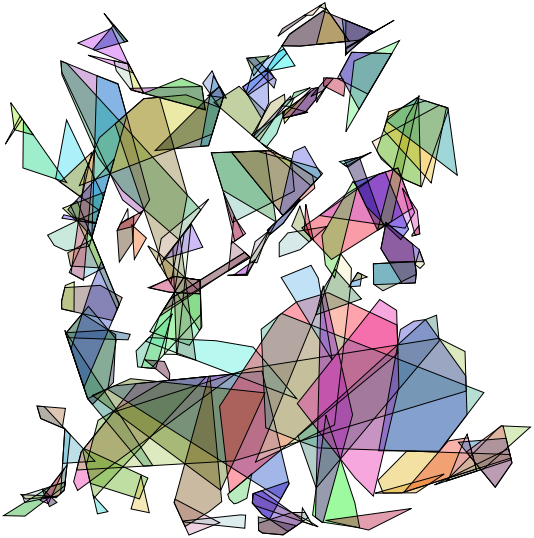




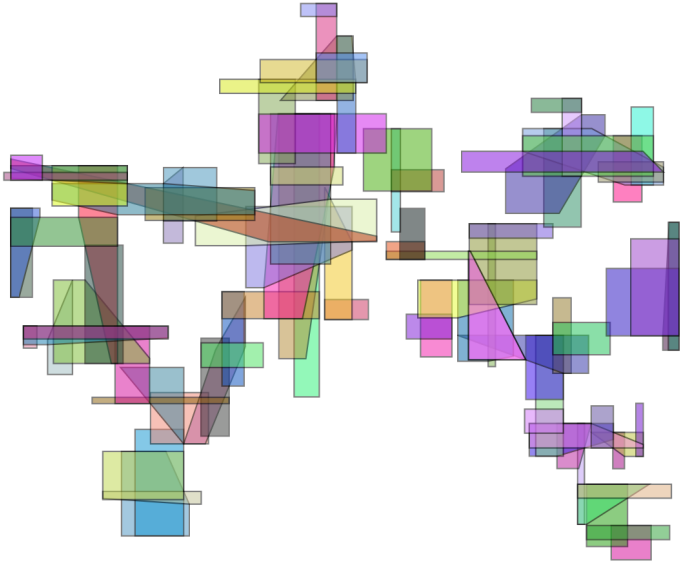
Each Color Class (Interior-Disjoint Segments)



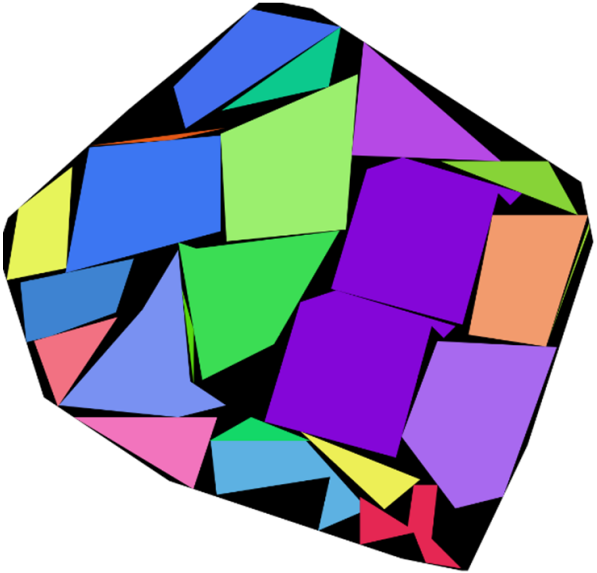
Convex Cover



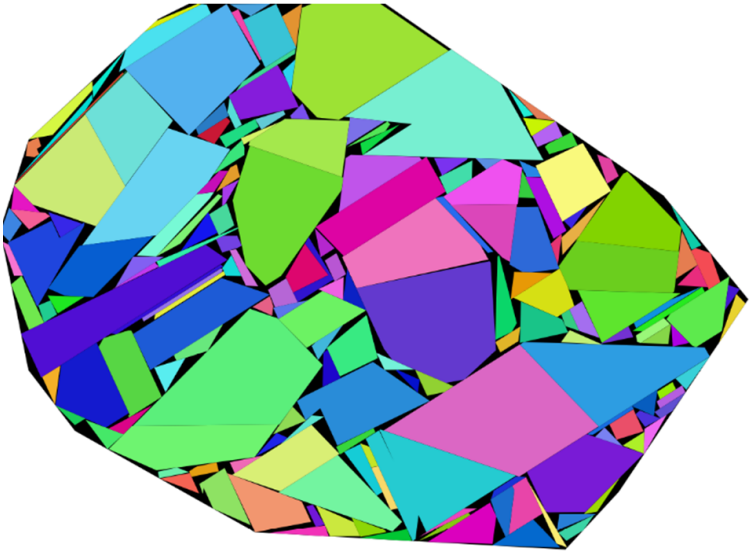
Convex Cover



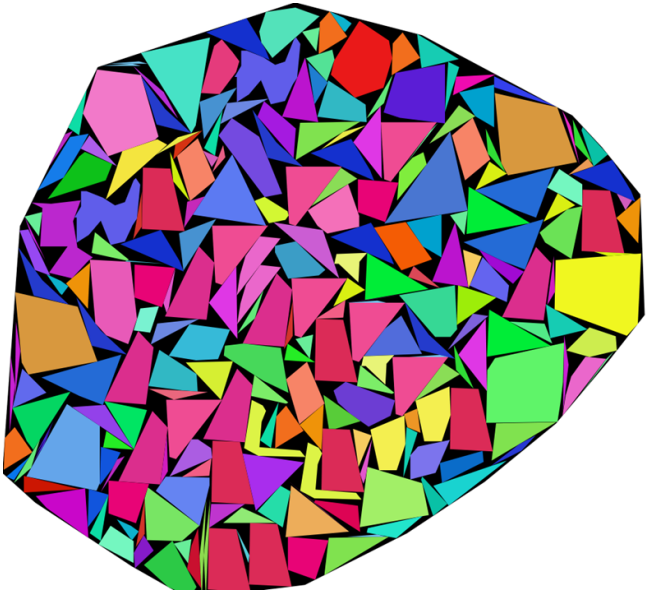
Packing (50 items)



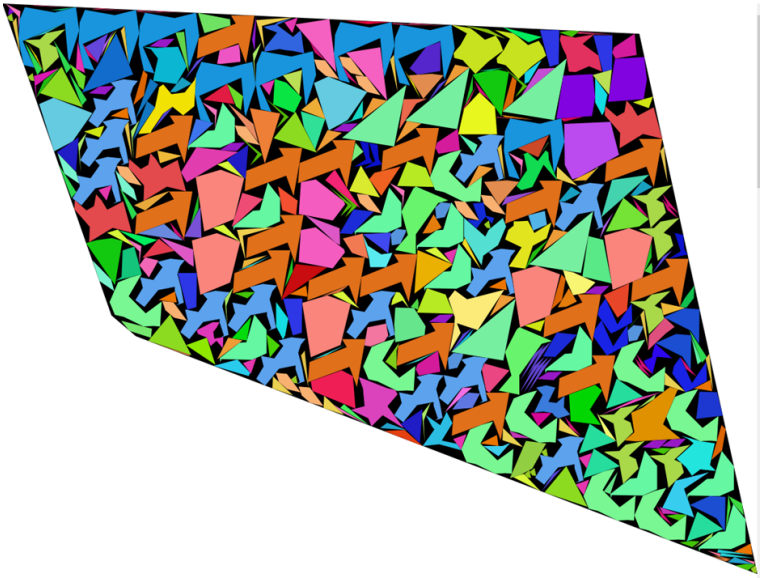
Packing (335 items)



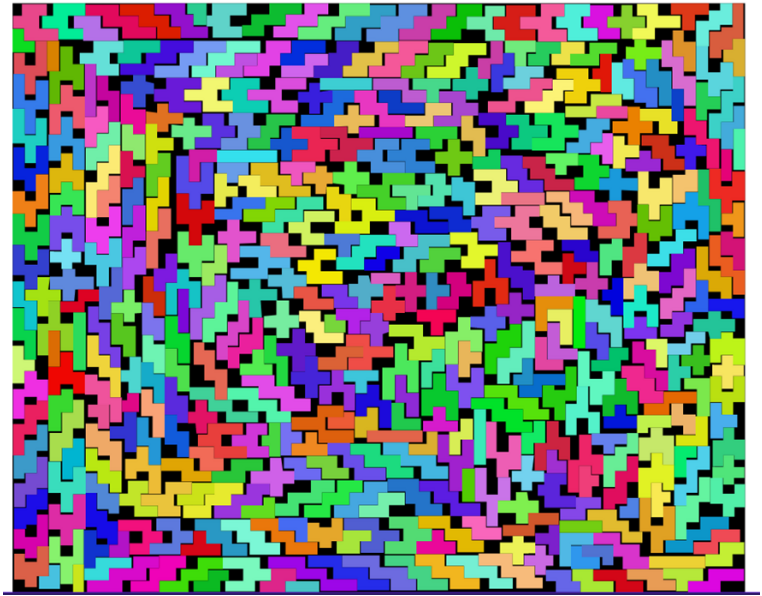
Packing (500 items)



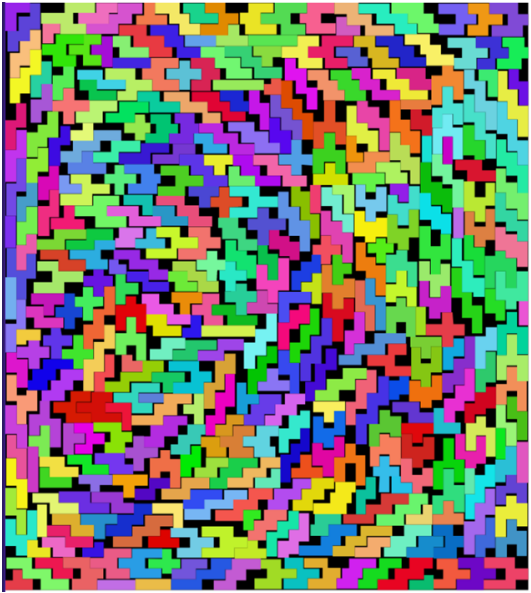
Packing 1000 items)



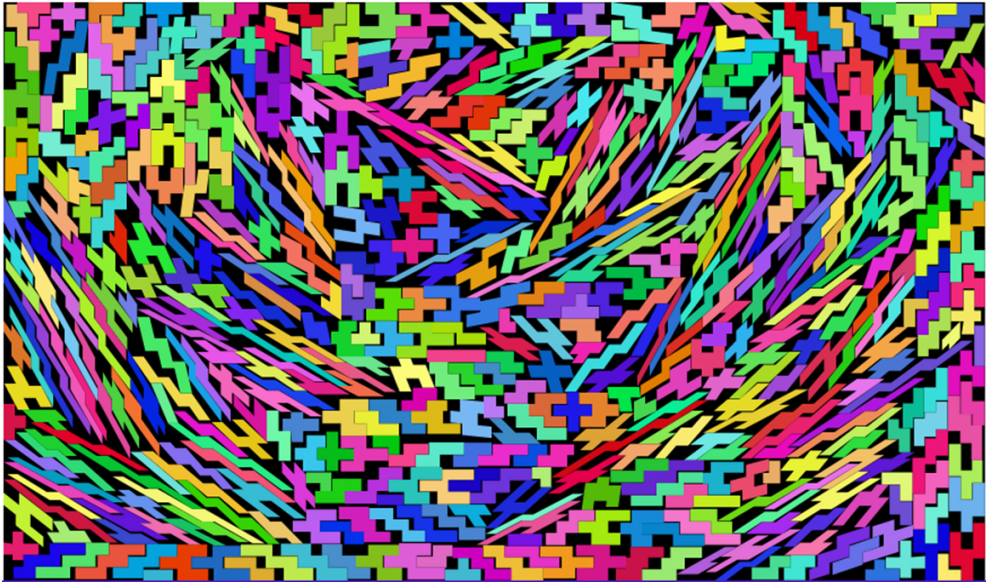
Packing (1240 items)



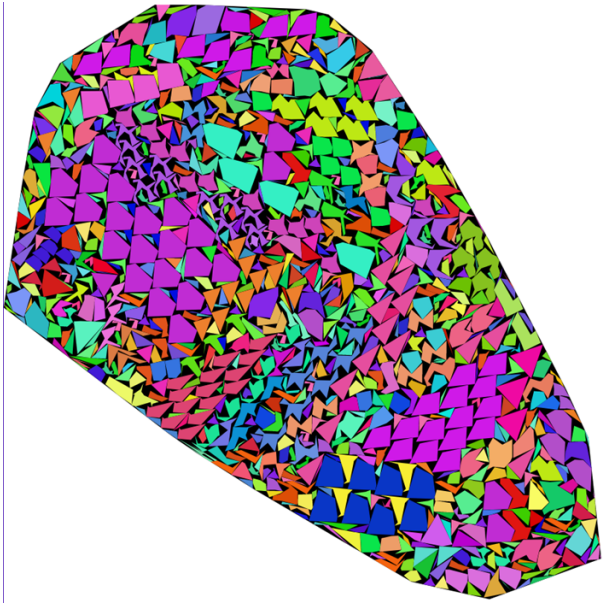
Packing (1672 items)



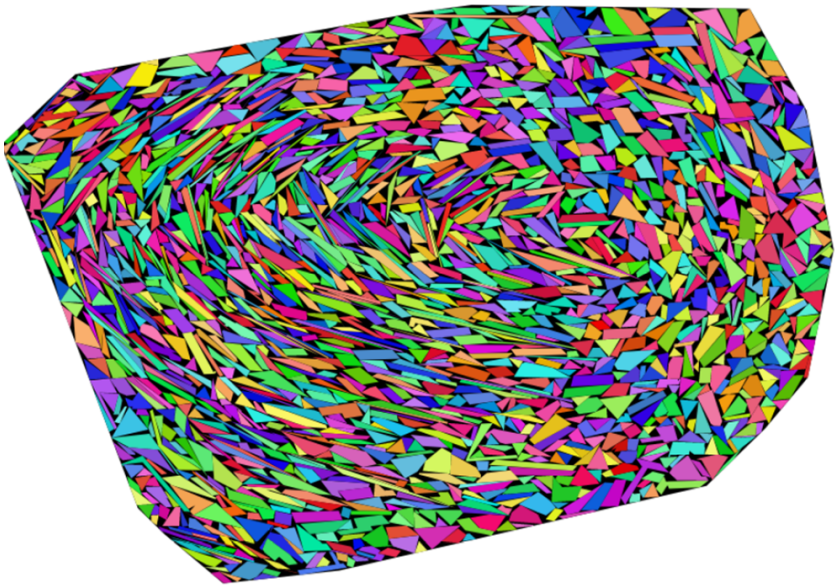
Packing (1685 items)



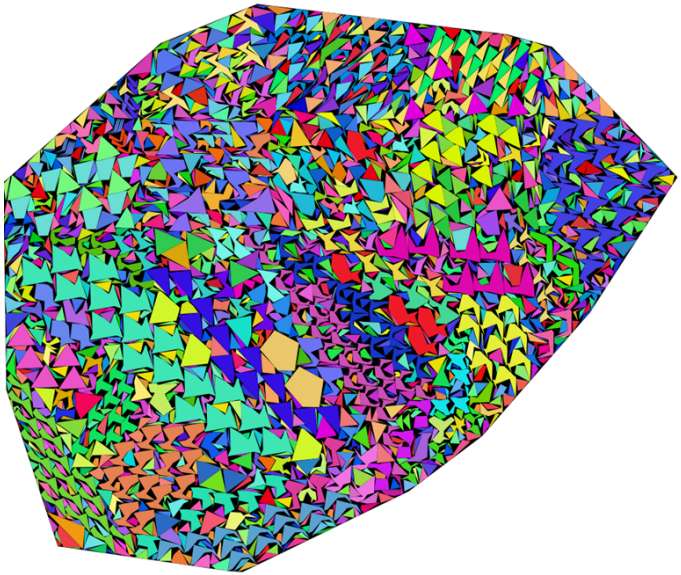
Packing (2000 items)



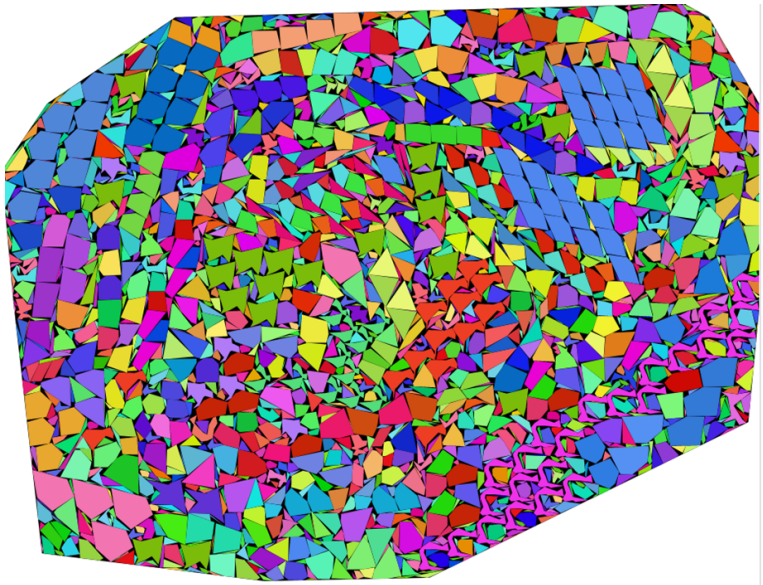
Packing (3000 items)



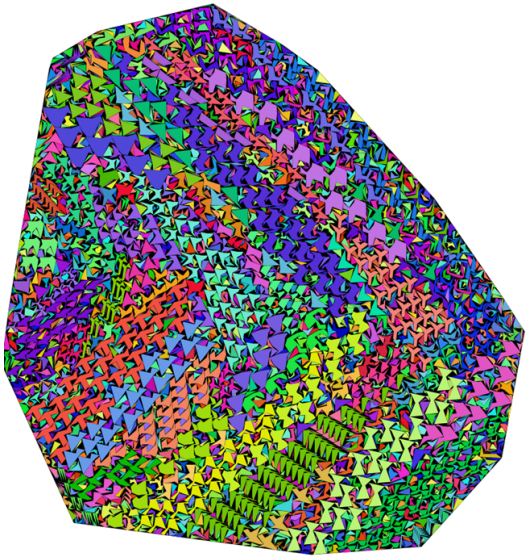
Packing (5000 items)



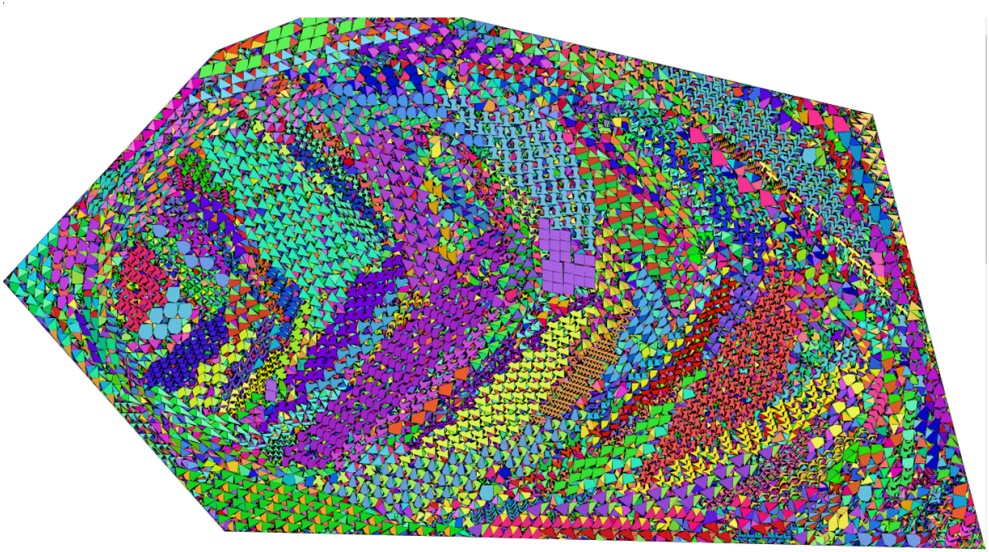
Packing (10000 items)



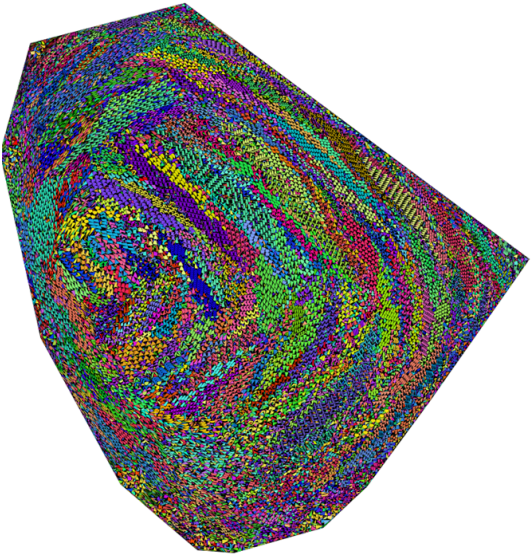
Packing (10000 items)



Packing (50000 items)



Packing (50000 items)



Moving Robots Video

Thank You!

Art by @maryanneshakyhand