Cassting

# Priced Timed Games with Negative Costs

Third Cassting Meeting, Brussels

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May 21, 2014

SEVENTH FRAMEWORK PROGRAMME

# Smart Houses on a Grid (Jadevej)


Eight houses
Electric local grid

Each house:

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- Electric heating
- Storage of energy

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Solar panel ON
- Selling energy: +2€/t.u.
- Consumption: 0€/t.u.
- Storing energy: 0€/t.u.

Solar panel OFF
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Solar panel OFF
- Selling energy: +1€/t.u.
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\+ fixed cost to start selling or buying energy

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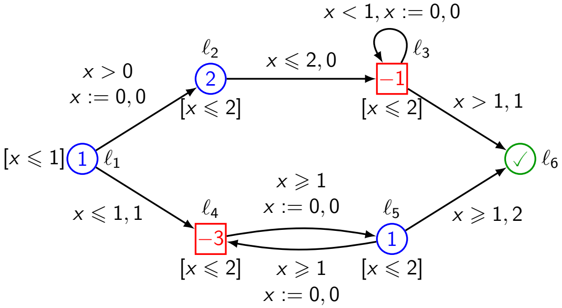
**Our contribution**: Synthesize optimal behaviors in each phase by solving priced timed games with a limited number of distinct rates

# Priced Timed Games




Timed Automaton
with partition of states
between 2 players
+ reachability objective
+ rates in locations
+ costs over transitions

Semantics in terms of
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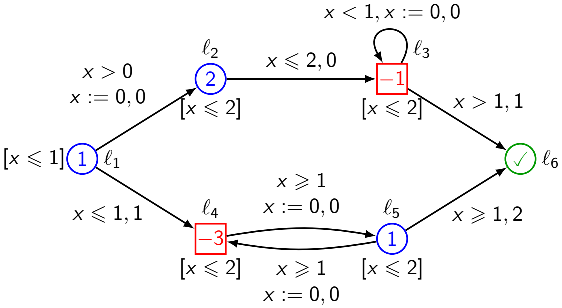


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$(\ell_1, 0)$

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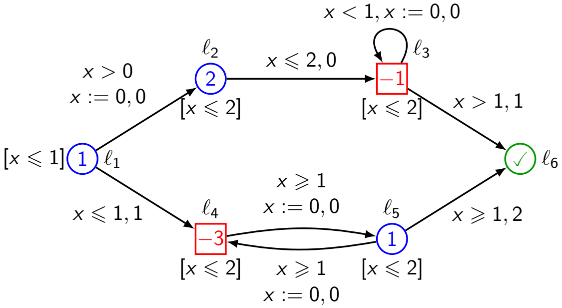


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$(\ell_1, 0) \xrightarrow{0.4, \searrow} (\ell_4, 0.4)$

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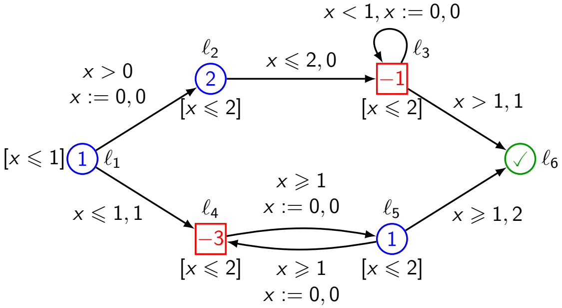


$(\ell_1, 0) \xrightarrow{0.4, \searrow} (\ell_4, 0.4) \xrightarrow{0.6, \rightarrow} (\ell_5, 0)$

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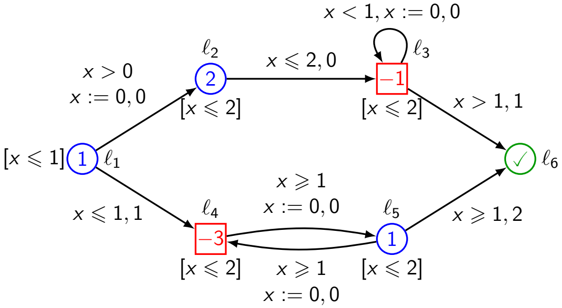


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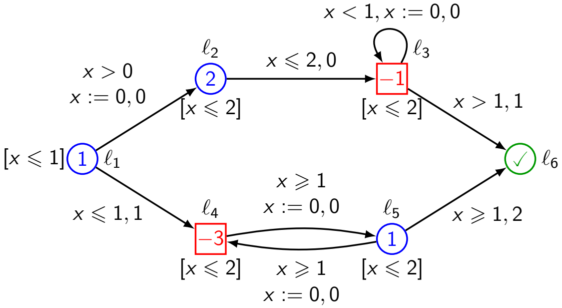


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$$0.4 + 1 \quad -3 \times 0.6 \quad +1.5 \quad -3 \times 1.1 \quad +2 \times 2 + 2 \quad = 3.8$$

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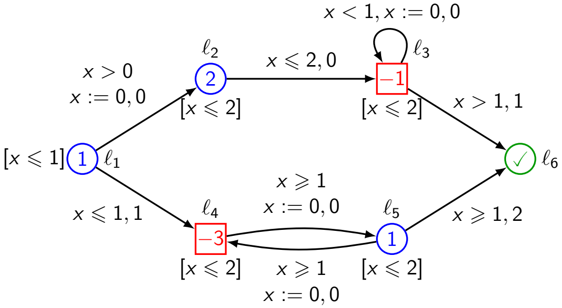


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$$(\ell_1, 0) \xrightarrow[0.4+1]{0.4, \searrow} (\ell_4, 0.4) \xrightarrow[-3 \times 0.6]{0.6, \rightarrow} (\ell_5, 0) \xrightarrow[+1.5]{1.5, \leftarrow} (\ell_4, 0) \xrightarrow[-3 \times 1.1]{1.1, \rightarrow} (\ell_5, 0) \xrightarrow[+2 \times 2 + 2]{2, \nearrow} (\checkmark, 2) \quad = 3.8$$

$$(\ell_1, 0) \xrightarrow[0.2]{0.2, \nearrow} (\ell_2, 0) \xrightarrow[+0.9]{0.9, \rightarrow} (\ell_3, 0.9) \xrightarrow[-0.2]{0.2, \circlearrowleft} (\ell_3, 0) \xrightarrow[-0.9]{0.9, \circlearrowleft} (\ell_3, 0) \quad \cdots \quad = +\infty \ (\checkmark \text{ not reached})$$

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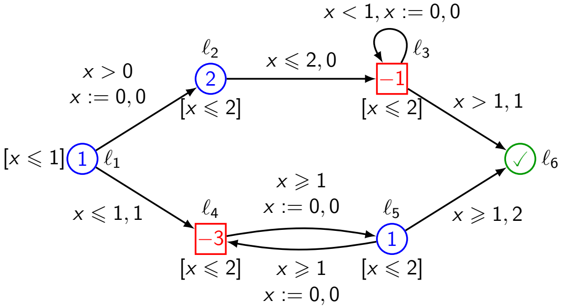


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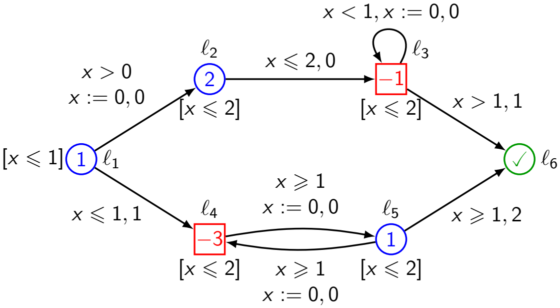
$$\text{Cost of a play: } \begin{cases} +\infty & \text{if } \checkmark \text{ not reached} \\ \text{total payoff up to } \checkmark & \text{otherwise} \end{cases}$$

# Strategies and objectives




Strategy for each player: mapping of finite runs to a delay and an action

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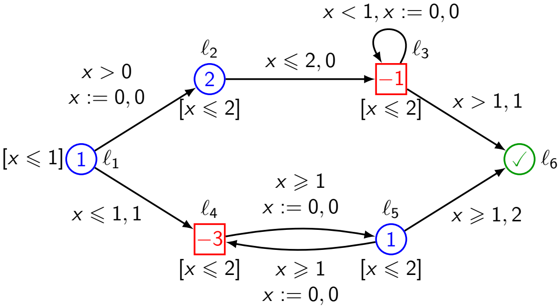


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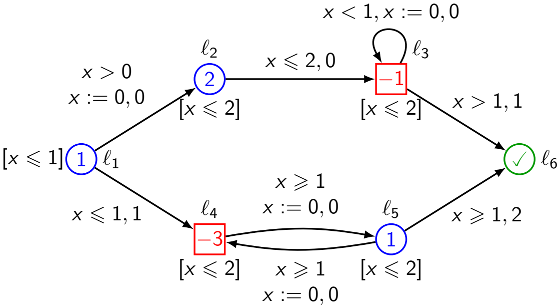


Strategy for each player: mapping of finite runs to a delay and an action

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Main object of interest:

$$\overline{\mathrm{Val}}(\ell, v) = \inf_{\sigma_\bigcirc \in \mathrm{Strat}_\bigcirc} \sup_{\sigma_\square \in \mathrm{Strat}_\square} \mathrm{Wt}(\mathrm{Play}((\ell, v), \sigma_\bigcirc, \sigma_\square)) \in \mathbf{R} \cup \{-\infty, +\infty\}$$

What player ○ can guarantee as a payoff? and design *good* strategies

# State of the art

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This talk: **PTGs with negative costs**

# Undecidability Results: Constrained-Price Reachability

- Known: $F_{\leqslant K}\checkmark$ undecidable for 3 or more clocks

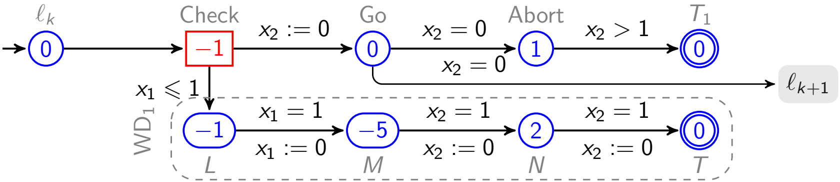
Proof by reduction of 2-counter machines: $x_1 = \frac{1}{2^{c_1}}$, $x_2 = \frac{1}{3^{c_2}}$, $x_3$ for work

**Theorem:**

$F_{\leqslant K}\checkmark$ undecidable for PTGs with 2 or more clocks
idem for $F_{\geqslant K}\checkmark$, $F_{>K}\checkmark$, $F_{=K}\checkmark$, $F_{<K}\checkmark$

New encoding: $x_1 = \frac{1}{5^{c_1}7^{c_2}}$, $x_2$ for work

Simulation of "$\ell_k$ : `decrement` $c_1$; `goto` $\ell_{k+1}$" for Reach$(= 1)$



# Other Undecidability Results

**Theorem: Time-bounded Reachability**

The following problem is undecidable for PTGs with 6 or more clocks:

Input: $K, T \in \mathbf{N}$
Question: $\mathsf{F}^{\leqslant T}_{\leqslant K}\checkmark$: $\exists$ strategy for $\bigcirc$ that reaches $\checkmark$ with cost $\leqslant K$ within time $T$?

**Theorem: Repeated Reachability**

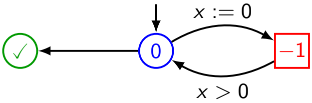
The following problem is undecidable for PTGs with 3 or more clocks:

Input: $\eta \geqslant 0$
Question: $\mathsf{GF}_{[-\eta,\eta]}\checkmark$: $\exists$ strategy for $\bigcirc$ that visits $\checkmark$ infinitely often with a cost in $[-\eta, \eta]$?

# Regain decidability?

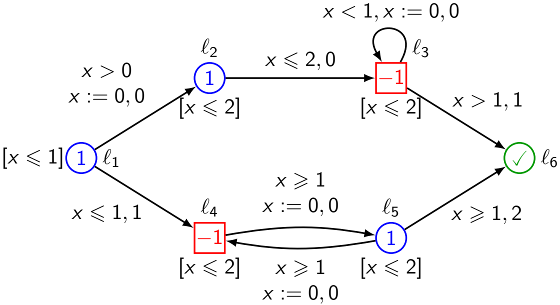
# More complex when negative costs

- Value $-\infty$: detection is as hard as mean-payoff. No hope for complexity better than **NP** $\cap$ **co-NP**, or pseudo-polynomial
- Memory complexity
  - Player ○ needs memory, even in the untimed setting: see Gilles' talk
  - Player □ may require infinite memory



# One-clock Bi-Valued PTGs (1BPTGs)

**Assumption: rates of locations** $\{p^-, p^+\}$ **included in** $\{0, +d, -d\}$ **(**$d \in \mathbf{N}$**)** (no assumption on costs of transitions)




- Techniques of [Bouyer, Cassez, Fleury, and Larsen, 2004, Alur, Bernadsky, and Madhusudan, 2004] not applicable, e.g., because of Zeno costs cycles
- Exponential algorithms of [Bouyer, Larsen, Markey, and Rasmussen, 2006b, Rutkowski, 2011, Hansen, Ibsen-Jensen, and Miltersen, 2013] not working because of presence of negative costs

# Results

**Theorem:**

- Computation of the value $\overline{\mathsf{Val}}(\ell, v)$ of states of a 1BPTG in pseudo-polynomial time
- Synthesis of $\varepsilon$-optimal strategies for player ○ in pseudo-polynomial time

**Theorem: Non-negative case**

In case of a 1BPTG with only non-negative costs, all complexities drop down to polynomial.

# First idea: symmetrize the viewpoint

Value for player $\bigcirc$: $\overline{\mathsf{Val}}(\ell, v) = \inf_{\sigma_\bigcirc \in \mathsf{Strat}_\bigcirc} \sup_{\sigma_\Box \in \mathsf{Strat}_\Box} \mathsf{Wt}(\mathsf{Play}((\ell, v), \sigma_\bigcirc, \sigma_\Box))$

Value for player $\Box$: $\underline{\mathsf{Val}}(\ell, v) = \sup_{\sigma_\Box \in \mathsf{Strat}_\Box} \inf_{\sigma_\bigcirc \in \mathsf{Strat}_\bigcirc} \mathsf{Wt}(\mathsf{Play}((\ell, v), \sigma_\bigcirc, \sigma_\Box))$

How to compare them? $\underline{\mathsf{Val}}(\ell, v) \leqslant \overline{\mathsf{Val}}(\ell, v)$

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Value for player □: $\underline{\mathsf{Val}}(\ell, v) = \sup_{\sigma_\square \in \mathsf{Strat}_\square} \inf_{\sigma_\bigcirc \in \mathsf{Strat}_\bigcirc} \mathsf{Wt}(\mathsf{Play}((\ell, v), \sigma_\bigcirc, \sigma_\square))$

How to compare them? $\underline{\mathsf{Val}}(\ell, v) \leqslant \overline{\mathsf{Val}}(\ell, v)$

**Theorem: (continued)**

- 1BPGs are determined: $\underline{\mathsf{Val}}(\ell, v) = \overline{\mathsf{Val}}(\ell, v)$
- Synthesis of $\varepsilon$-optimal strategies for player □ in pseudo-polynomial time (and polynomial in case of non-negative weights)

# Sketch of proof

1. **Reduce the space of strategies in the 1BPTG**
   - restrict to uniform strategies w.r.t. timed behaviors

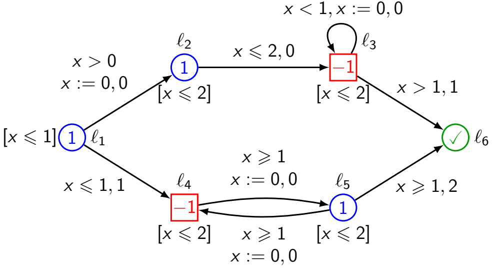
2. **Build a finite priced game $\mathcal{G}$**
   - based on a refinement of the region abstraction

3. **Study $\mathcal{G}$**

4. **Lift results of $\mathcal{G}$ to the original 1BPTG**

# 1. Reduce the space of strategies

Intuition: no need for both players to play far from borders of regions




Regions:
$\{0\}, (0,1), \{1\}, (1,2), \{2\}, (2,+\infty)$

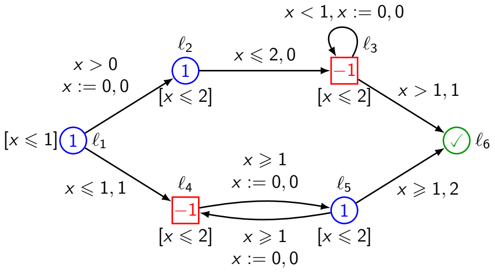
*Player ○ wants to leave as soon as possible a state with rate $p^+$, and wants to stay as long as possible in a state with rate $p^-$: so, he will always play $\eta$-close to a border...*

## Lemma:

Both players can play arbitrarily close to borders w.l.o.g.: for every $\eta$

$$\underline{\mathsf{Val}}^\eta(\ell, v) \leqslant \underline{\mathsf{Val}}(\ell, v) \quad \leqslant \quad \overline{\mathsf{Val}}(\ell, v) \leqslant \overline{\mathsf{Val}}^\eta(\ell, v)$$

# 2. Finite priced game abstraction




$\eta$-regions: $\{0\}, (0, \eta), (1 - \eta, 1), \{1\}, (1, 1 + \eta), (2 - \eta, 2), \{2\}, (2, +\infty)$

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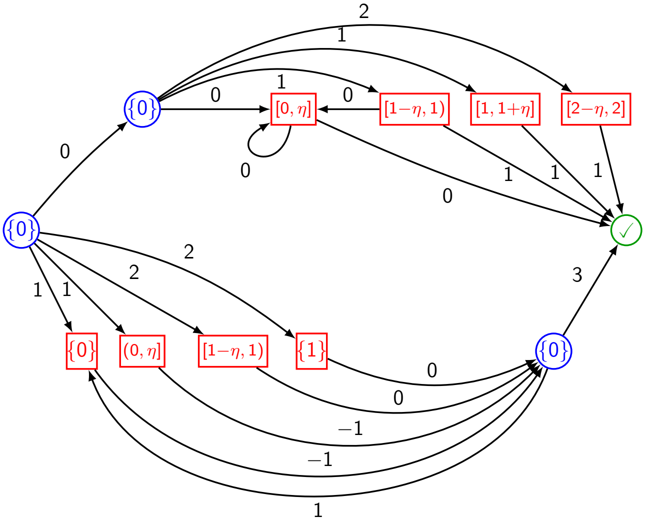
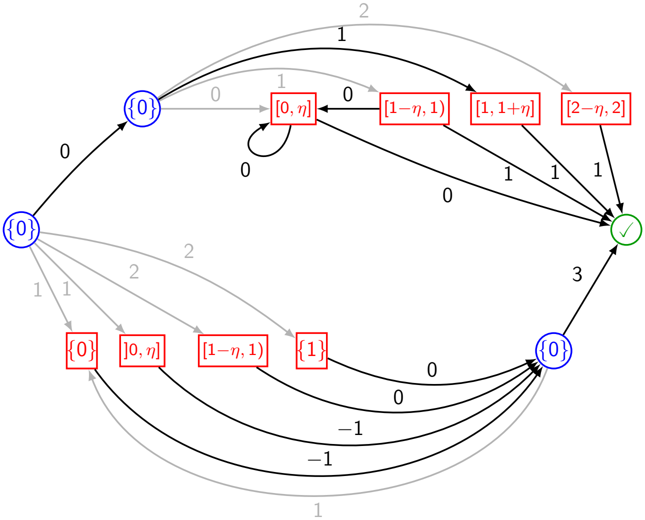


# 3. Study $\mathcal{G}$: values, optimal strategies




Optimal value: $\mathsf{Val}_{\mathcal{G}}(\ell_1, \{0\}) = +2$ (for both players)

# 4. Lift results to the original 1BPTG

Reconstruct strategies in the 1BPTG from optimal strategies of $\mathcal{G}$

**Lemma:**

For all $\varepsilon > 0$, there exists $\eta > 0$ such that:

$$\mathsf{Val}_{\mathcal{G}}(\ell, \{0\}) - \varepsilon \leqslant \underline{\mathsf{Val}}^{\eta}(\ell, 0) \leqslant \underline{\mathsf{Val}}(\ell, 0) \leqslant \overline{\mathsf{Val}}(\ell, 0) \leqslant \overline{\mathsf{Val}}^{\eta}(\ell, 0) \leqslant \mathsf{Val}_{\mathcal{G}}(\ell, \{0\}) + \varepsilon$$

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- So $\underline{\mathsf{Val}}(\ell, 0) = \overline{\mathsf{Val}}(\ell, 0)$, i.e., determination
- $\varepsilon$-optimal strategies for both players
  - Finite memory for player ○ (finite memory in finite priced games)
  - Infinite memory for player □ (even though memoryless in finite priced games), because it needs to ensure convergence of its differences between the 1BPTG and $\mathcal{G}$
- Overall complexity: pseudo-polynomial (polynomial if non-negative costs) in the size of $\mathcal{G}$, which is polynomial in the 1BPTG (because 1 clock)

# Summary and Future Work

## Results

- More undecidability results due to the presence of negative costs
- 1BPTGs are determined: $\underline{\mathsf{Val}}(\ell, v) = \overline{\mathsf{Val}}(\ell, v)$
- Computation of the values, and synthesis of $\varepsilon$-optimal strategies for both players, in pseudo-polynomial time
- Strategy complexity: finite memory for $\bigcirc$, infinite memory for $\square$
- In case of $\geqslant 0$ prices, non-trivial class of 1-clock PTGs in PTIME
- Lifting of corner point abstraction to game setting

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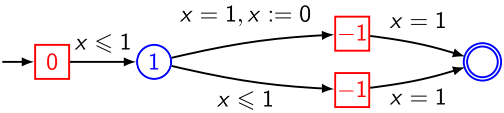
- Implementation and test of this algorithm for real instances
- Decidability results for a bigger subset of PTGs with negative weights? careful since players may need to play far from boundaries in case of 2 clocks, or 1 clock and 3 distinct rates...



$x = 1, x := 0$
$x \leqslant 1$
$x = 1$
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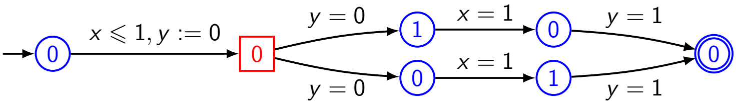
$x = 1$
0
1
−1
−1

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$x = 1$
$y = 1$
0
0
1
0
0
1
0

Thank you for your attention

Questions?

Rajeev Alur, Mikhail Bernadsky, and P. Madhusudan. Optimal reachability for weighted timed games. In *Proceedings of the 31st International Colloquium on Automata, Languages and Programming (ICALP'04)*, volume 3142 of *Lecture Notes in Computer Science*, pages 122–133. Springer, 2004.

Patricia Bouyer, Franck Cassez, Emmanuel Fleury, and Kim G. Larsen. Optimal strategies in priced timed game automata. In *Proceedings of the 24th Conference on Foundations of Software Technology and Theoretical Computer Science (FSTTCS'04)*, volume 3328 of *Lecture Notes in Computer Science*, pages 148–160. Springer, 2004.

Patricia Bouyer, Thomas Brihaye, and Nicolas Markey. Improved undecidability results on weighted timed automata. *Information Processing Letters*, 98(5):188–194, 2006a.

Patricia Bouyer, Kim G. Larsen, Nicolas Markey, and Jacob Illum Rasmussen. Almost optimal strategies in one-clock priced timed games. In *Proceedings of the 26th Conference on Foundations of Software Technology and Theoretical Computer Science (FSTTCS'06)*, volume 4337 of *Lecture Notes in Computer Science*, pages 345–356. Springer, 2006b.

Patricia Bouyer, Thomas Brihaye, Véronique Bruyère, and Jean-François Raskin. On the optimal reachability problem of weighted timed automata. *Formal Methods in System Design*, 31(2):135–175, 2007.

John Fearnley and Marcin Jurdziński. Reachability in two-clock timed automata is pspace-complete. In *Proceedings of ICALP'13*, volume 7966 of *Lecture Notes in Computer Science*, pages 212–223. Springer, 2013.

Christoph Haase, Joël Ouaknine, and James Worrell. On the relationship between reachability problems in timed and counter automata. In *Proceedings of RP'12*, pages 54–65, 2012.

Thomas Dueholm Hansen, Rasmus Ibsen-Jensen, and Peter Bro Miltersen. A faster algorithm for solving one-clock priced timed games. In *Proceedings of the 24th International Conference on Concurrency Theory (CONCUR'13)*, volume 8052 of *Lecture Notes in Computer Science*, pages 531–545. Springer, 2013.

Michał Rutkowski. Two-player reachability-price games on single-clock timed automata. In *Proceedings of the Ninth Workshop on Quantitative Aspects of Programming Languages (QAPL'11)*, volume 57 of *Electronic Proceedings in Theoretical Computer Science*, pages 31–46, 2011.