Interval Iteration Algorithm for MDPs and IMDPs

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Mixing non-determinism and probabilities

- Acting in an *uncertain* world
  - non-determinism: *decisions* of an agent;
  - probabilities: effects of the decisions;
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  - probabilities: distributed *randomized* algorithm;
  - non-determinism: behavior of the network;
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Optimization problems

Markov Decision Processes

- What?
  - (Finite) *stochastic* process with *non-determinism*
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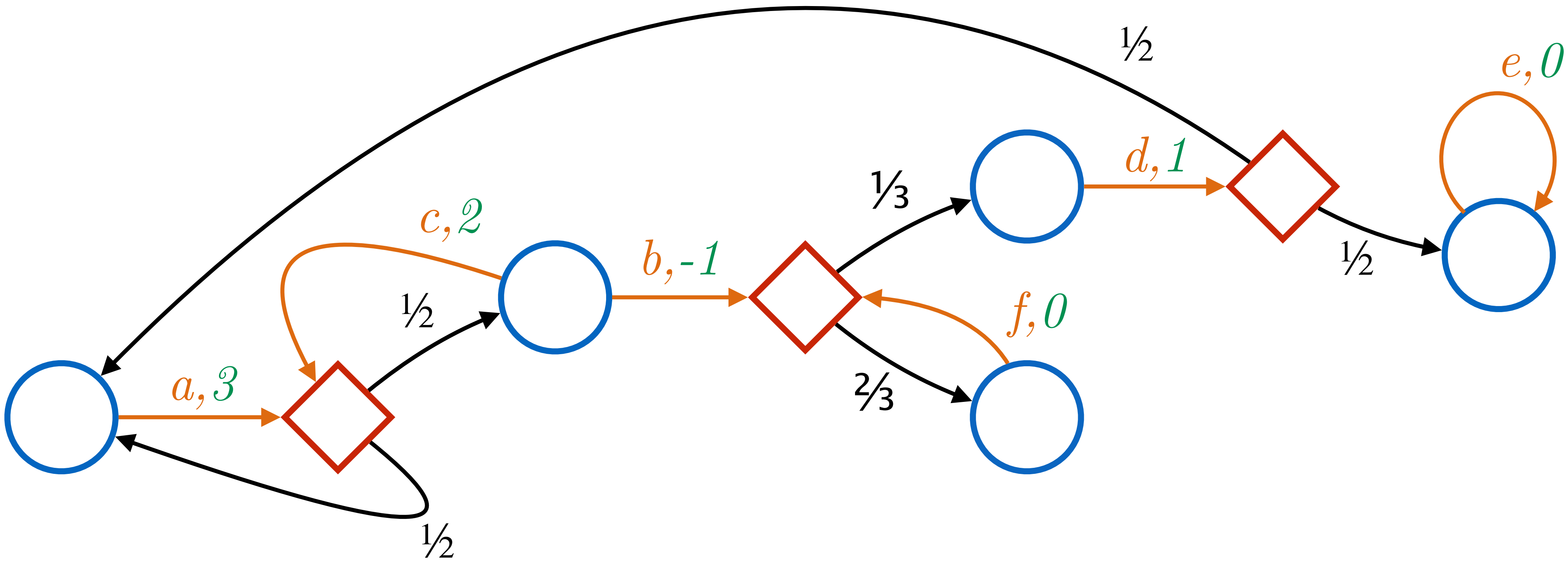
- Where?
  - *Optimization*
  - *Program verification*: PCTL model-checking...
  - *Game theory*: 1+½ players

MDPs with discounted rewards



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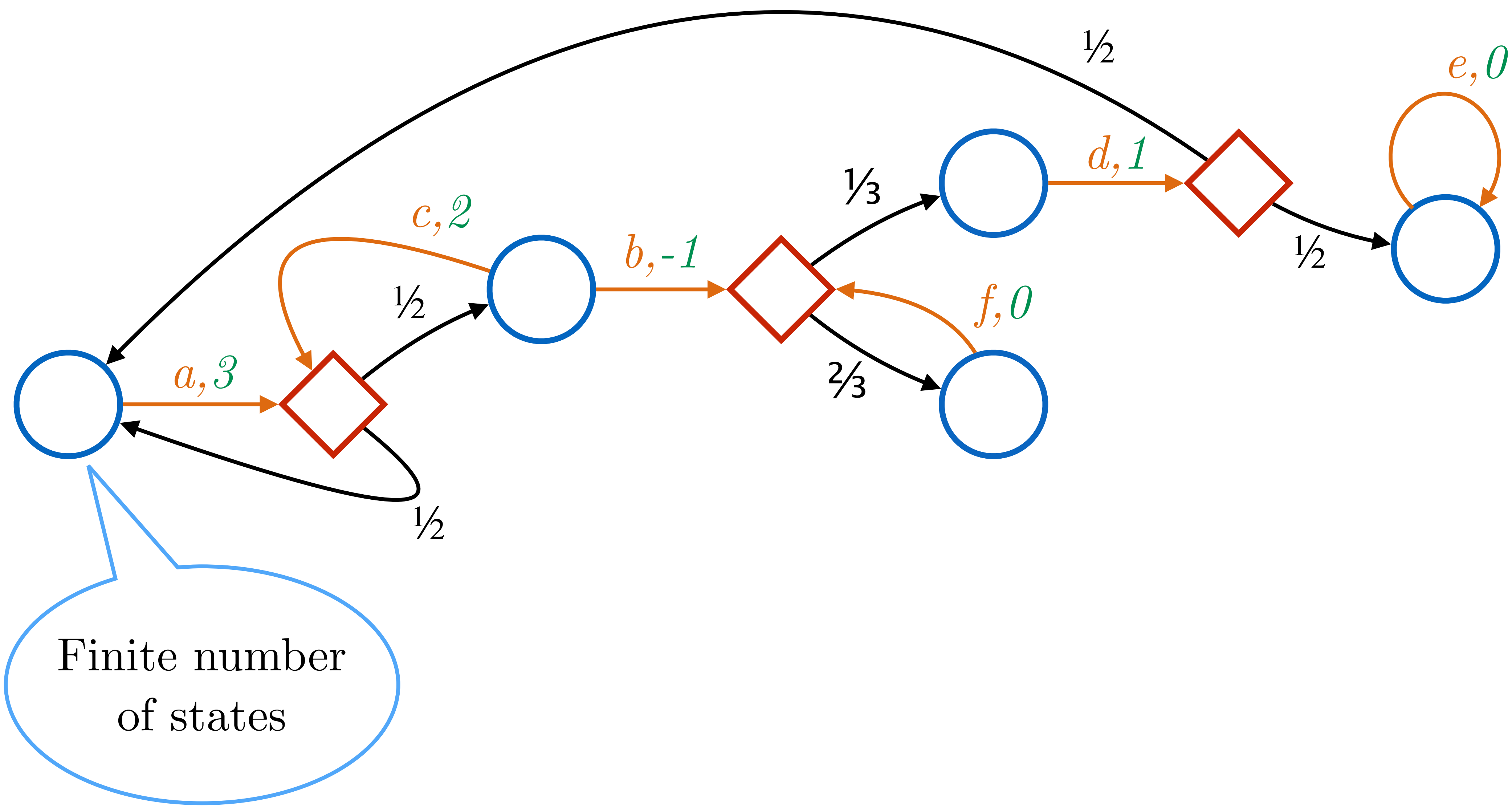

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$\delta : S \times \alpha \longrightarrow Dist(S)$

$r : S \times \alpha \longrightarrow \mathbb{R}$

Policy $\sigma : (S \cdot \alpha)^{\star} \cdot S \longrightarrow Dist(\alpha)$

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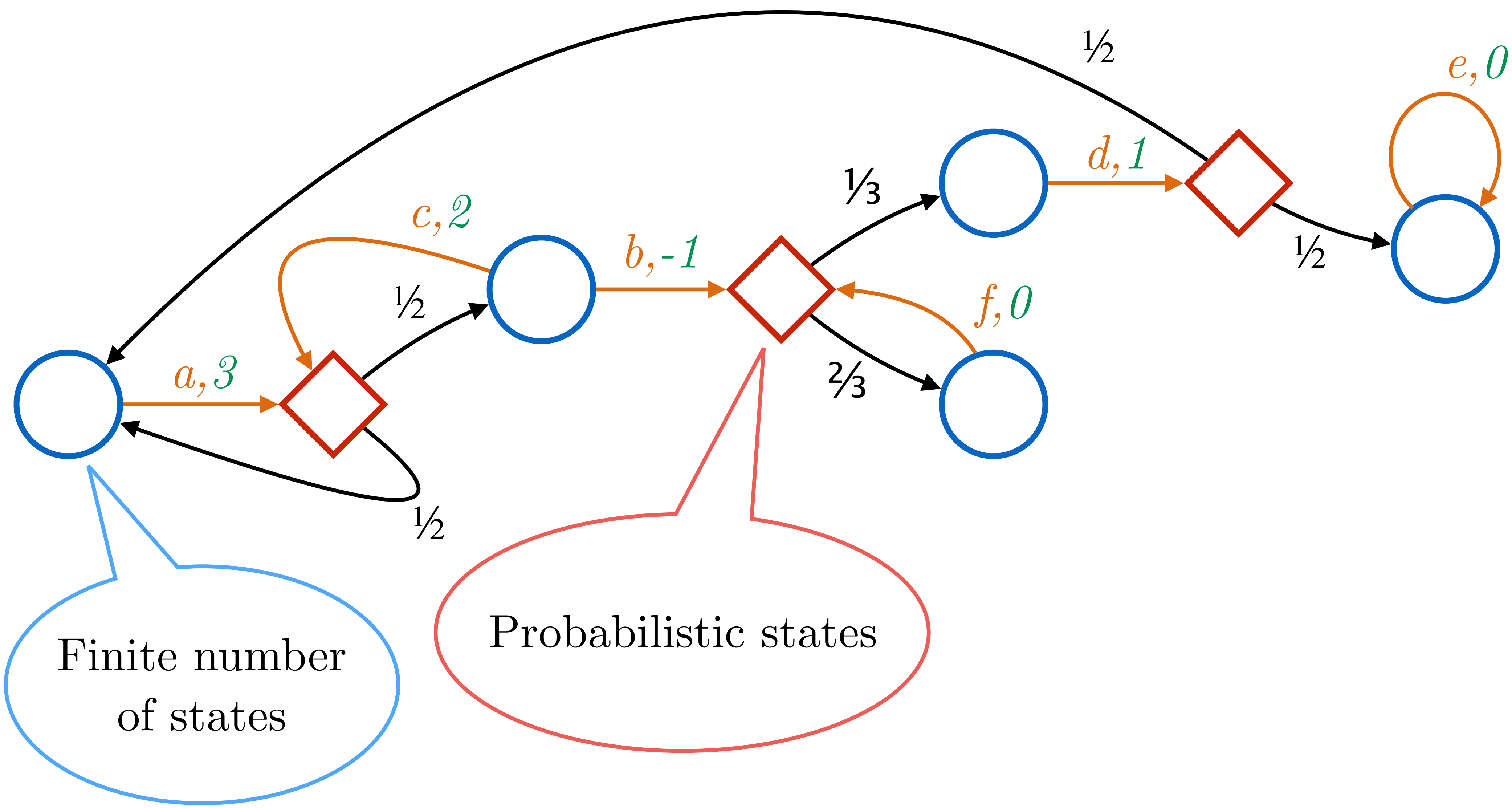

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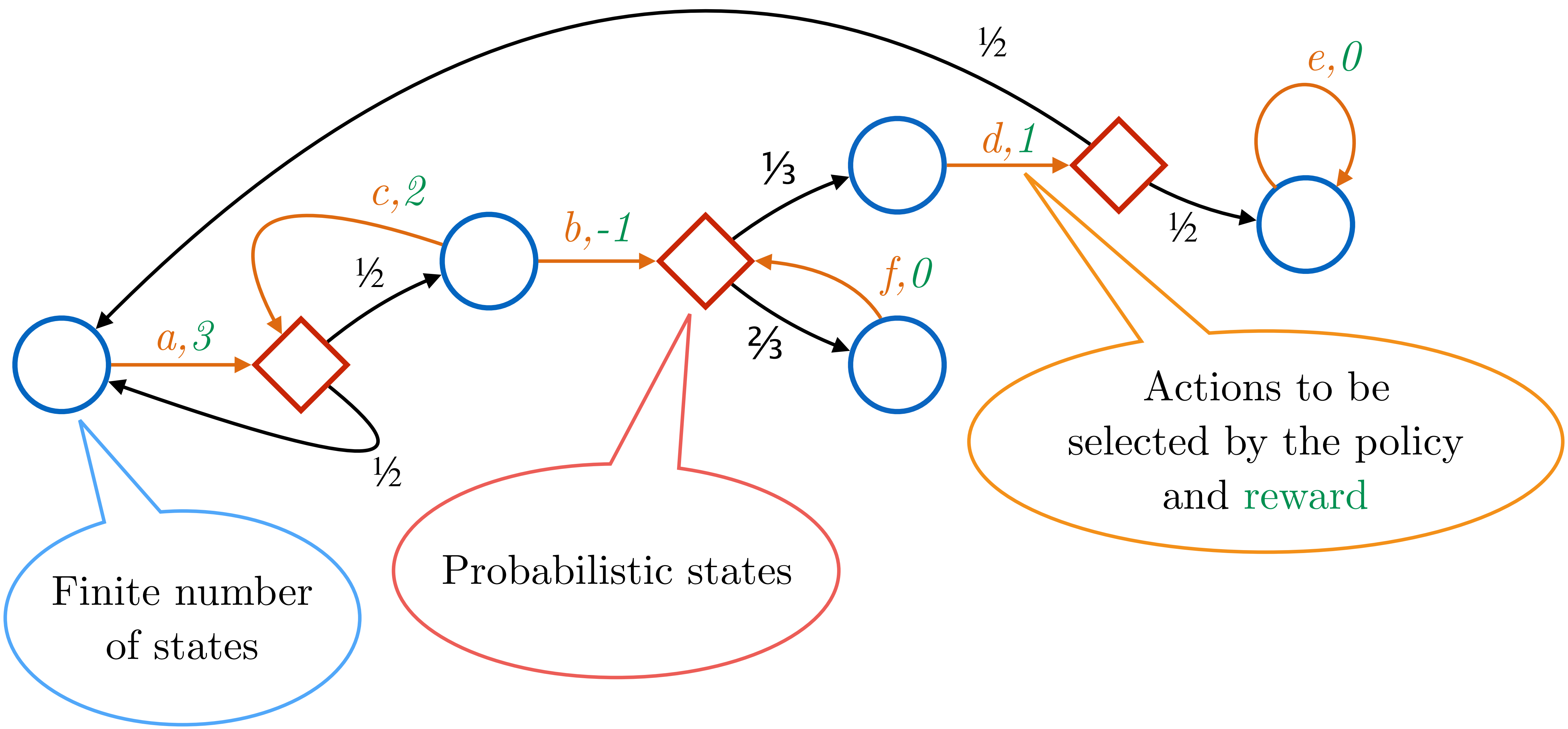

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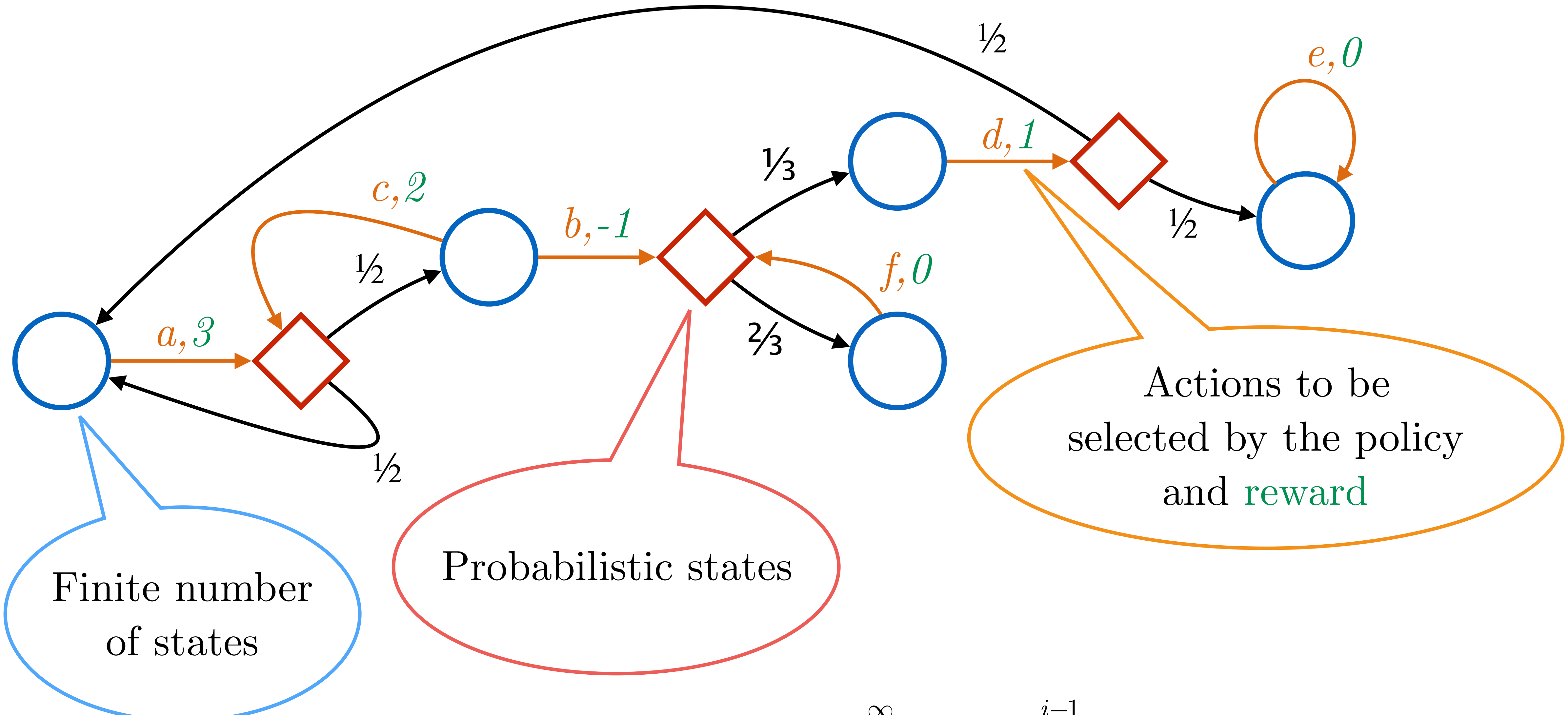

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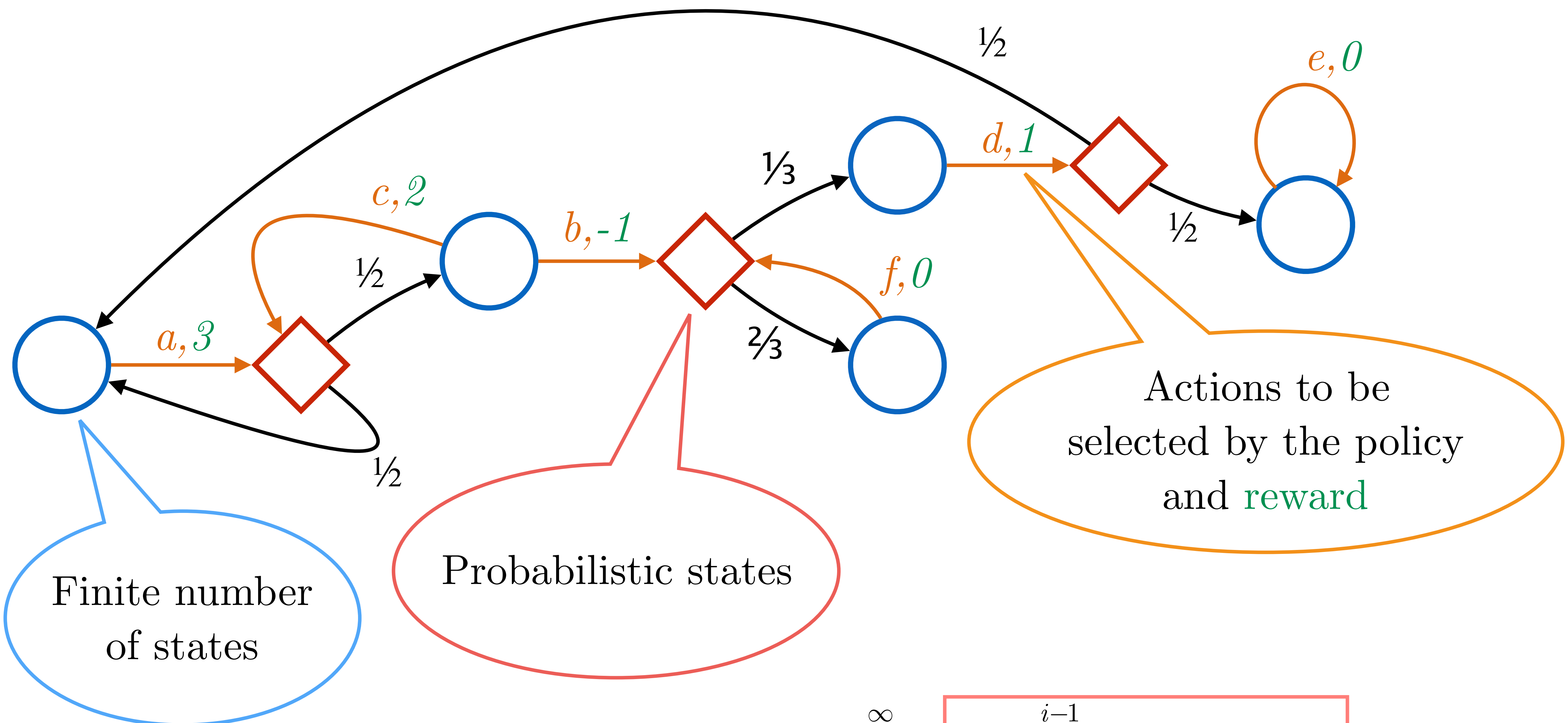

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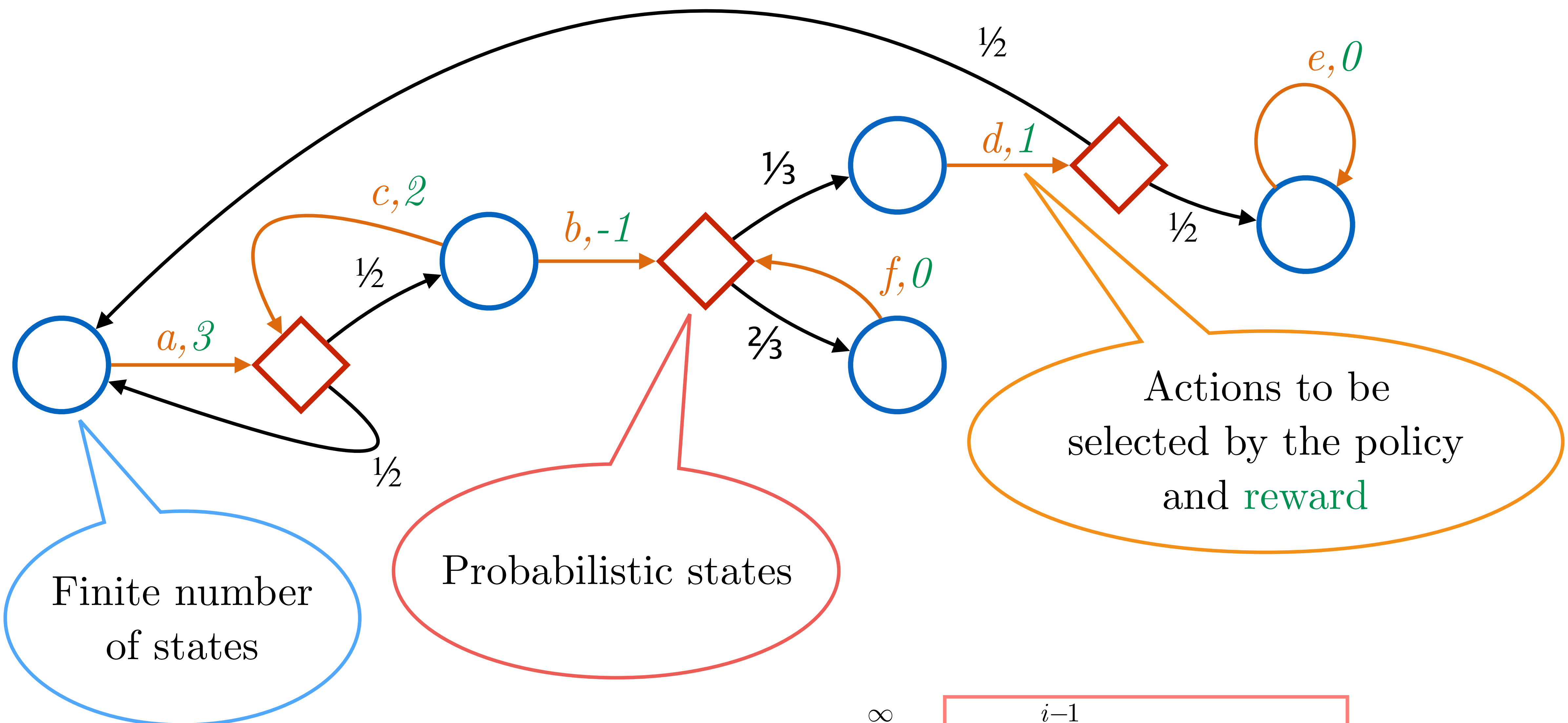
probability to arrive in s_i after i steps

MDPs with discounted rewards



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Objective: compute $\sup_\sigma v^\sigma(s_0)$ and good policies

Resolution of MDPs with discounted rewards

$$v^{\sigma}(s_0) = \sum_{i=0}^{\infty} \lambda^i \sum_{s_1,\ldots,s_i} \prod_{j=0}^{i-1} \delta\Big(s_j, \sigma(\ldots s_j)\Big)\, r\Big(s_i, \sigma(\ldots s_i)\Big)$$

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time horizon 1
with « terminal rewards » λv^σ.

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Function $L : \mathbb{R}^S \to \mathbb{R}^S$ defined by

$$L(v)_s = \max_{a \in \alpha} r(s,a) + \lambda \sum_{s' \in S} \delta(s,a)(s') v_{s'}$$

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speed of convergence +
stopping criterion for algorithm

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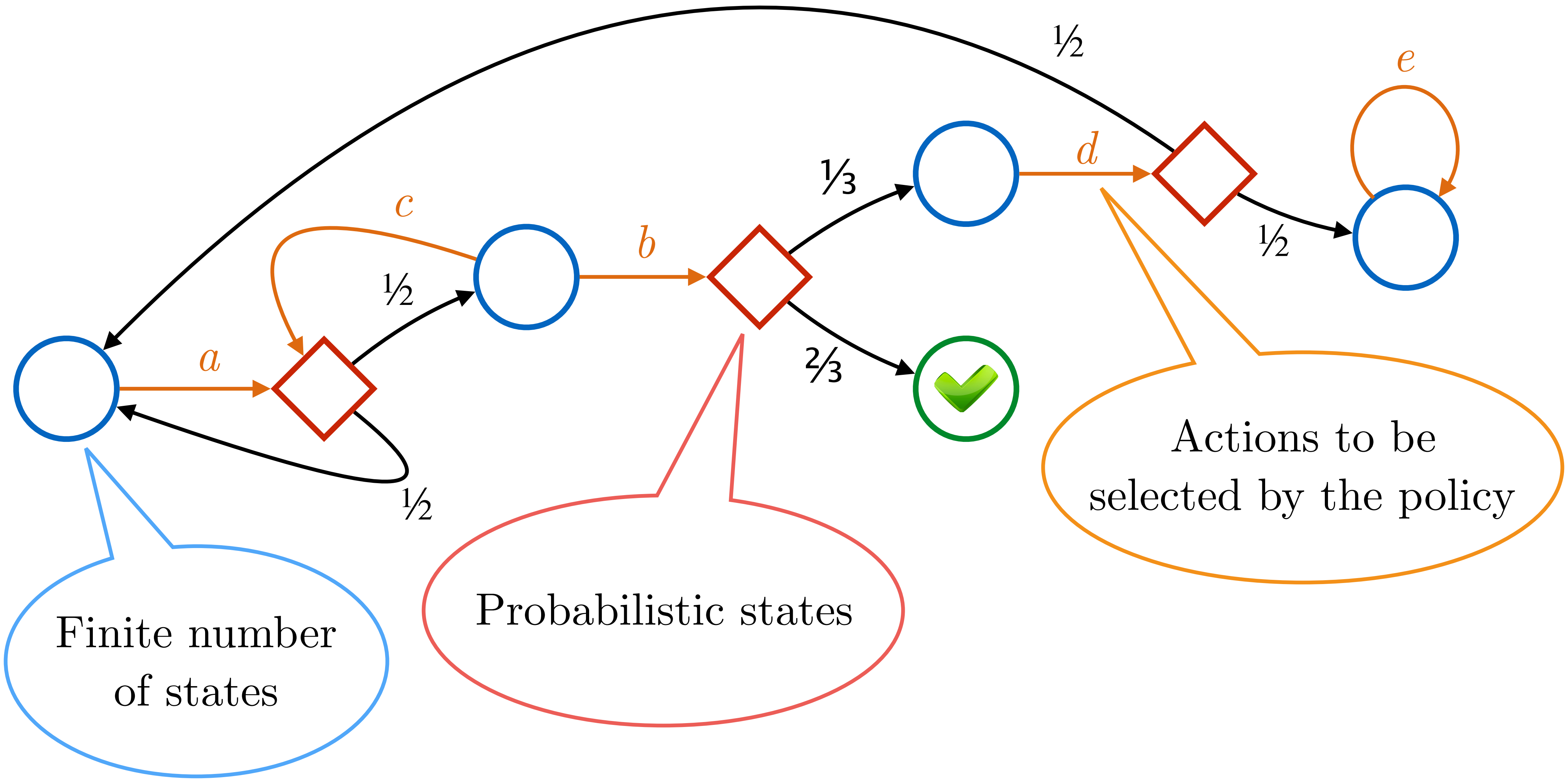
MDPs with reachability objectives



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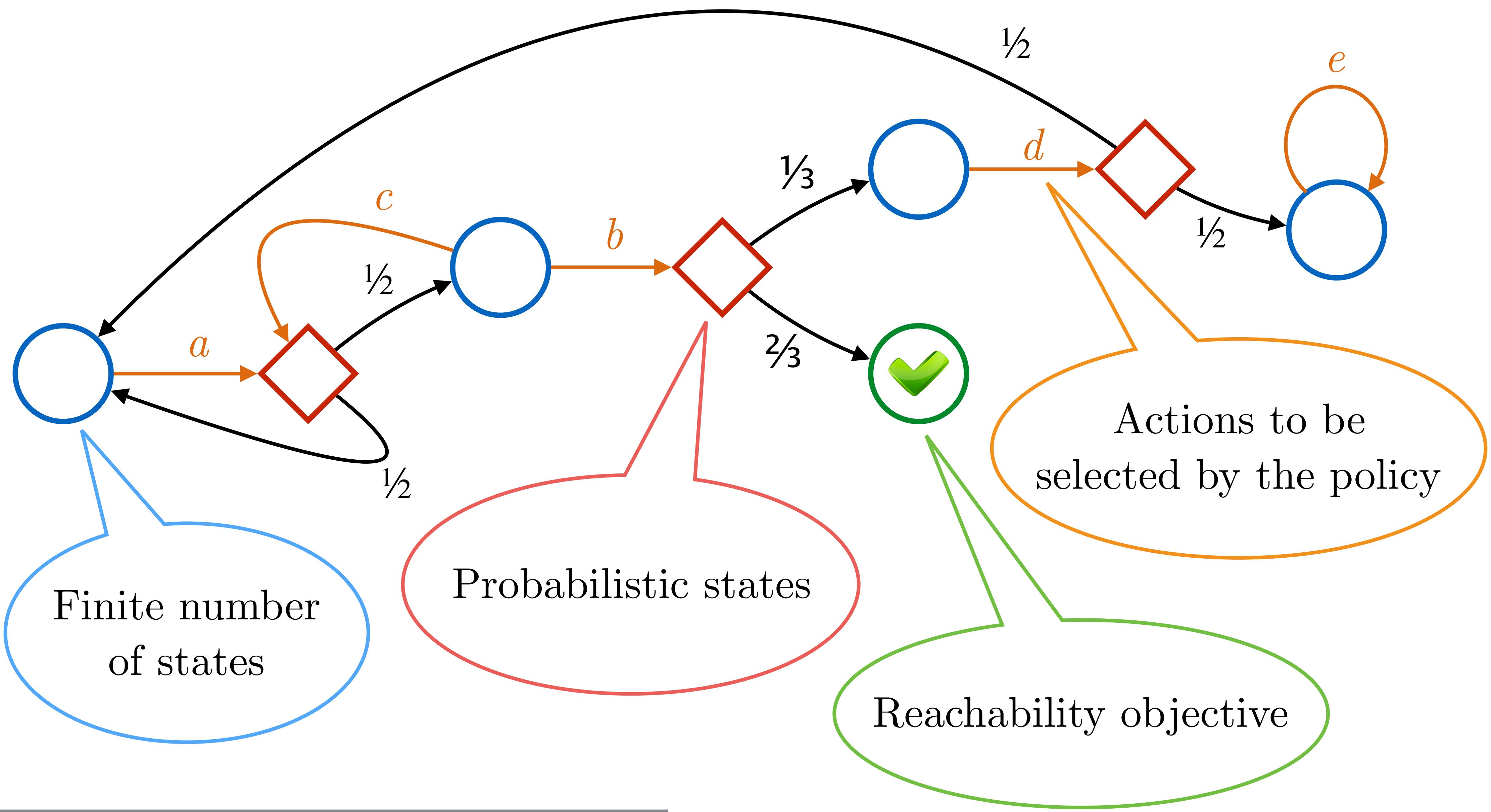

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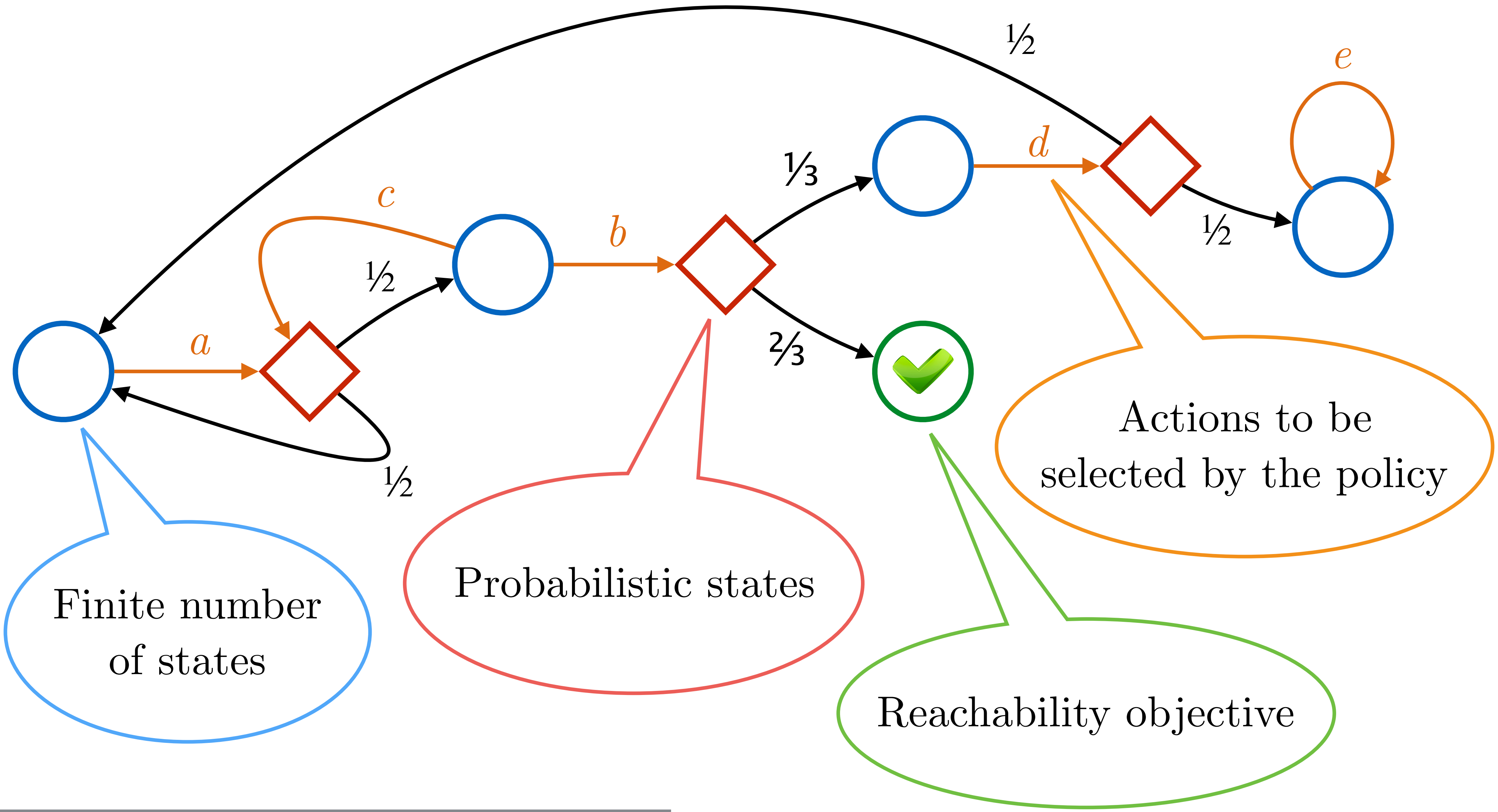

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Probability to reach: $\Pr_s^{\sigma}(\mathsf{F}$ ✔ $)$

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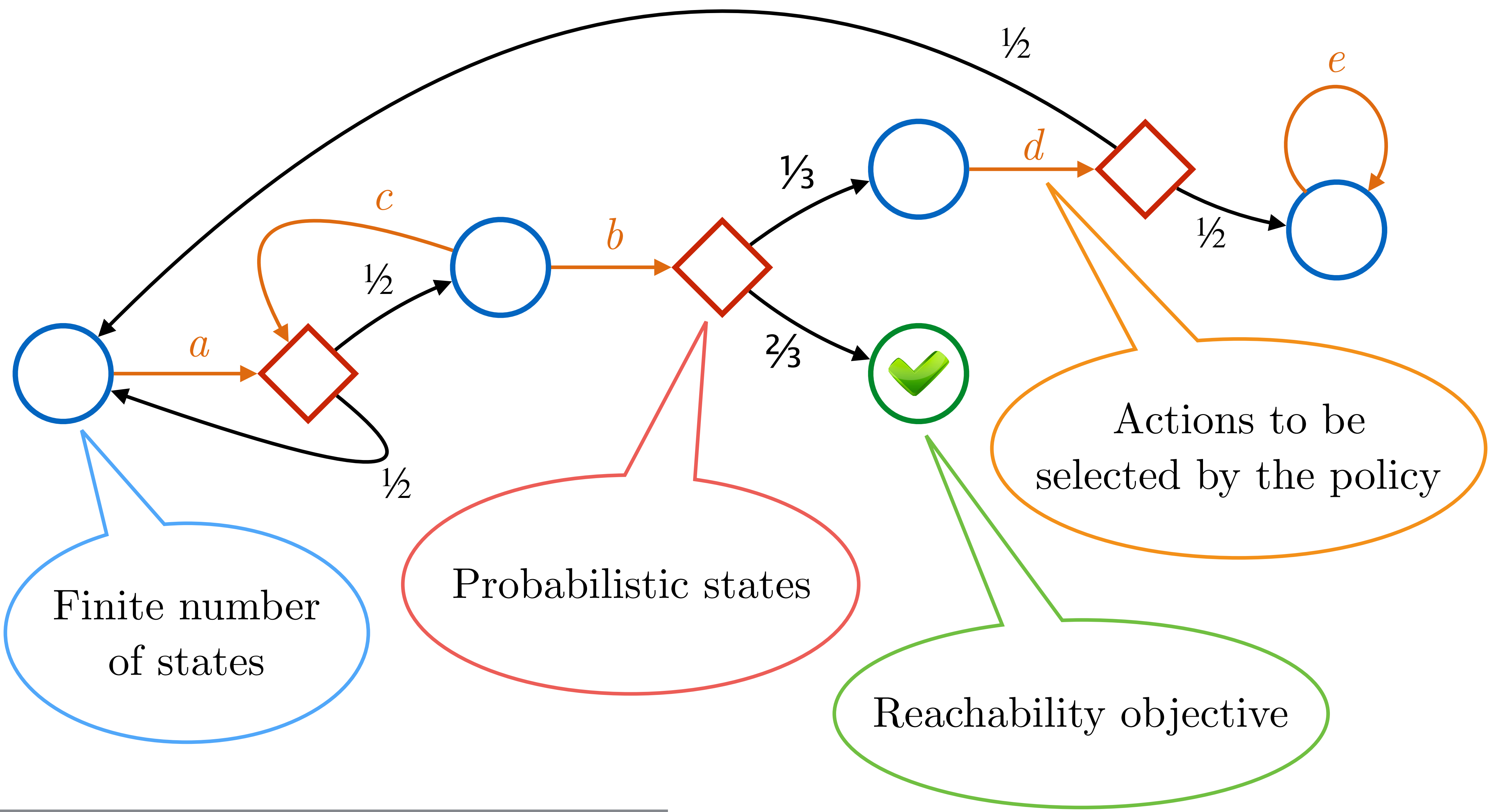

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Probability to reach: $\Pr_s^{\sigma}(\mathsf{F}\ ✓)$

Maximal probability to reach: $\Pr_s^{\max}(\mathsf{F}\ ✓) = \sup_{\sigma} \Pr_s^{\sigma}(\mathsf{F}\ ✓)$

Optimal reachability probabilities of MDPs

- How?
  - *Linear programming*
  - *Policy iteration*
  - *Value iteration*: numerical scheme that scales well and works in practice

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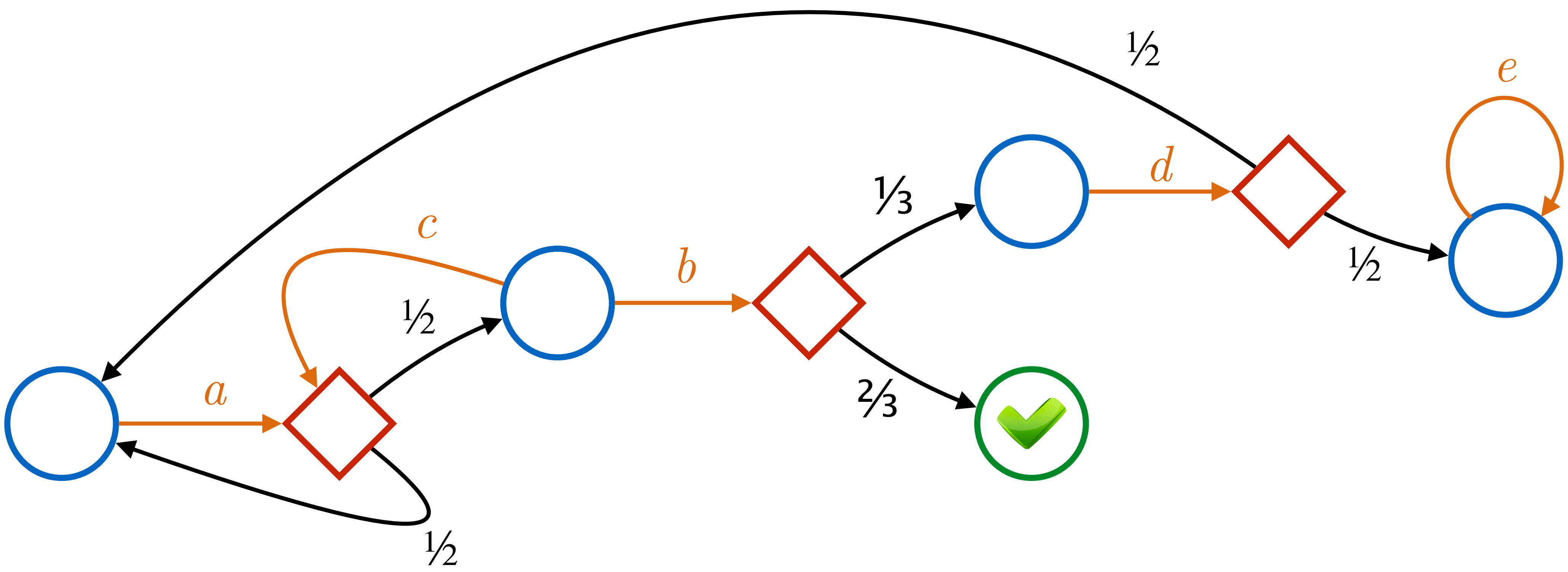
used in the numerical PRISM model checker
[Kwiatkowska, Norman, Parker, 2011]

Value iteration



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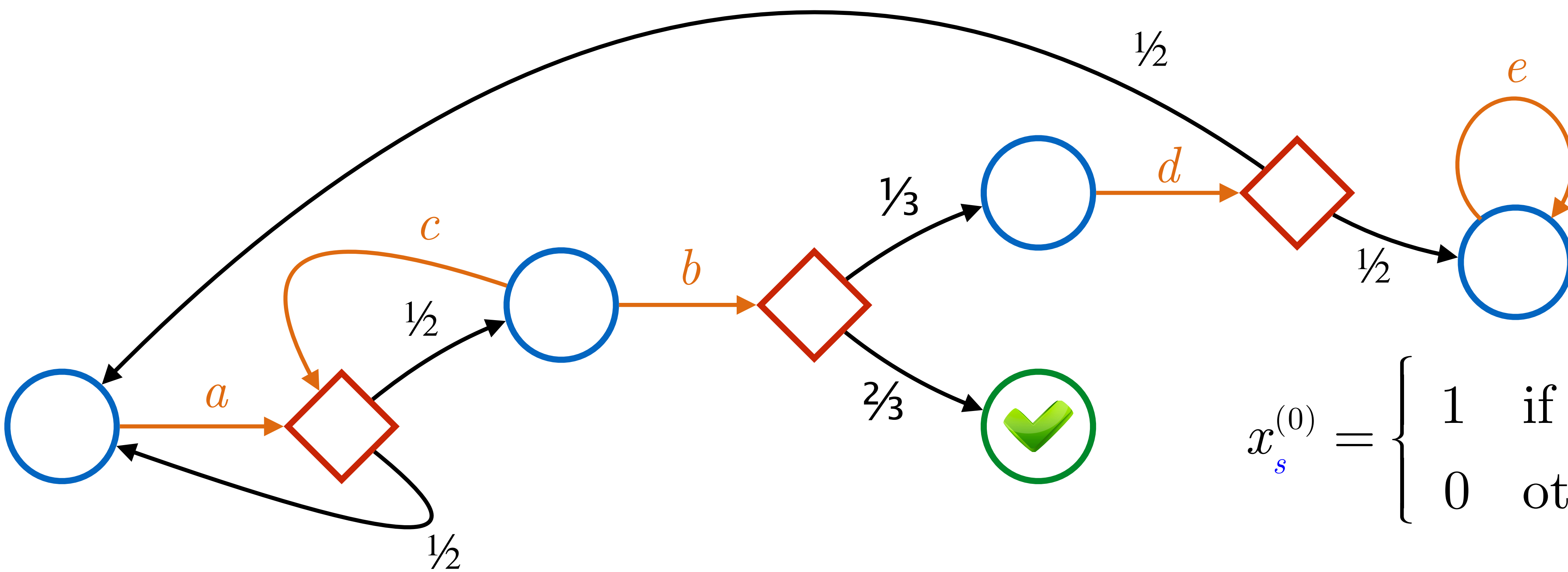

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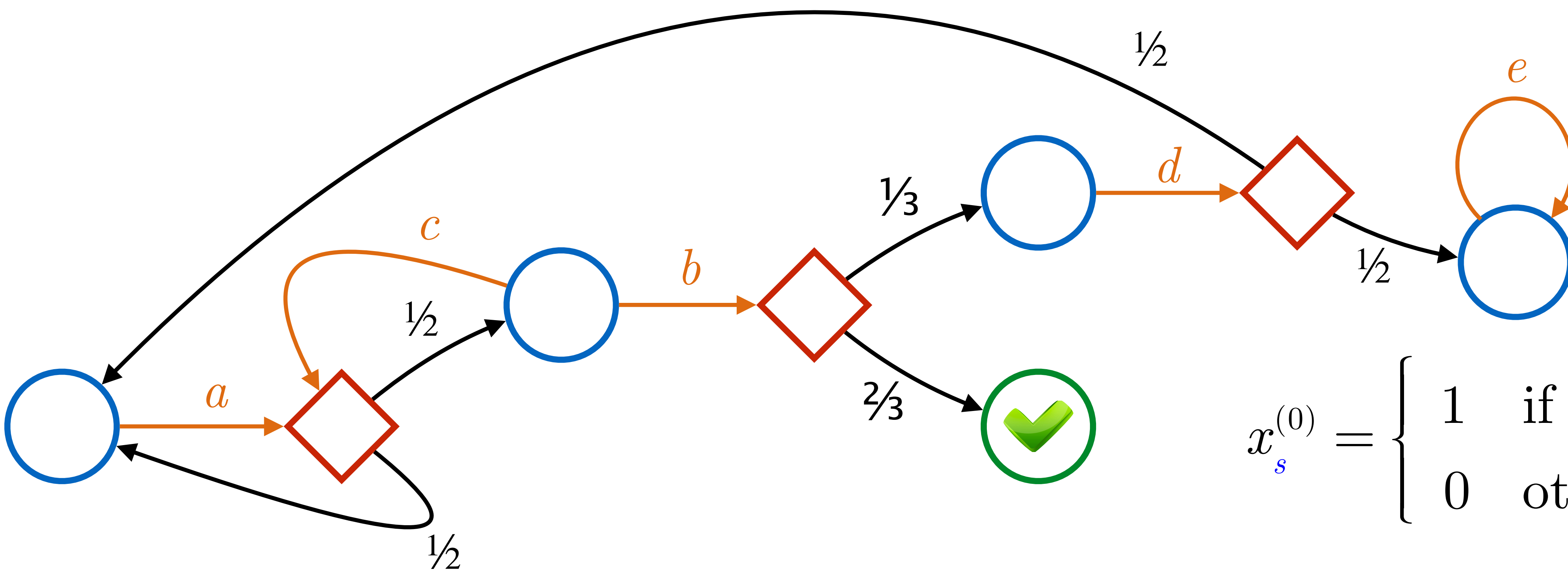
$$x_s^{(n+1)} = \max_{a \in \alpha} \sum_{s' \in S} \delta(s, a)(s') \times x_{s'}^{(n)}$$

Value iteration

0	0	0	0



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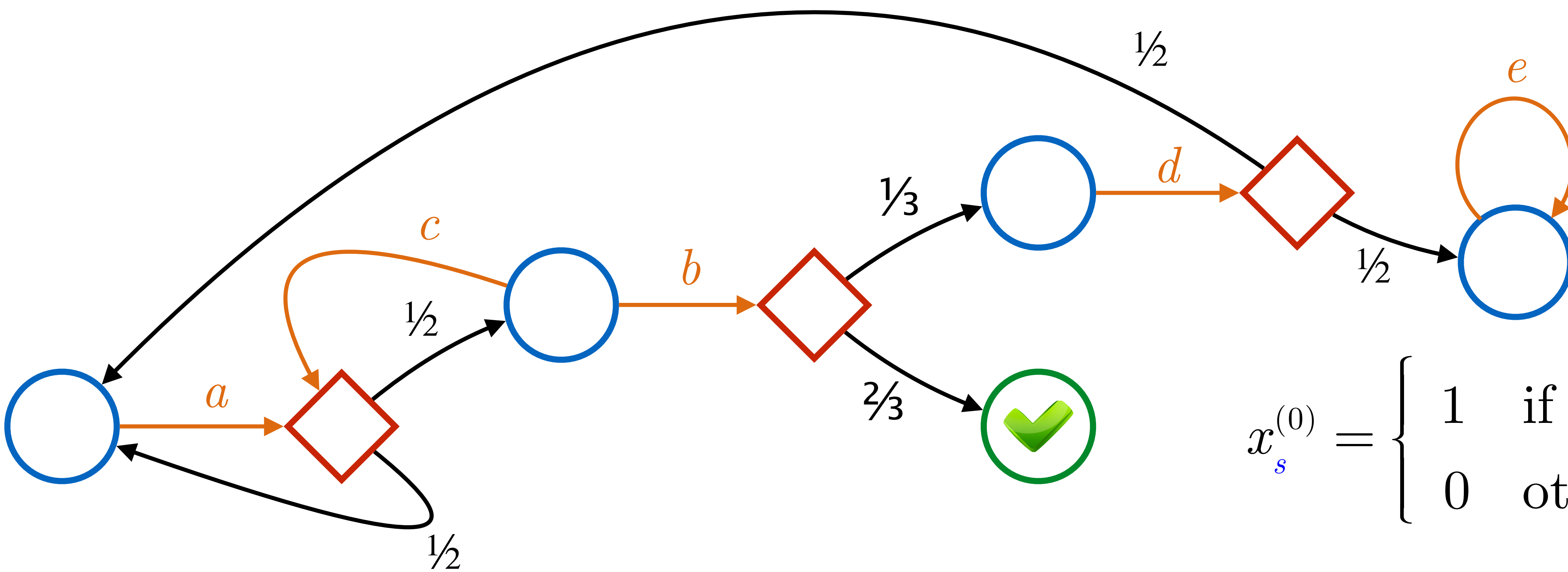
Value iteration

0	0	0	0
0	$2/3\ (b)$	0	0



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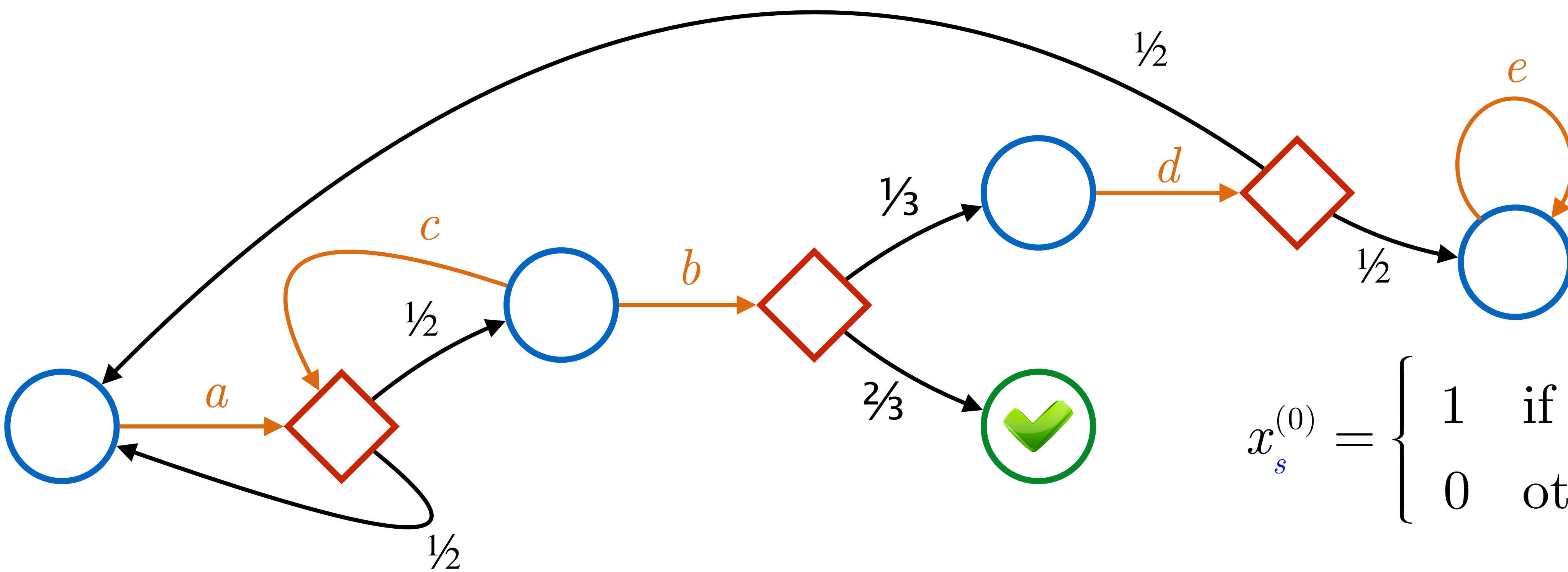
$1/3$	$2/3\ (b)$	0	0



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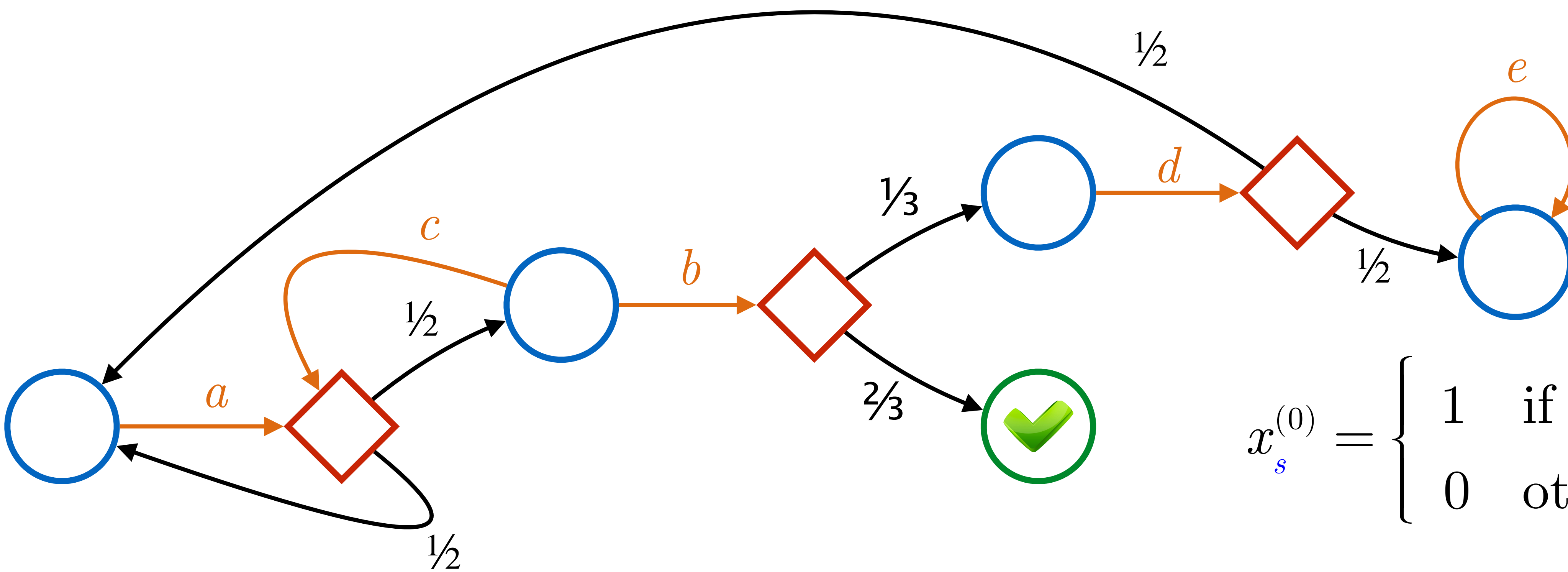
0	0	0	0
0	$2/3\ (b)$	0	0
$1/3$	$2/3\ (b)$	0	0
$1/2$	$2/3\ (b)$	$1/6$	0



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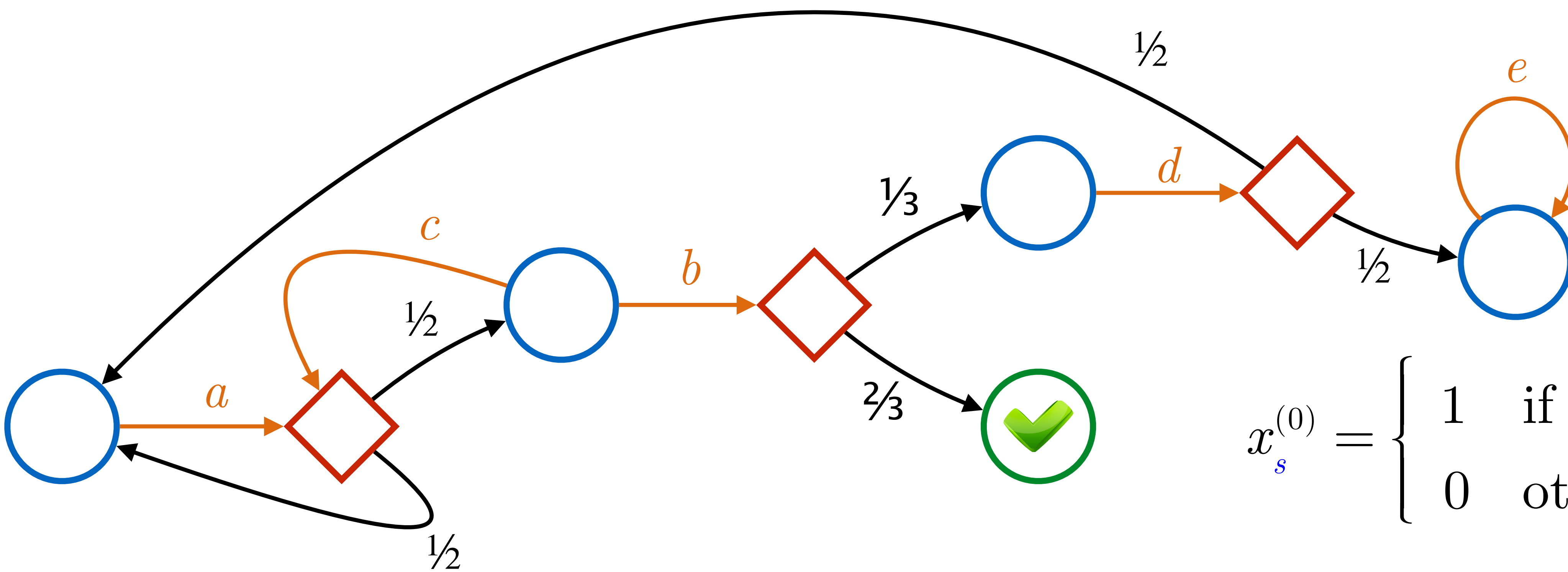
0	2/3 (b)	0	0
1/3	2/3 (b)	0	0
1/2	2/3 (b)	1/6	0
7/12	13/18 (b)	1/4	0



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1/3	2/3 (*b*)	0	0
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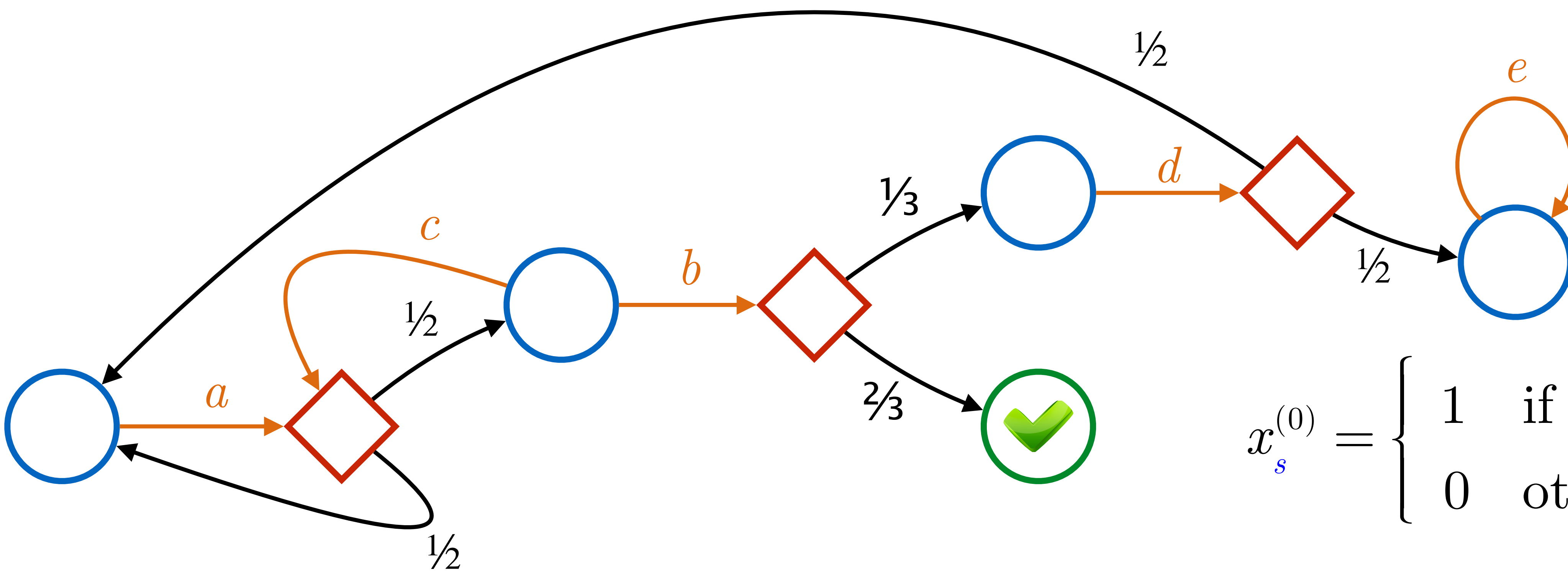
...	...	...	...



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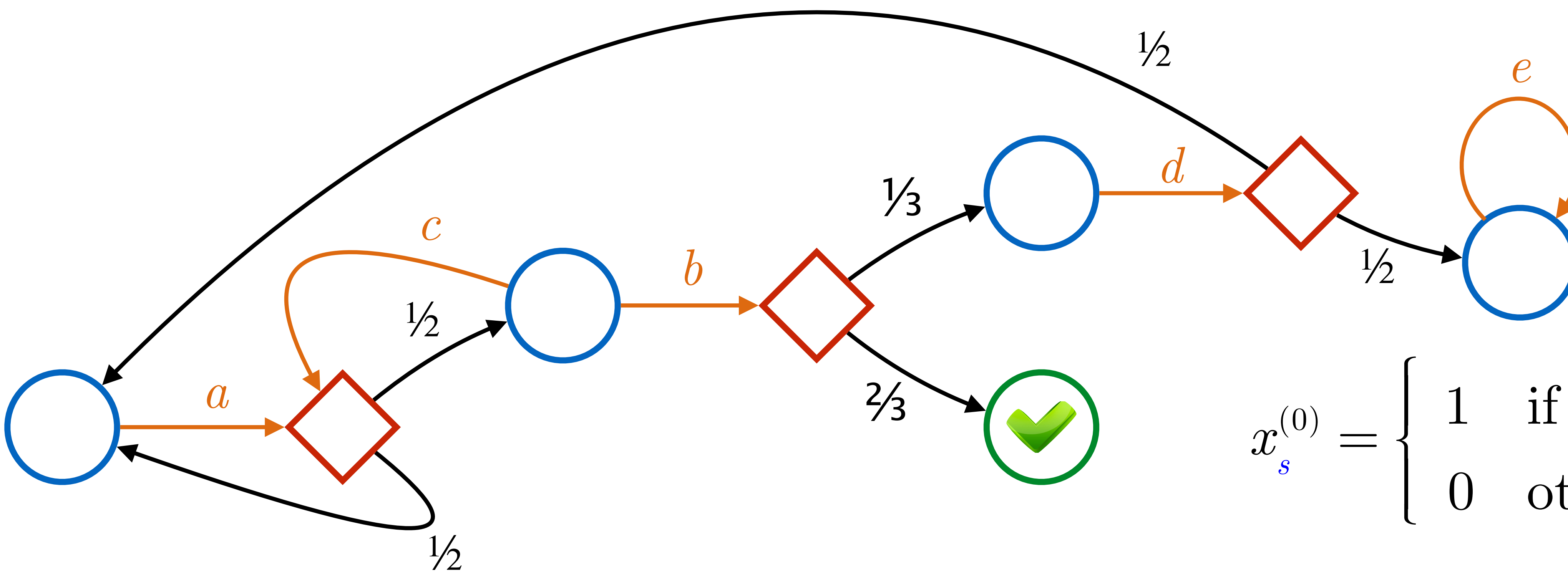
...	...	...	...
0.7969	0.7988 (b)	0.3977	0



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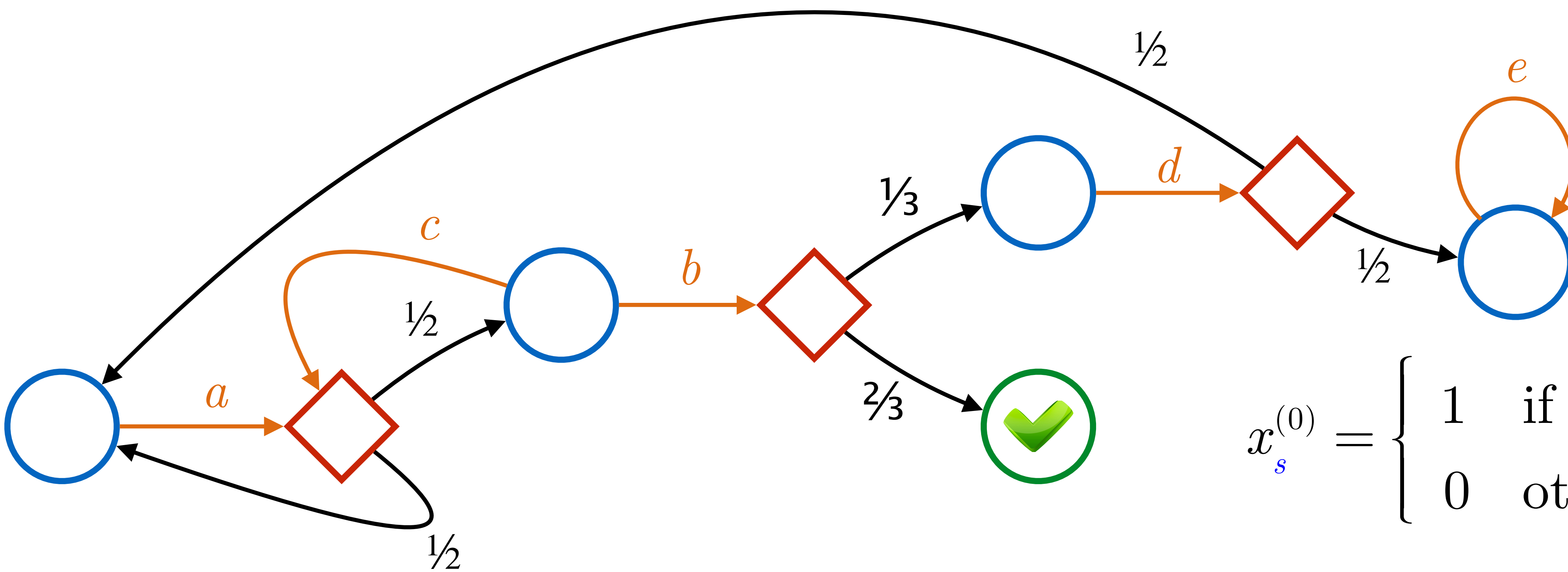
0.7978	$0.7992\ (b)$	0.3984	0



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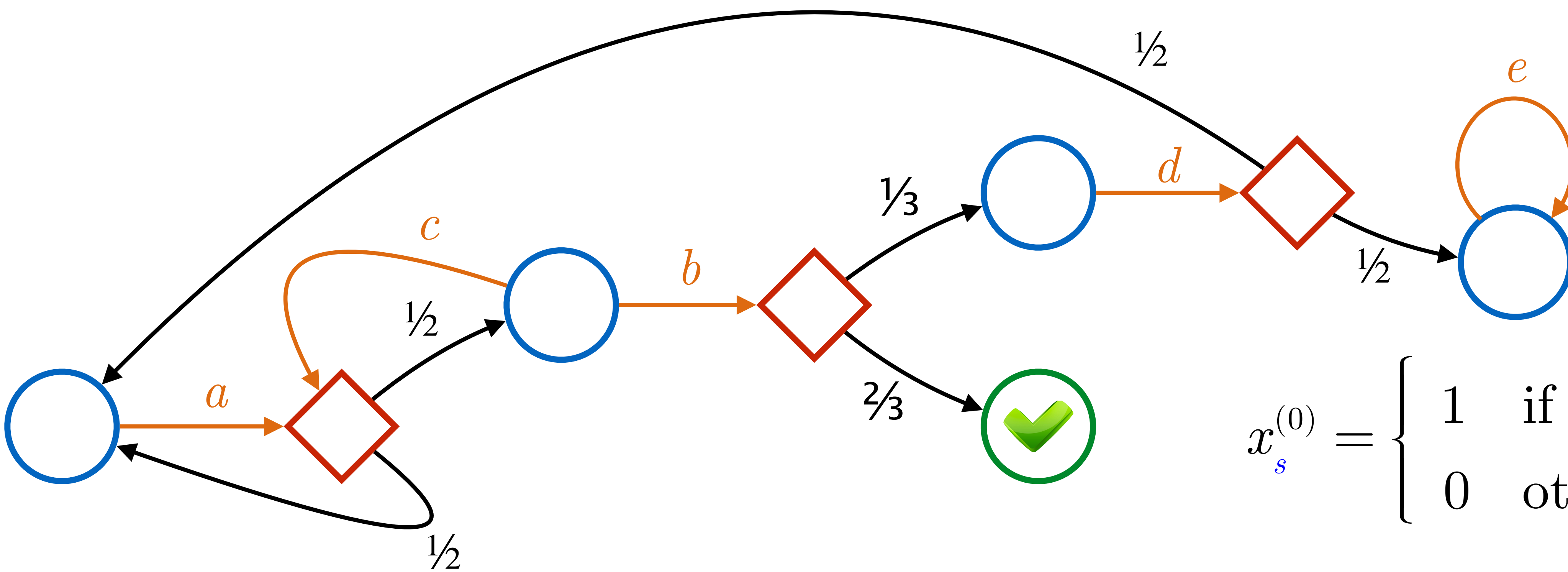
≤ 0.001



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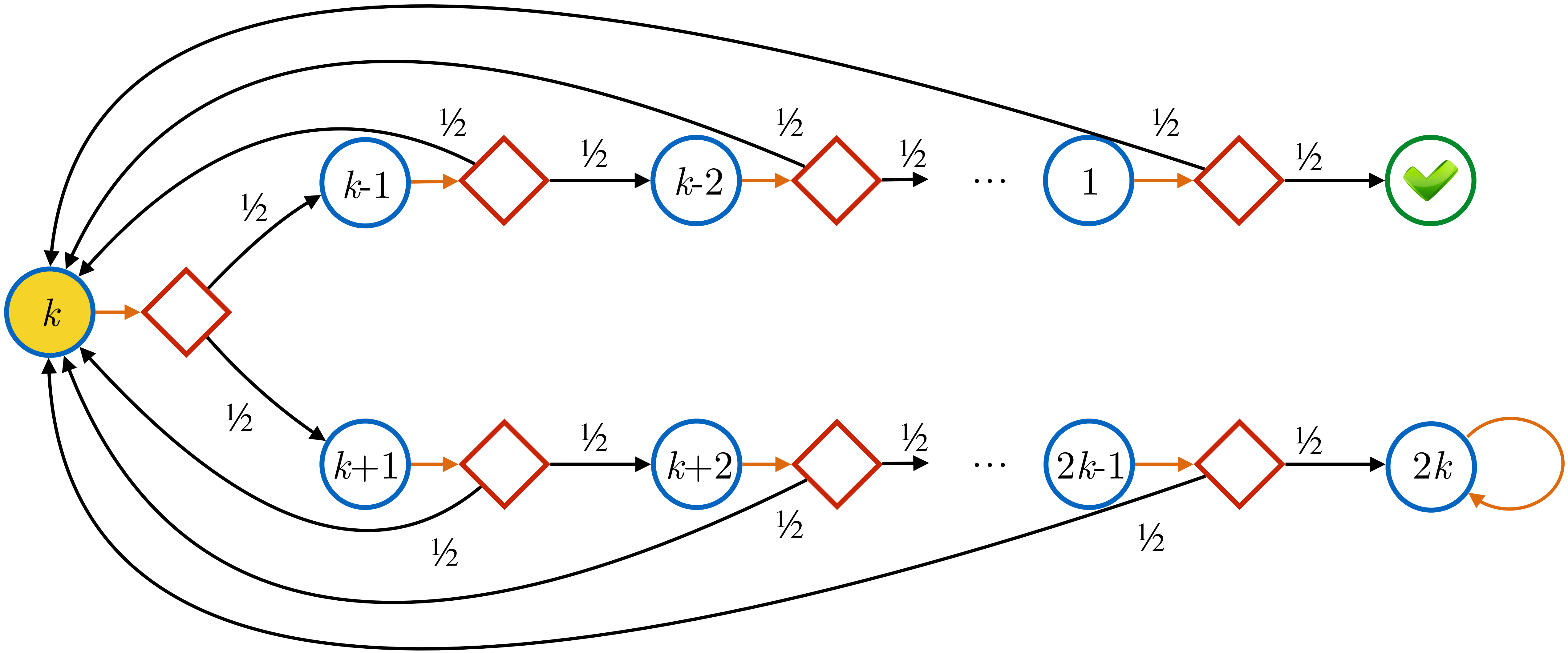
Value iteration: which guarantees?



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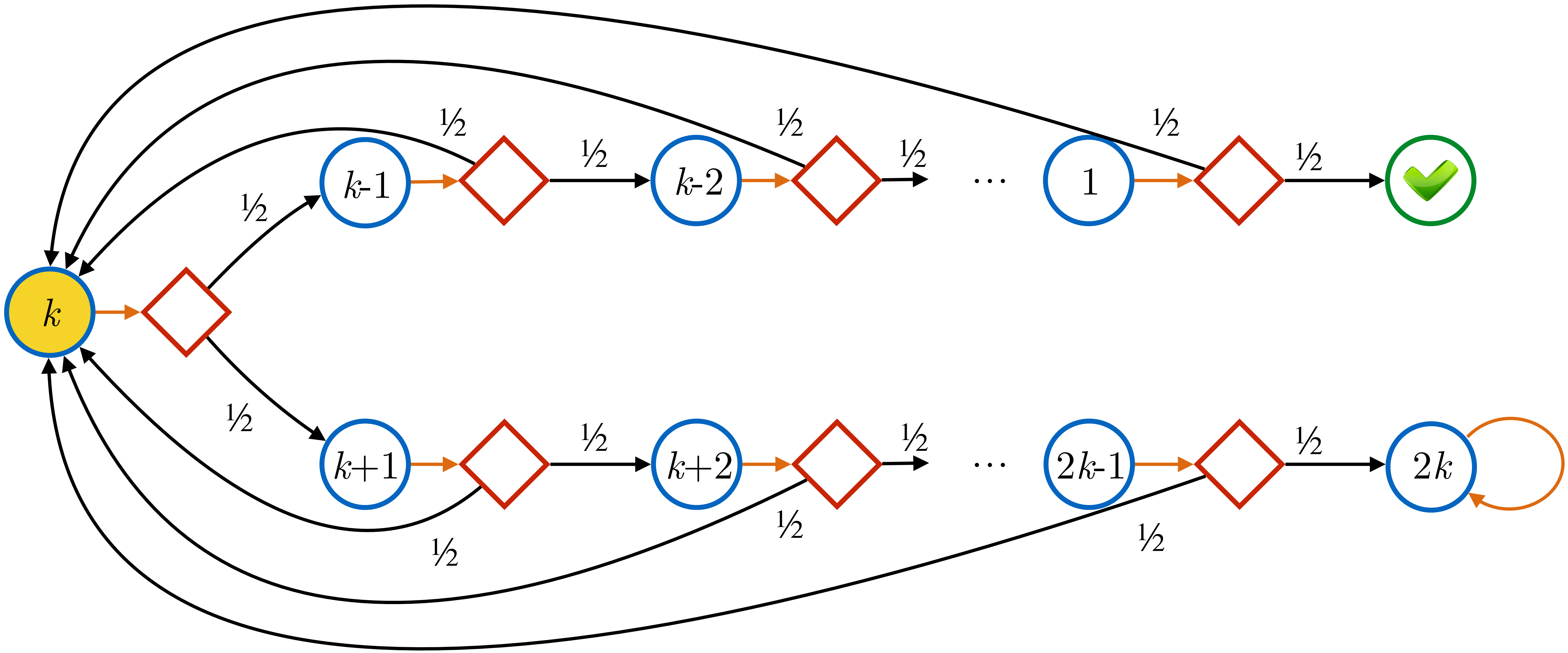

State	0	1	2	3	…	k-1	k	k+1	…	2k

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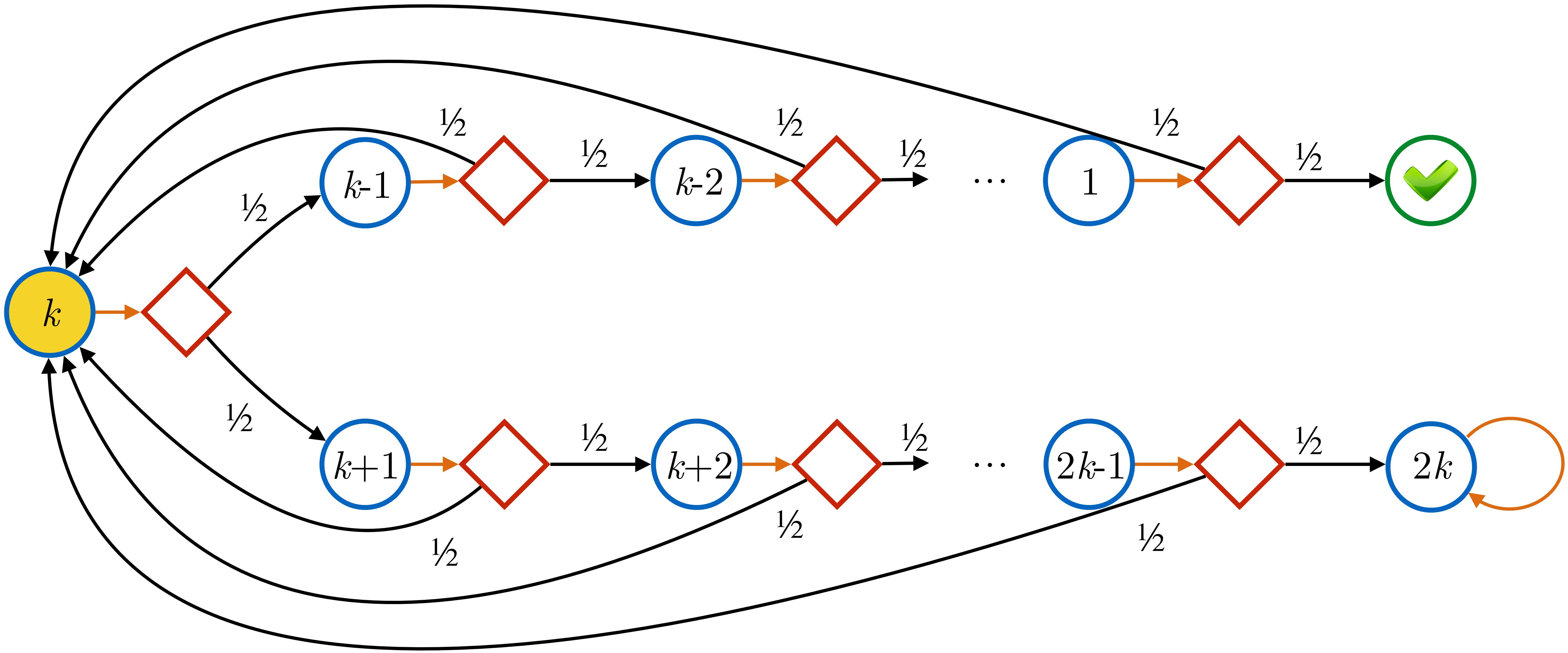

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Step 1	1	0	0	0	…	0	0	0	…	0

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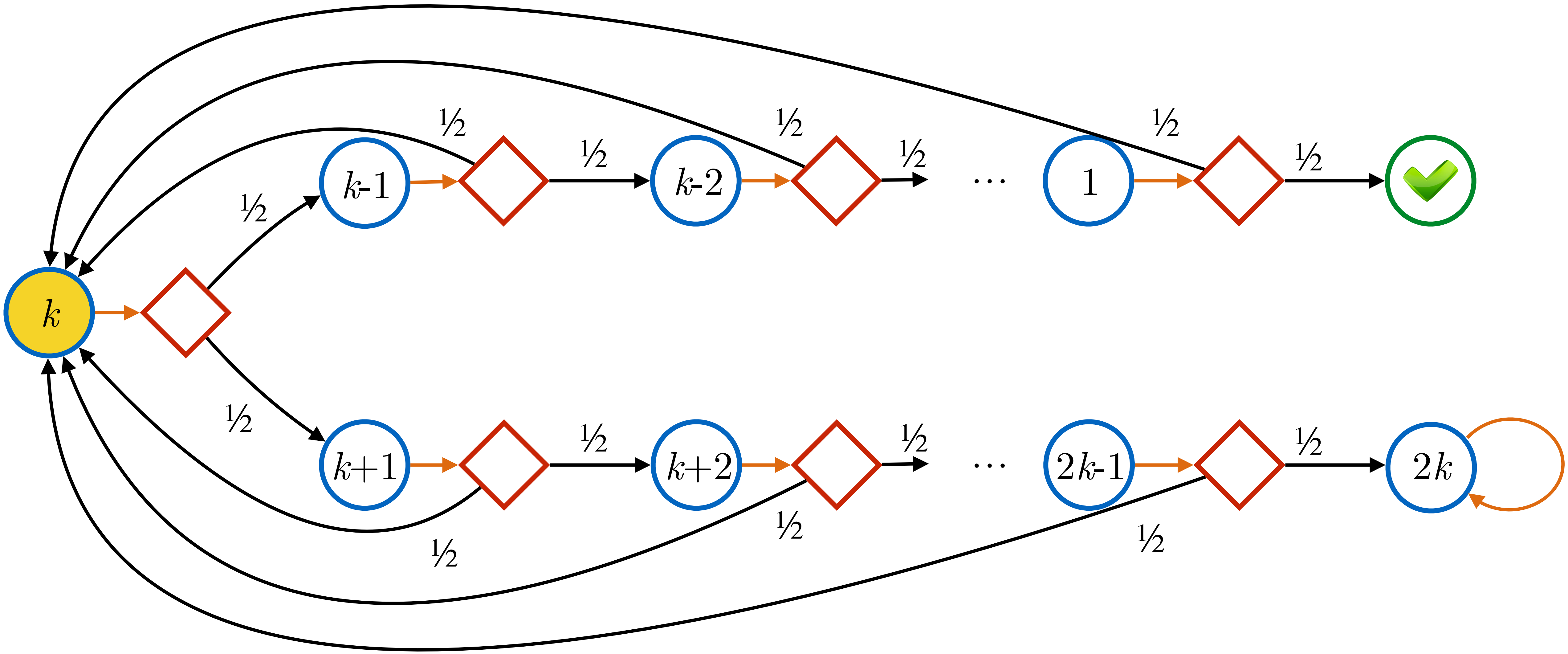

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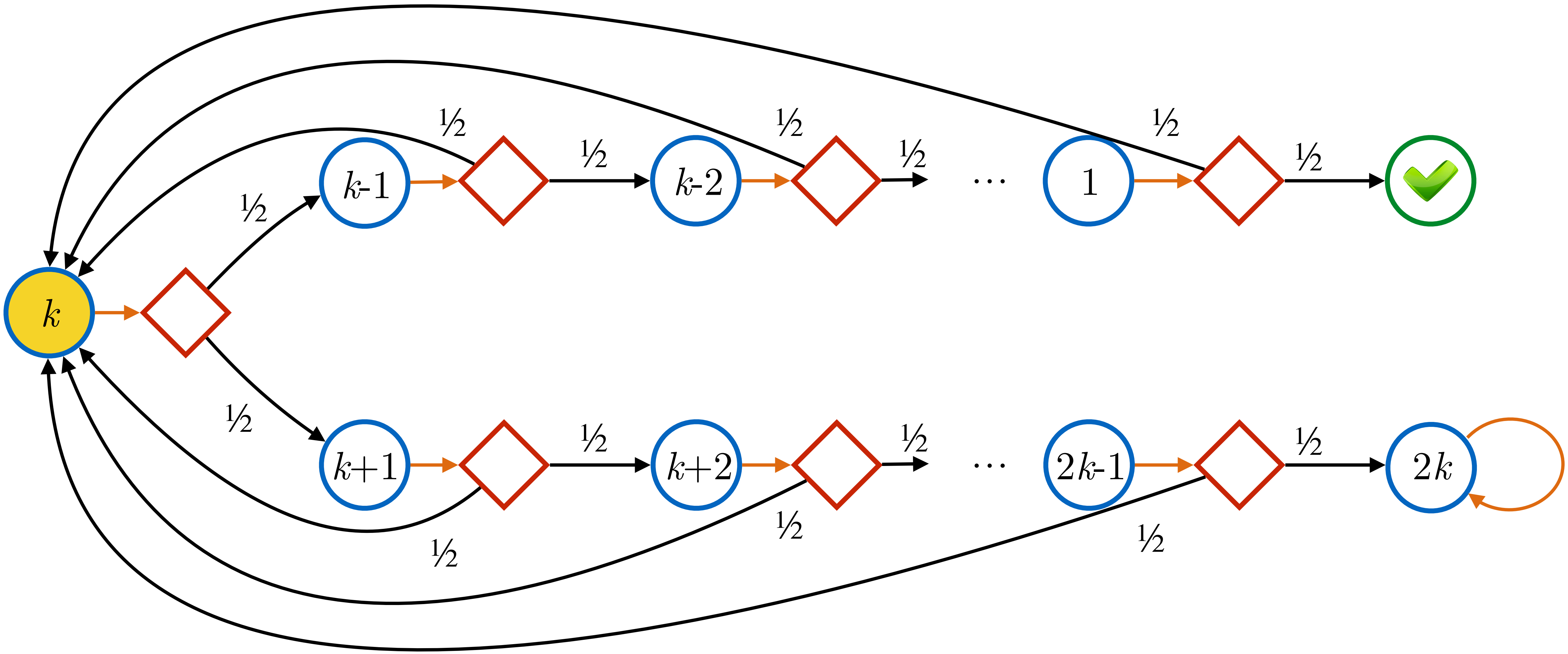
Step 1	1	0	0	0	…	0	0	0	…	0
Step 2	1	1/2	0	0	…	0	0	0	…	0

Value iteration: which guarantees?



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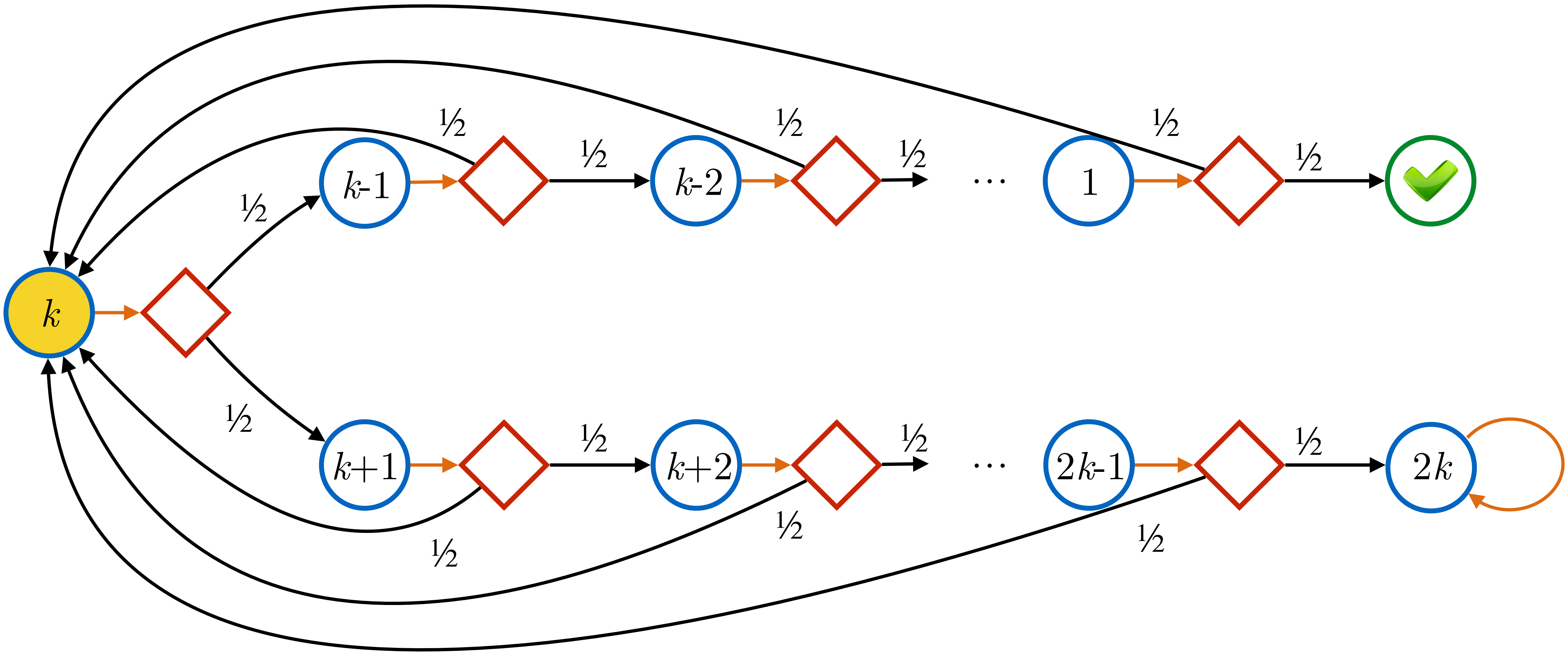

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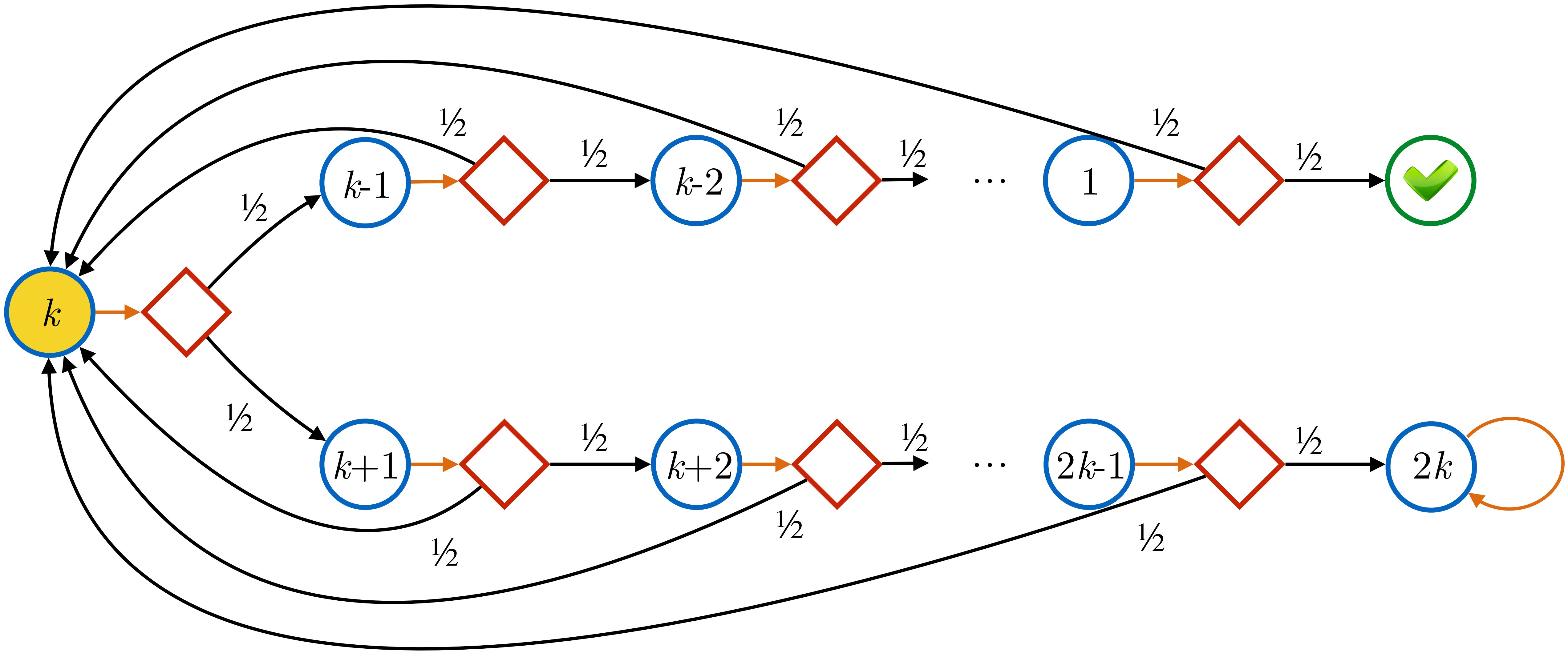

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…	…	…	…	…	…	…	…	…	…	…
Step k	1	1/2	1/4	1/8	…	$1/2^{k-1}$	0	0	…	0

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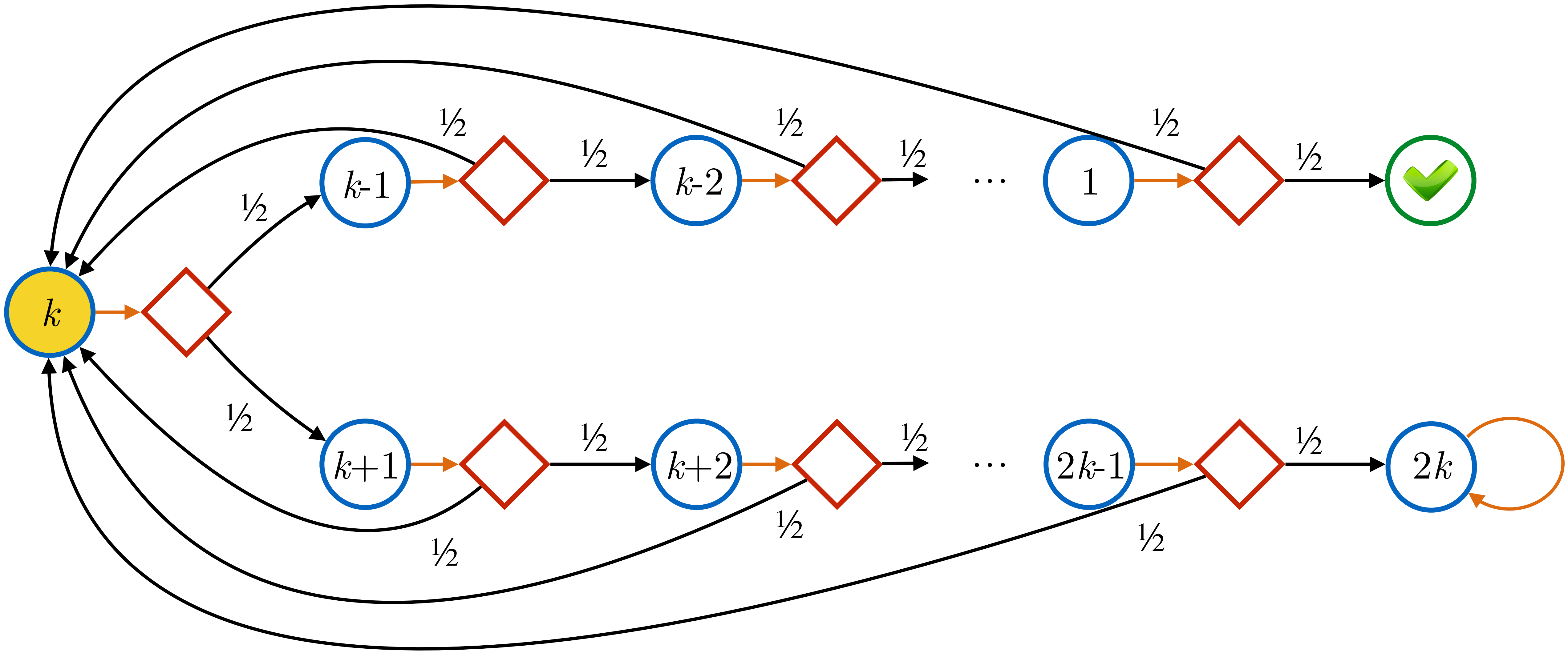

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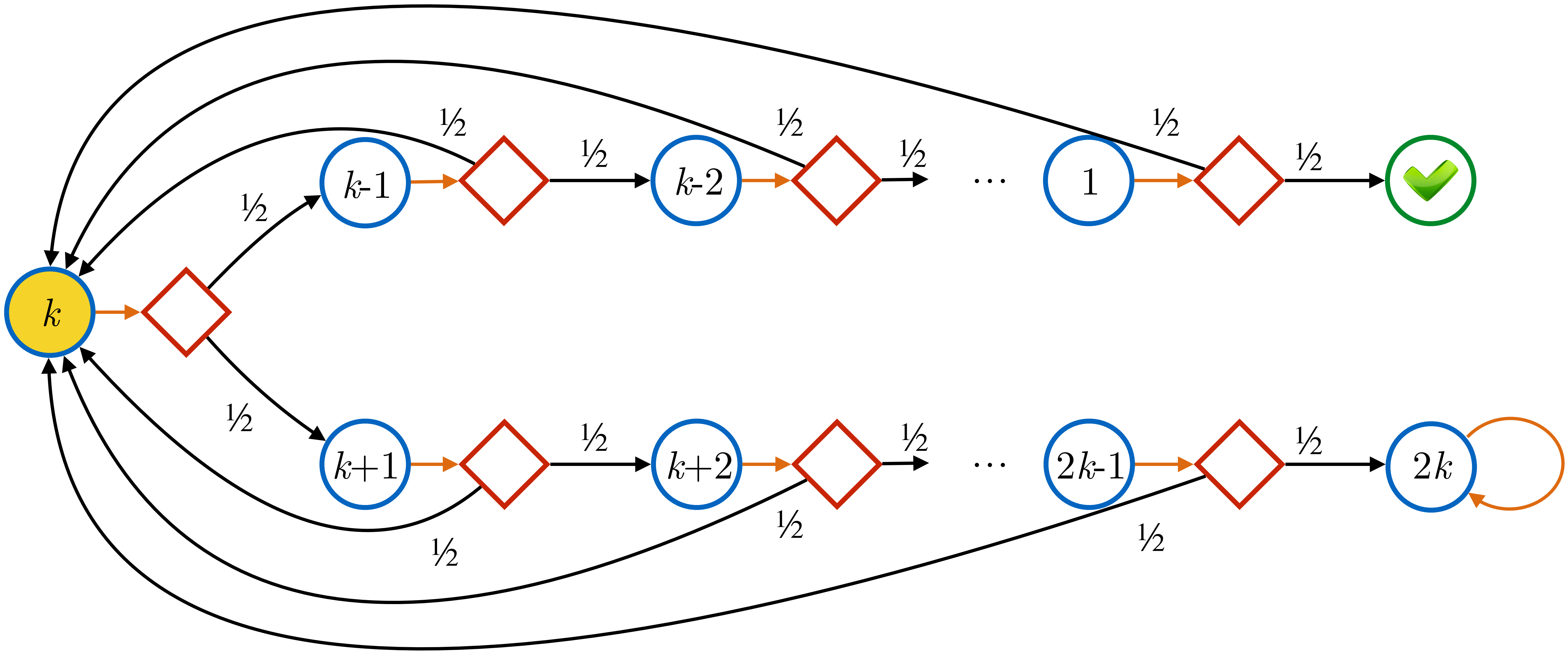

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$\leq 1/2^k$

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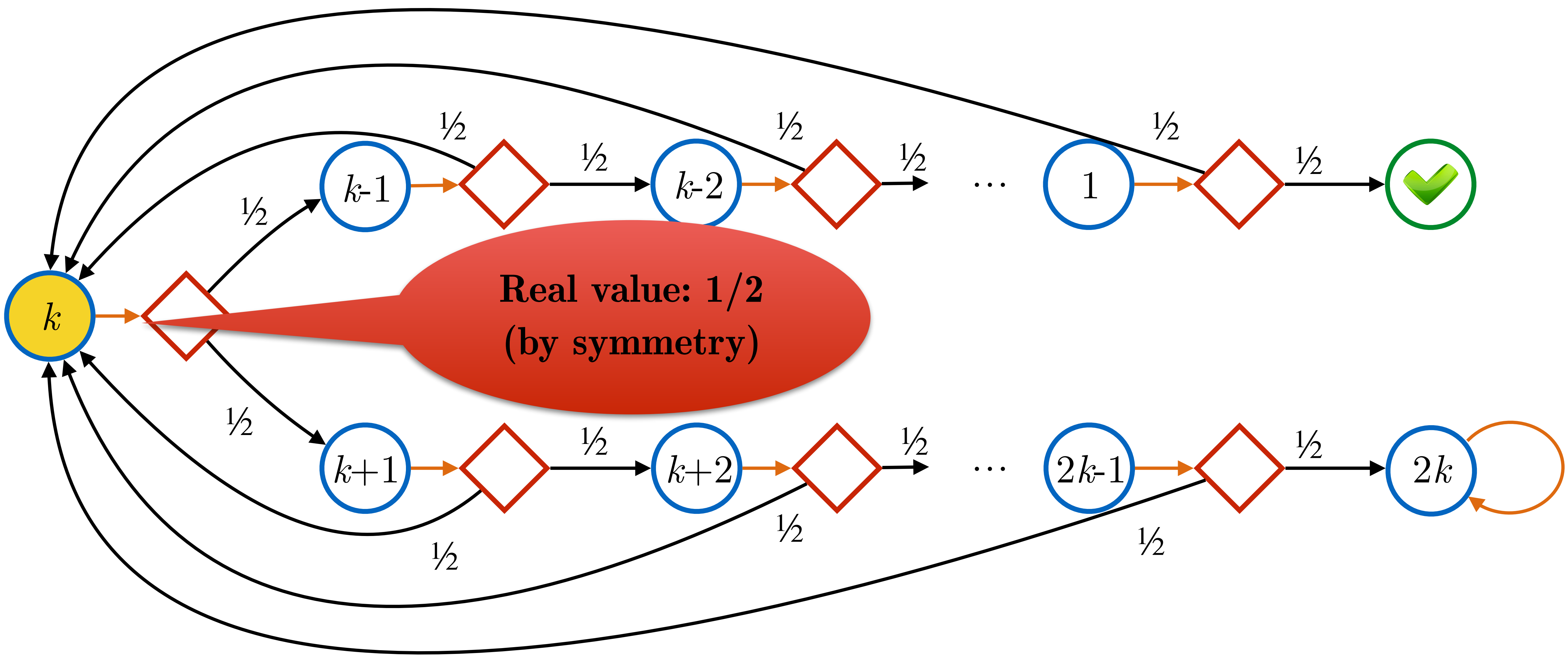

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…	…	…	…	…	…	…	…	…	…	…
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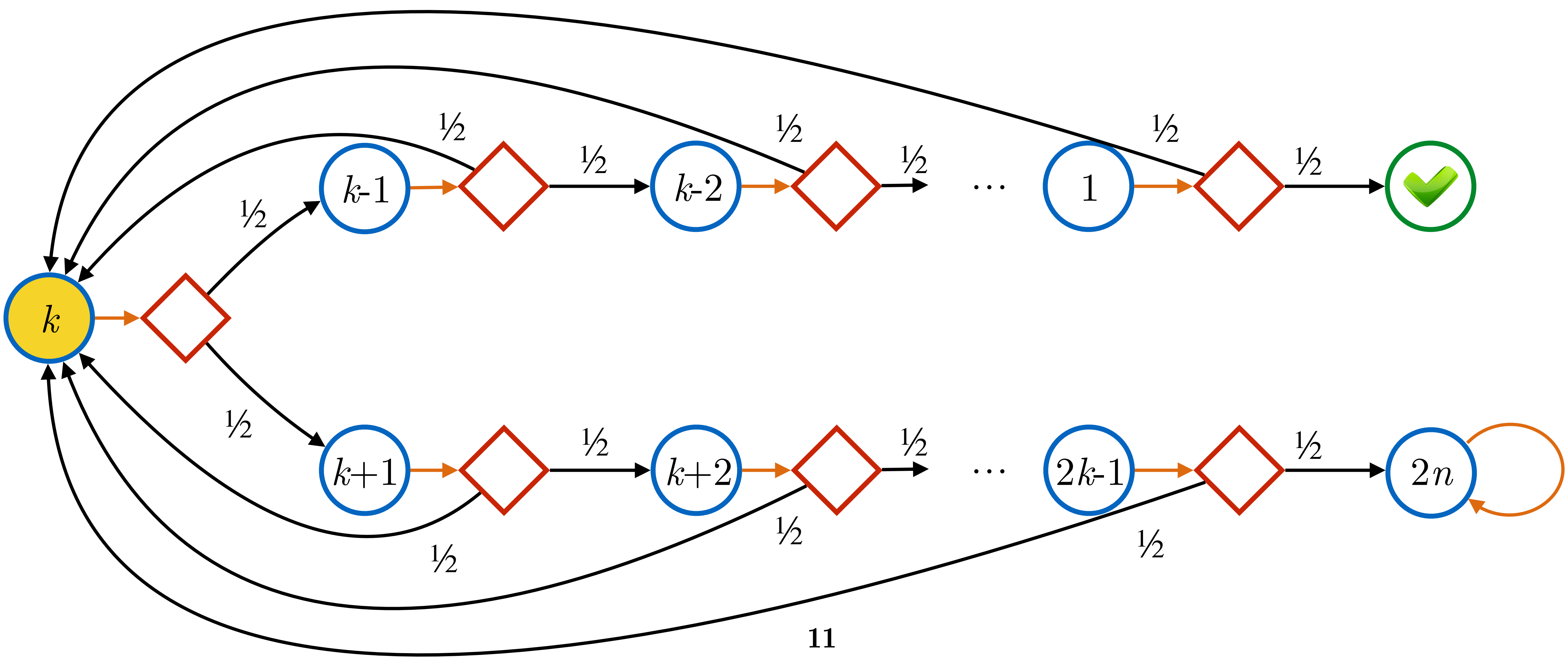
2. Study of the **speed of convergence**
    - also applies to classical value iteration
3. Improved **rounding** procedure for **exact** computation

Interval iteration

$$x_s^{(0)} = \begin{cases} 1 & \text{if } s = \text{✔} \\ 0 & \text{otherwise} \end{cases}$$

$$x_s^{(n+1)} = \max_{a \in \alpha} \sum_{s' \in S} \delta(s,a)(s') \times x_{s'}^{(n)}$$



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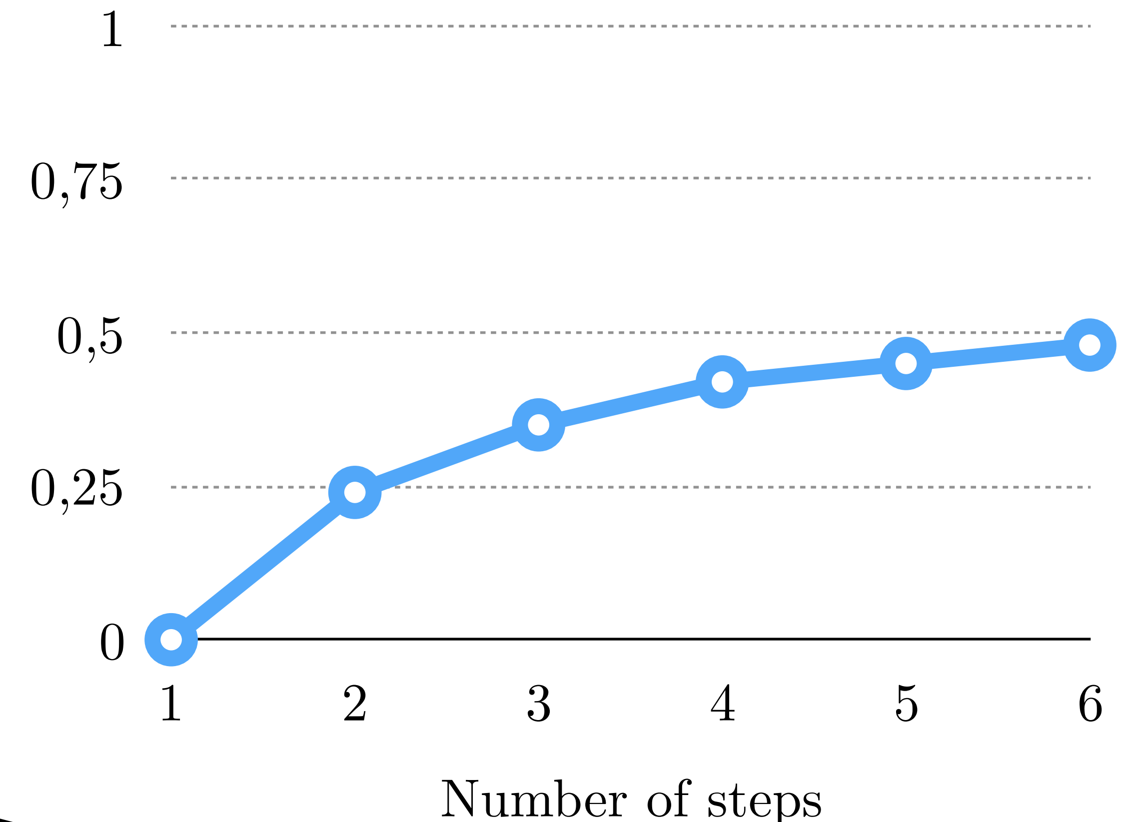

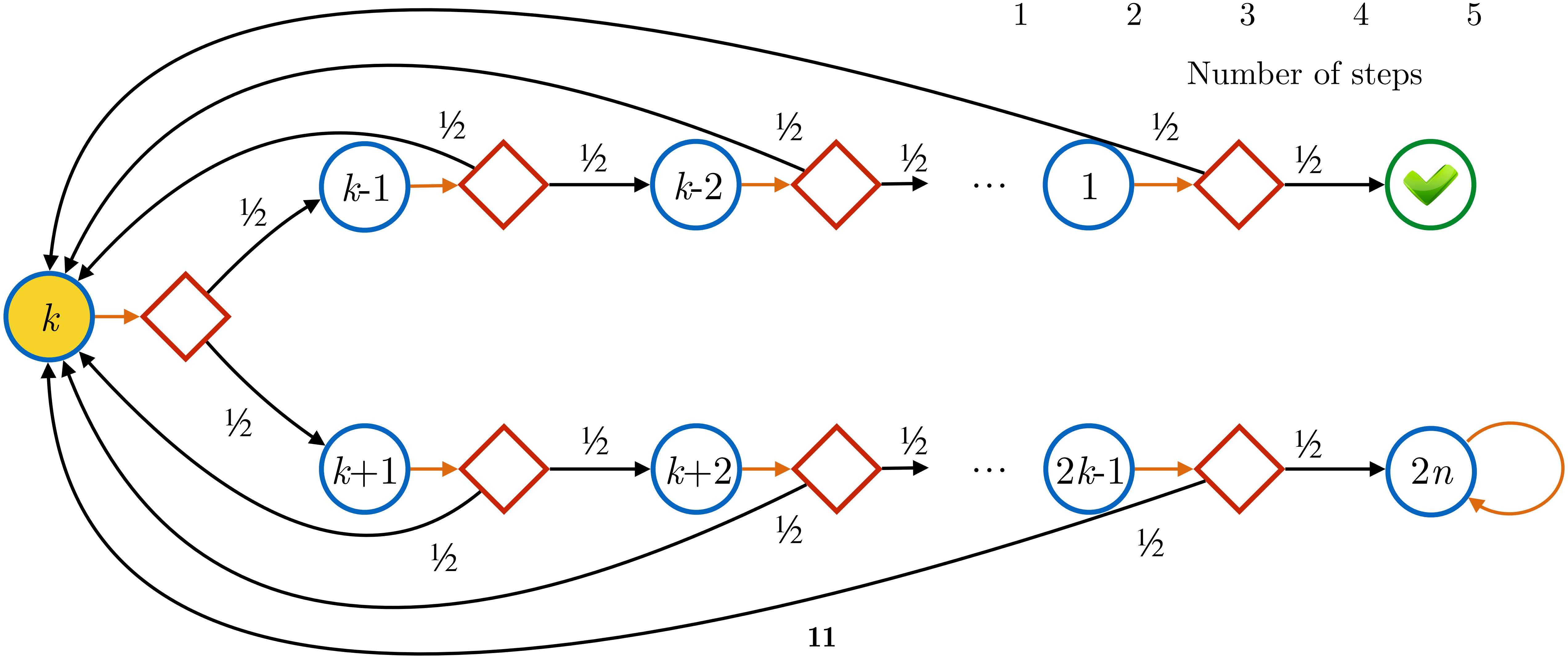

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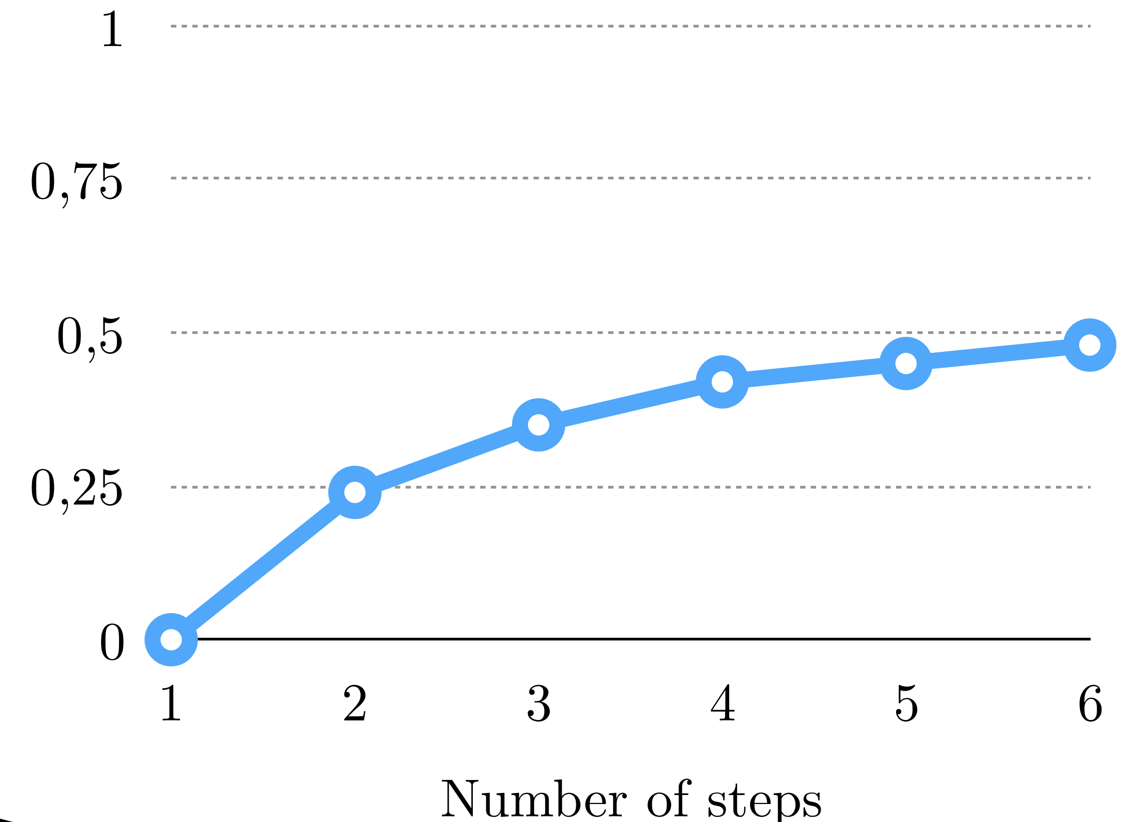

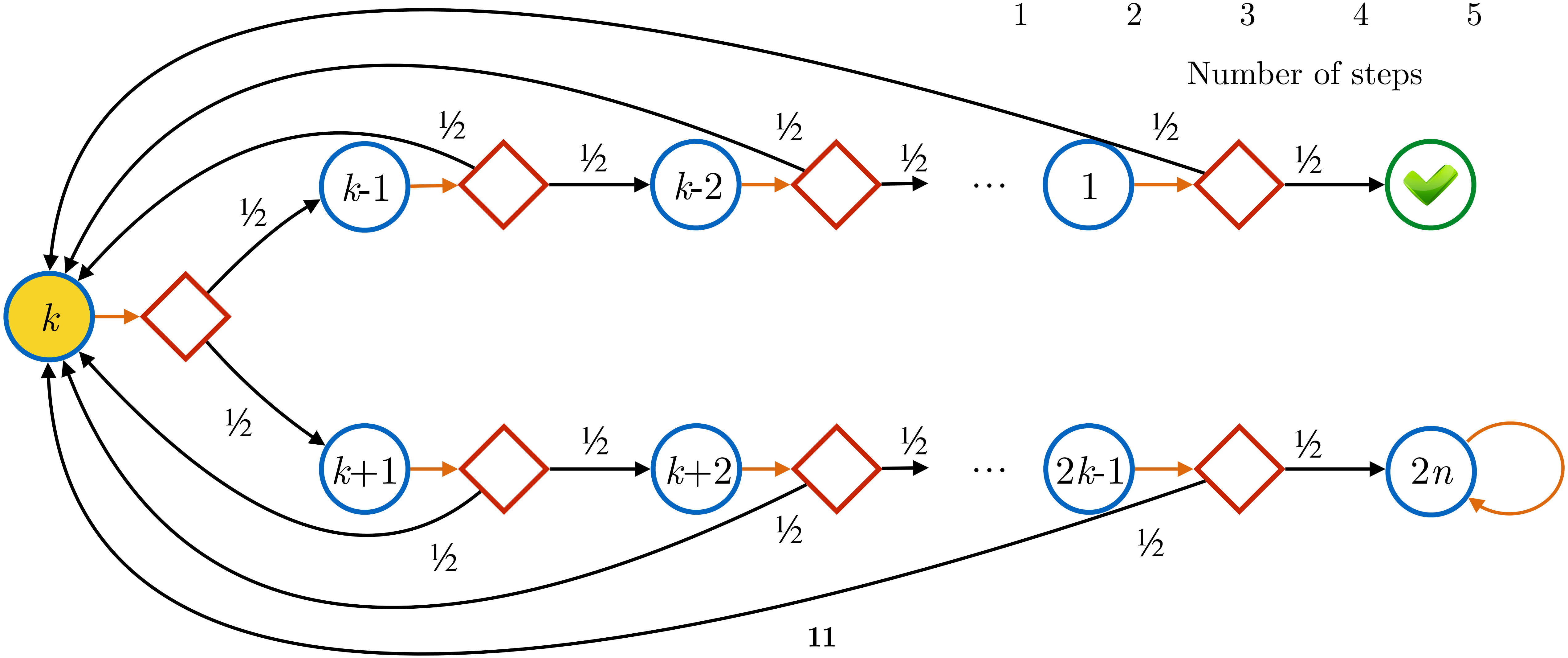

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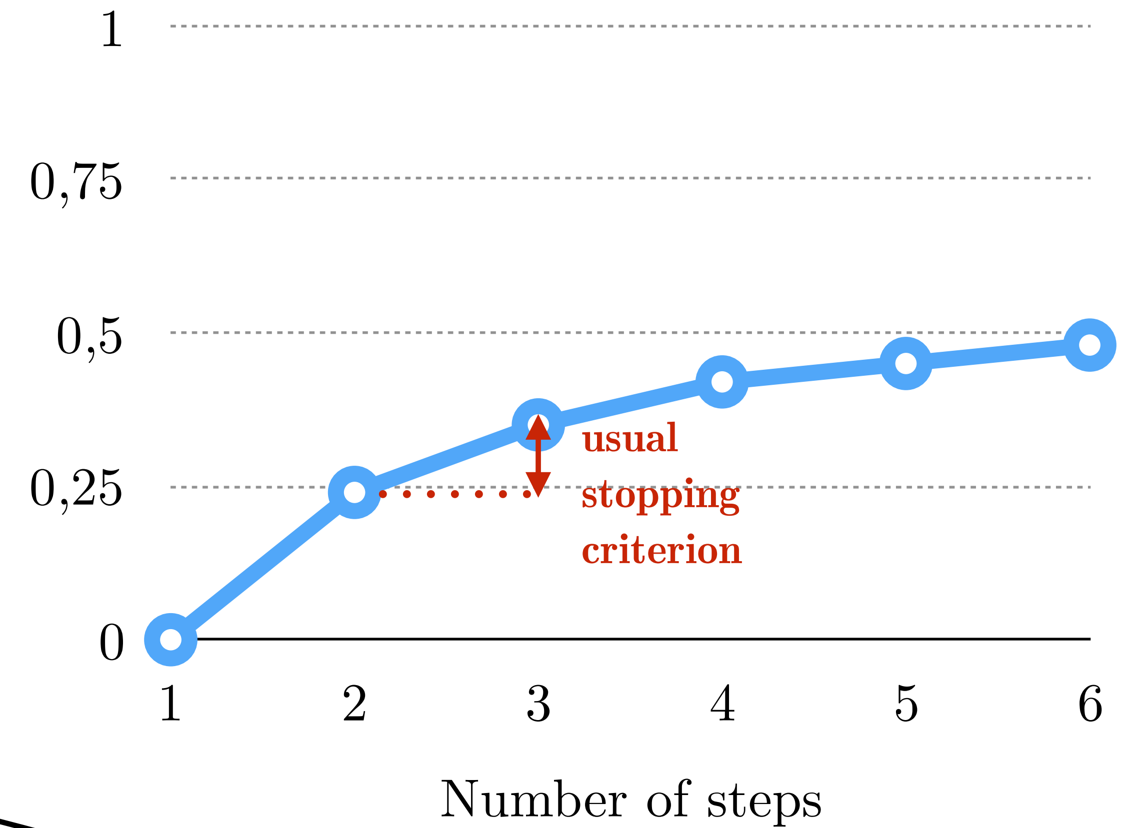

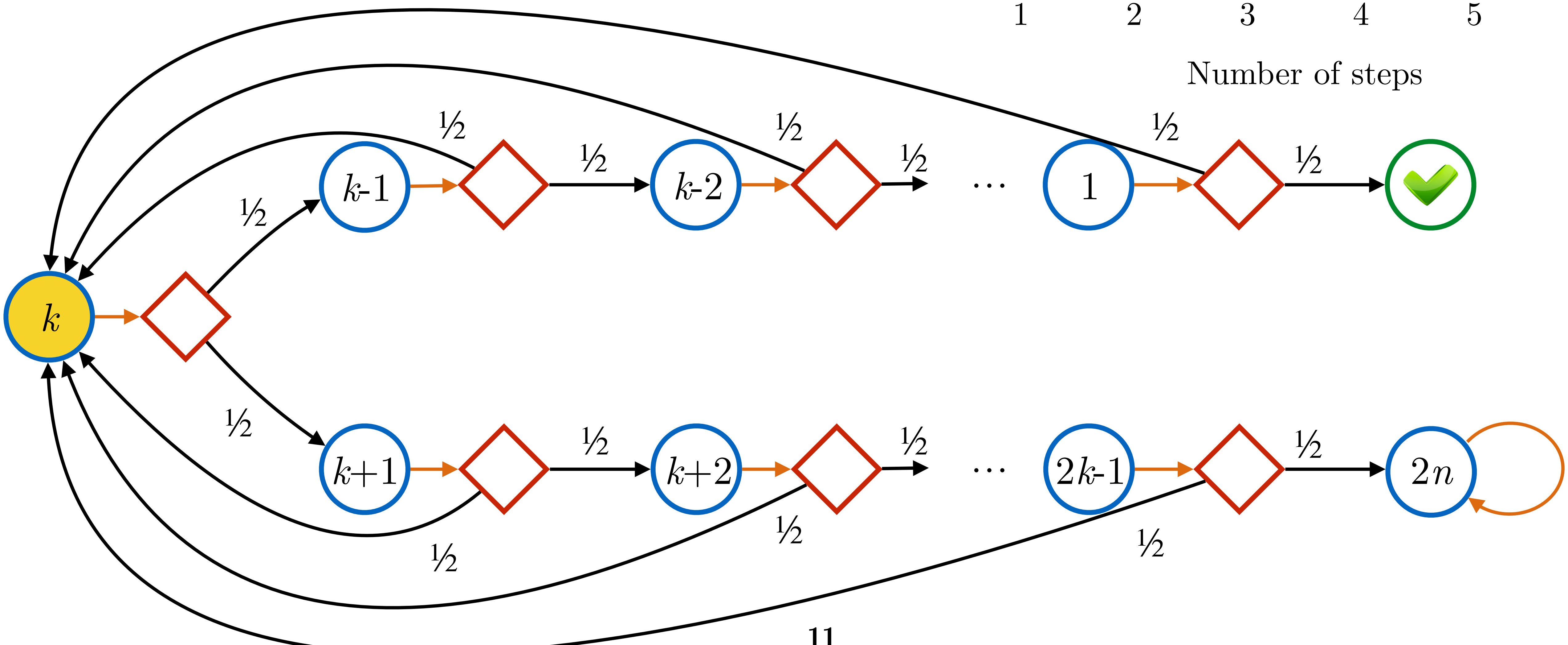

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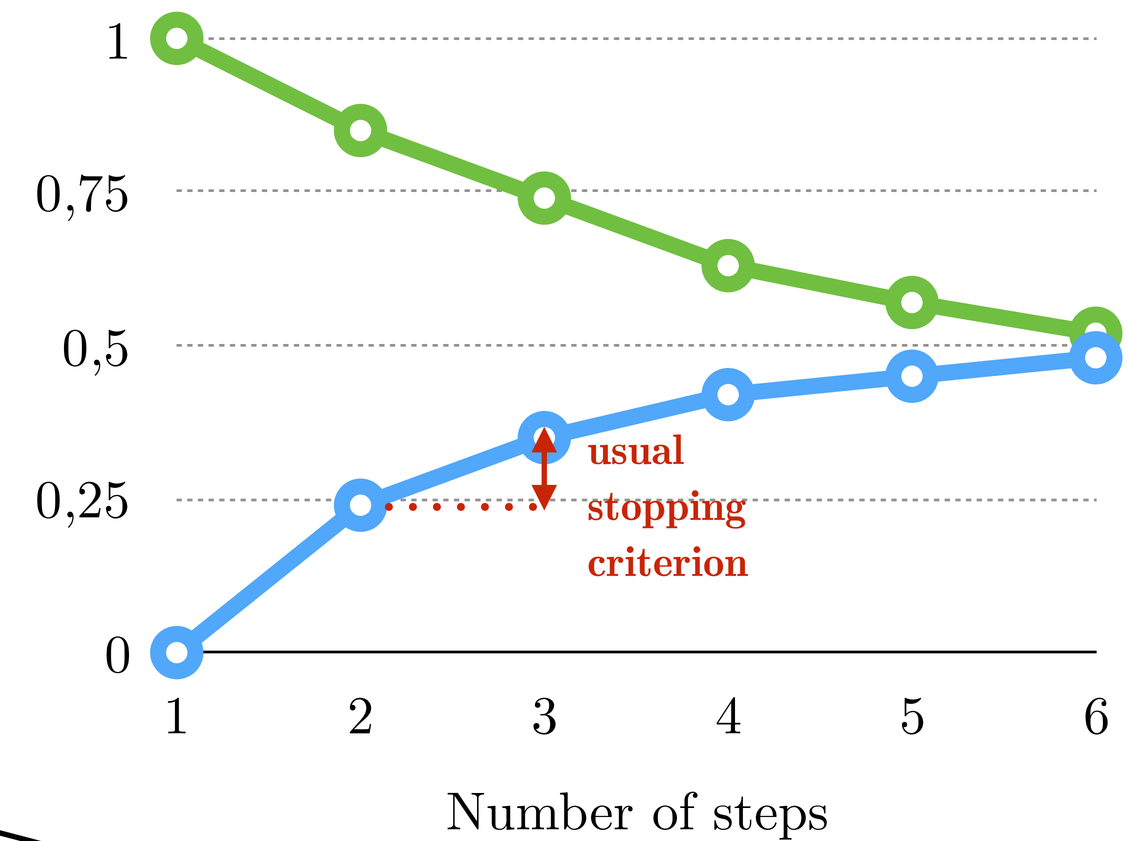

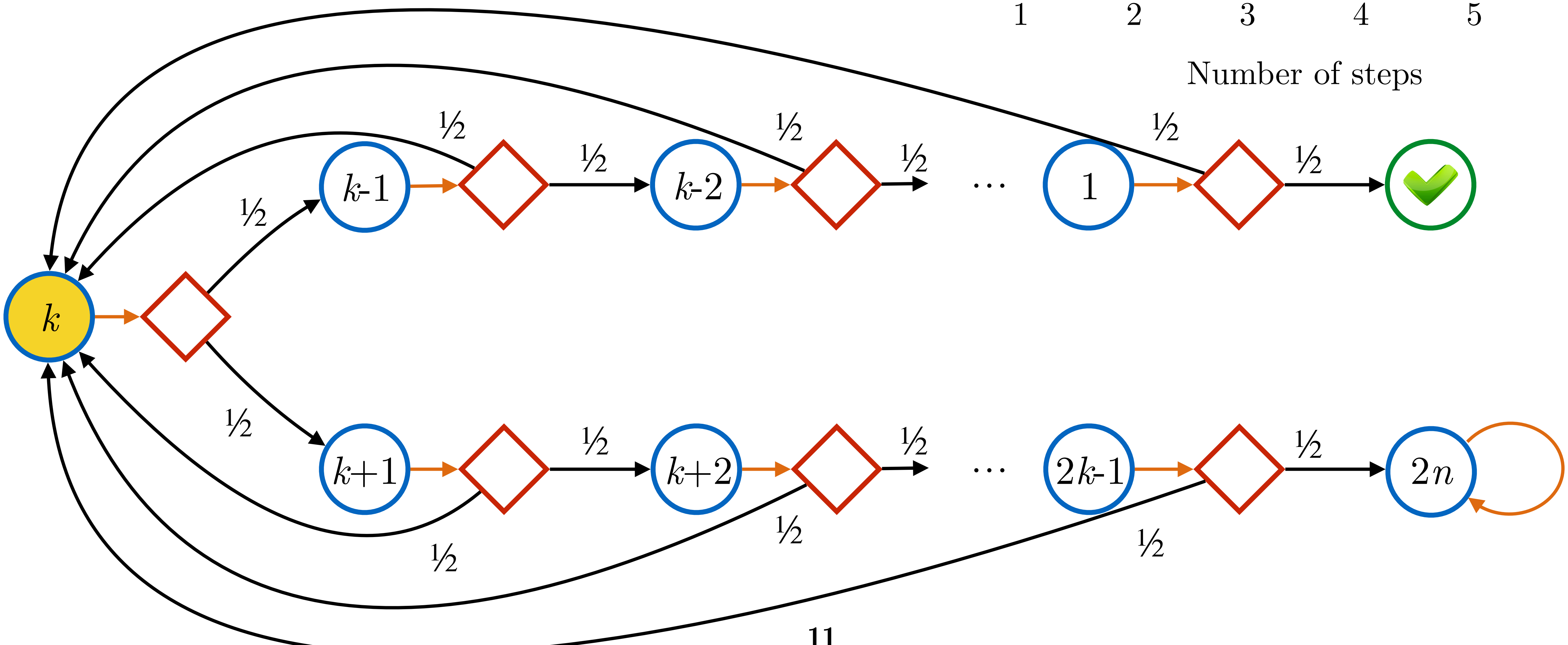

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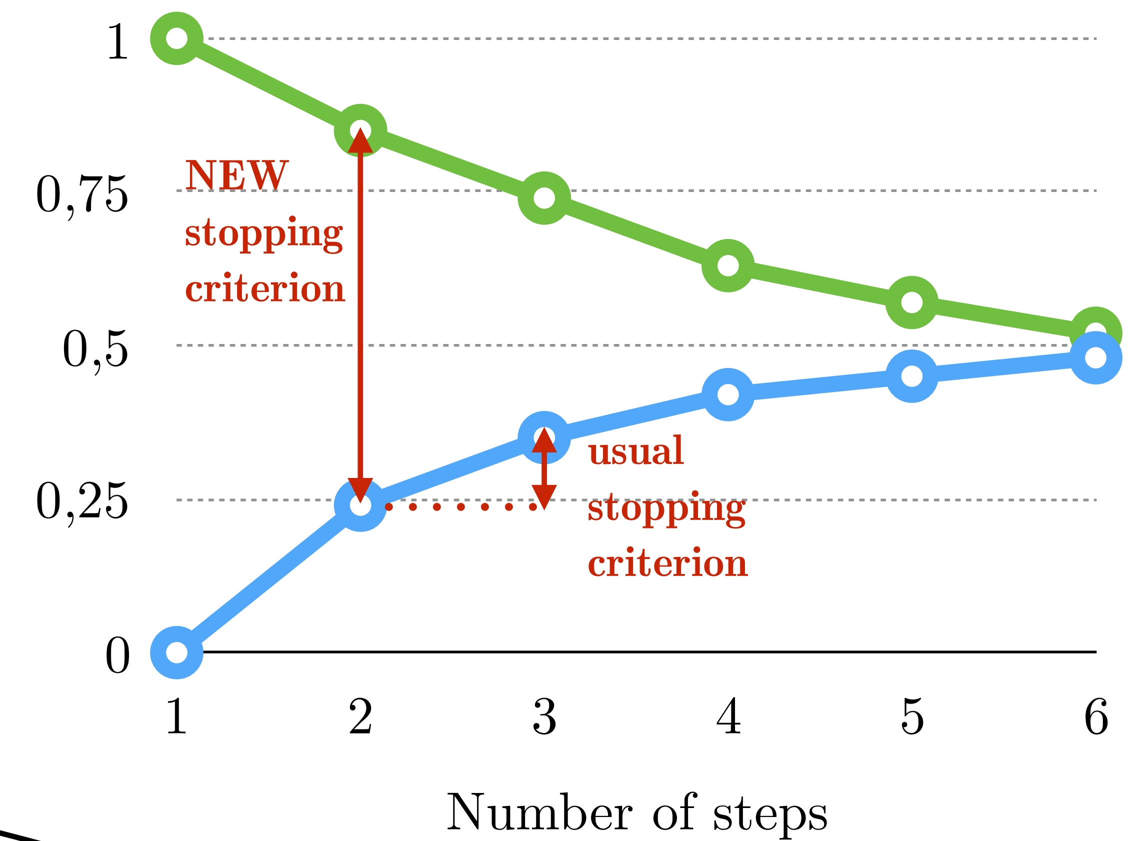

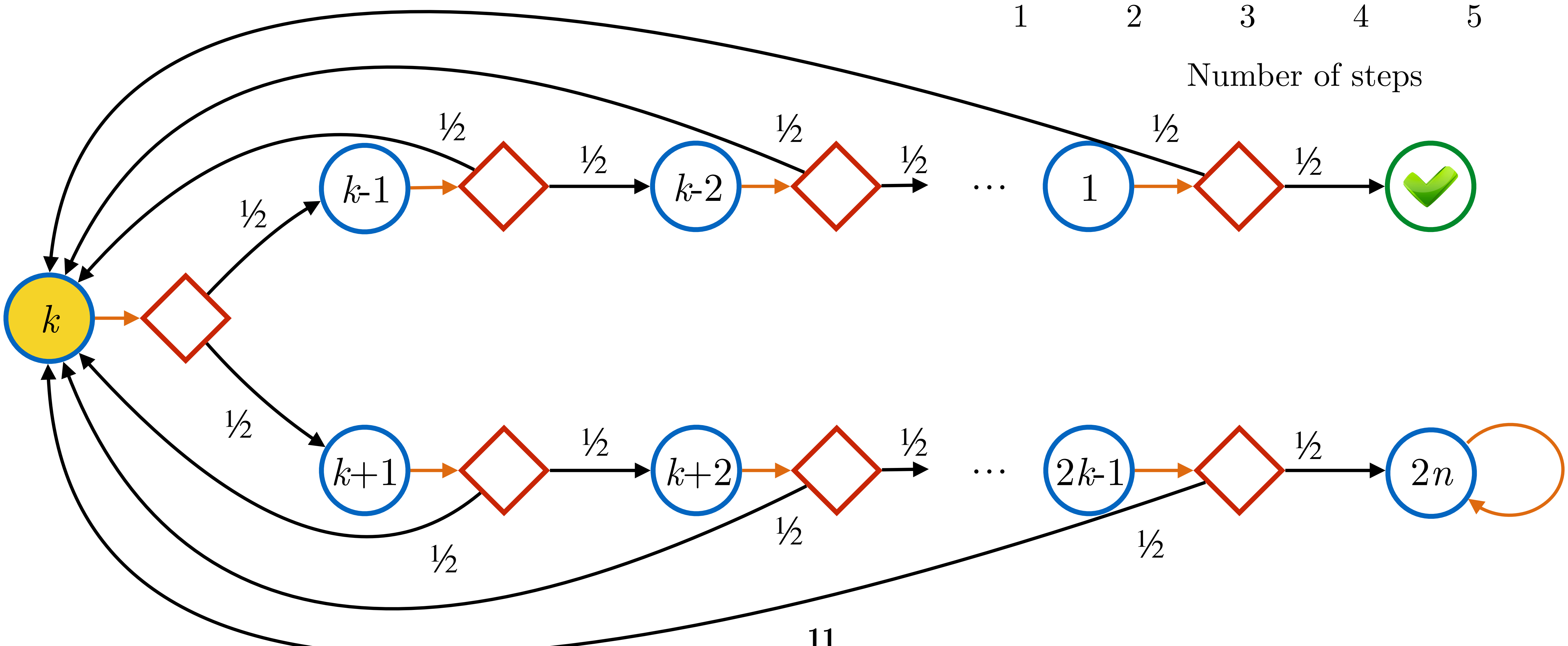

Interval iteration

$$x_s^{(0)} = \begin{cases} 1 & \text{if } s = \checkmark \\ 0 & \text{otherwise} \end{cases} \qquad y_s^{(0)} = \begin{cases} 0 & \text{if } s = \text{✗} \\ 1 & \text{otherwise} \end{cases}$$

$$x^{(n+1)} = f_{\max}(x^{(n)}) \qquad y^{(n+1)} = f_{\max}(y^{(n)})$$

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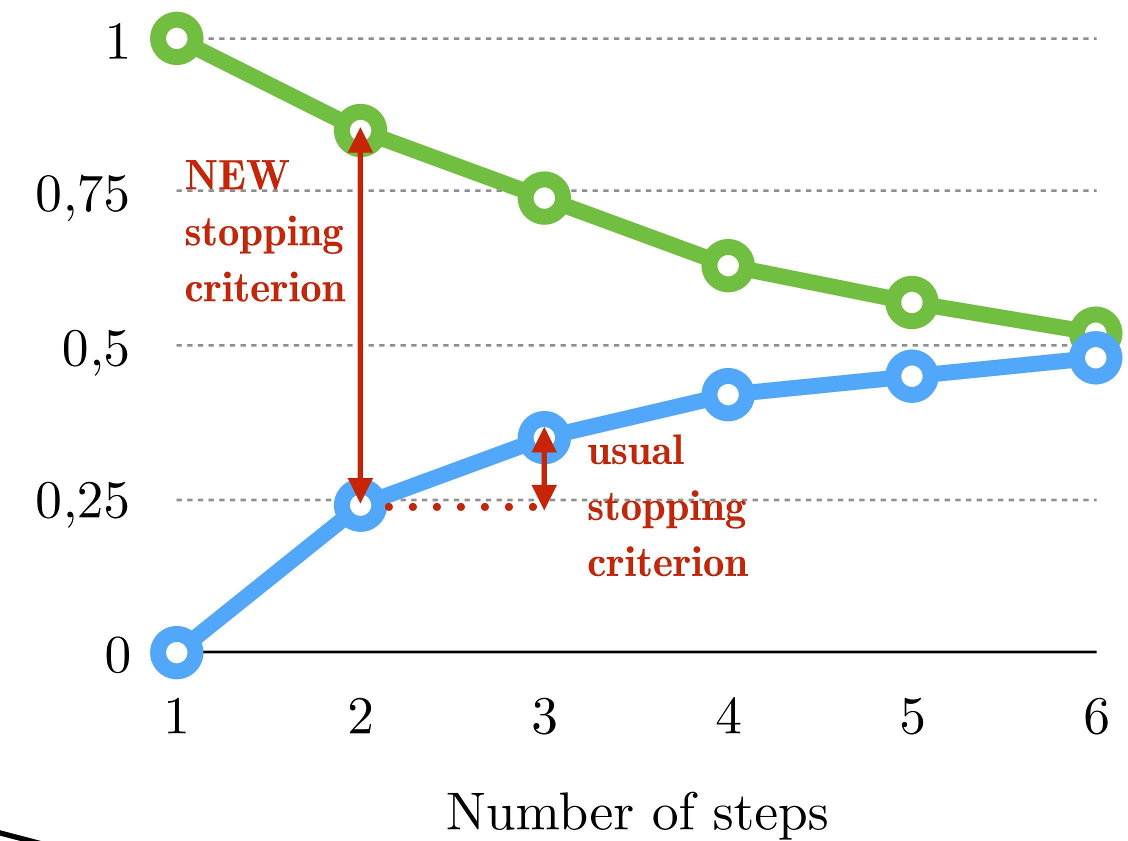

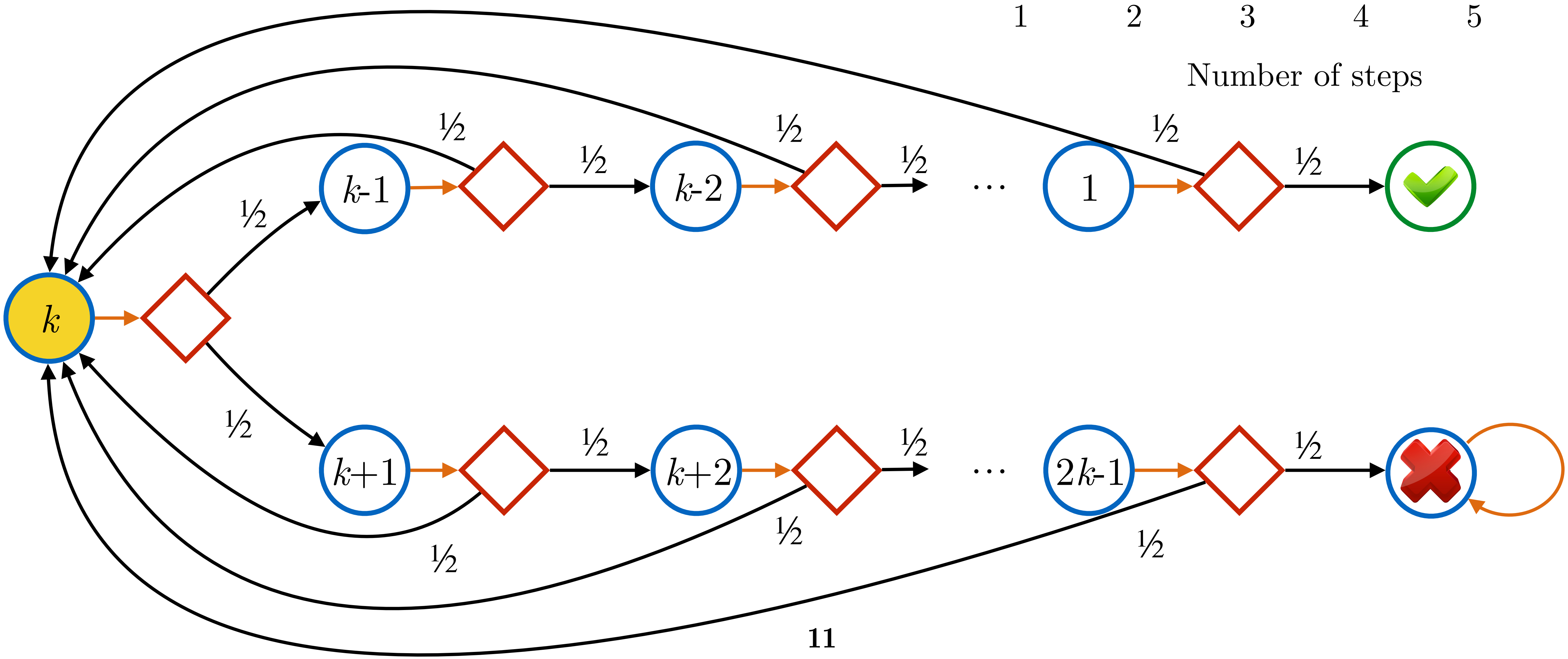

Fixed point characterization

$$\left(\Pr_s^{\max}(\mathsf{F}\,\checkmark)\right)_{s\in S} \text{ is the smallest fixed point of } f_{\max}.$$

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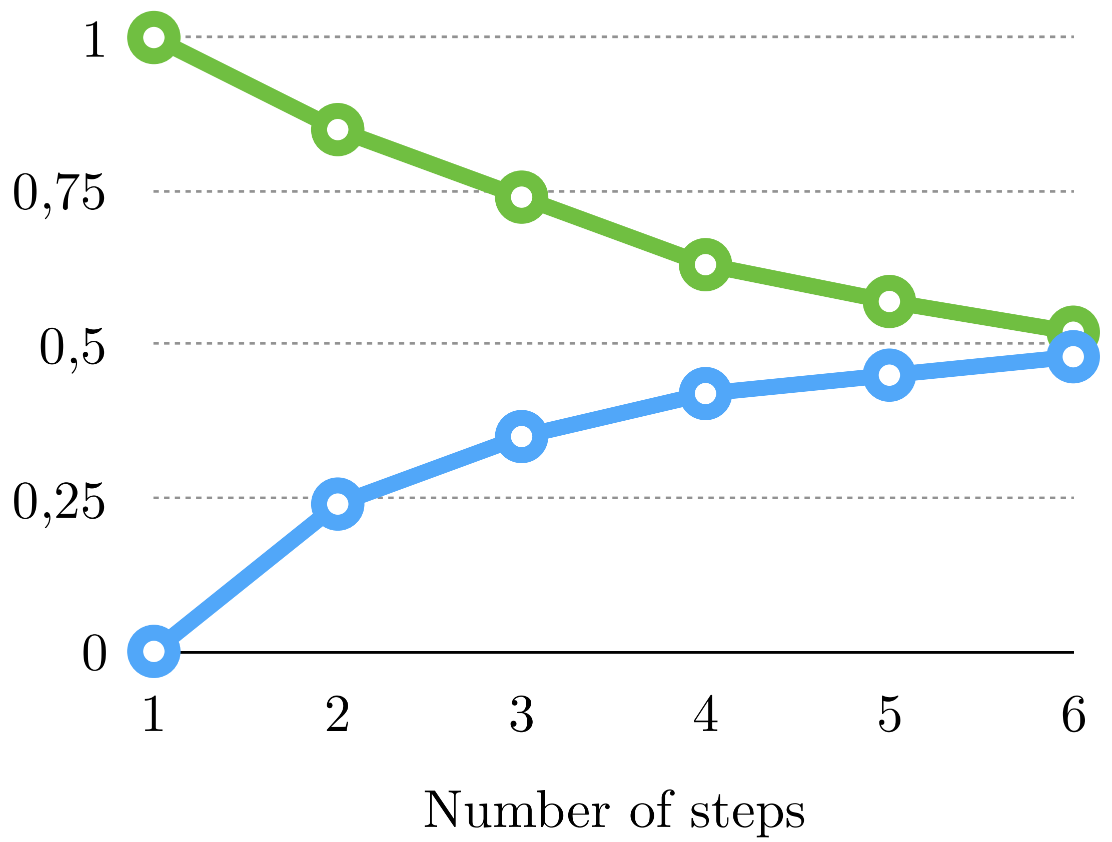

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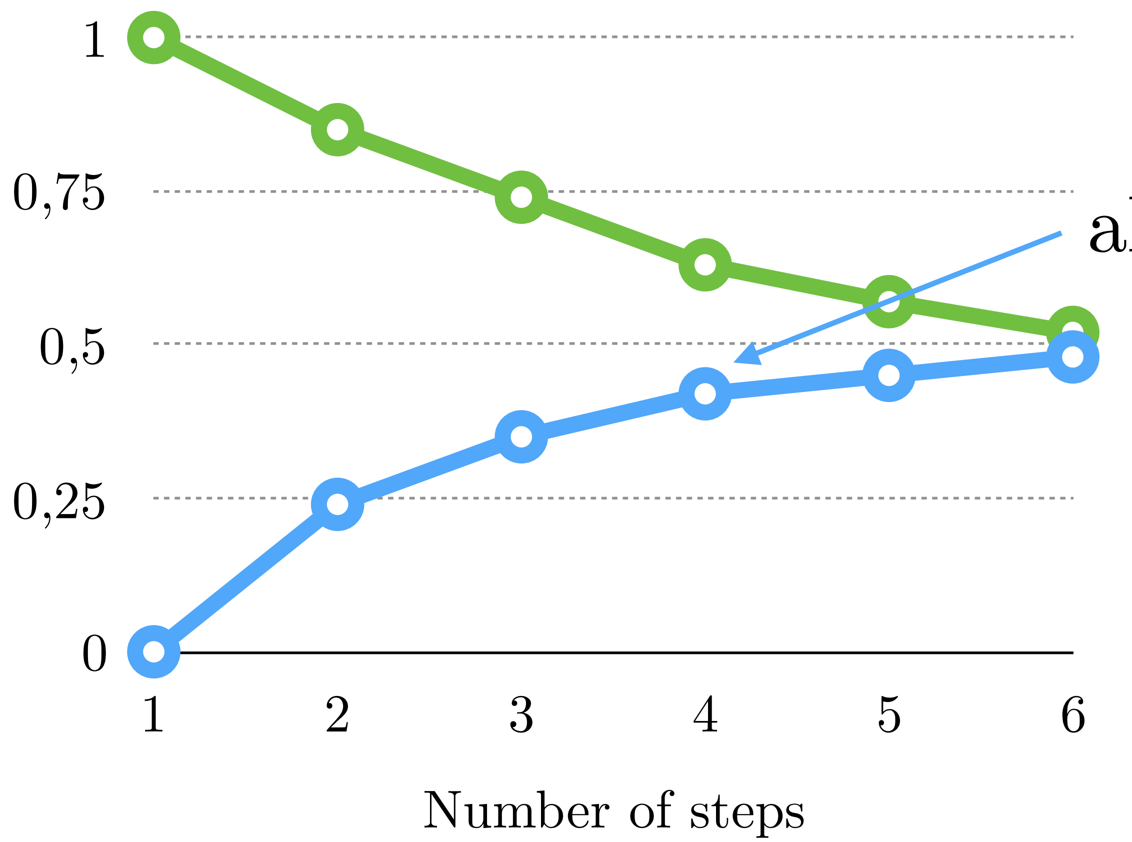

always converges towards $\left(\Pr_s^{\max}(\mathsf{F}\ ✔)\right)_{s\in S}$

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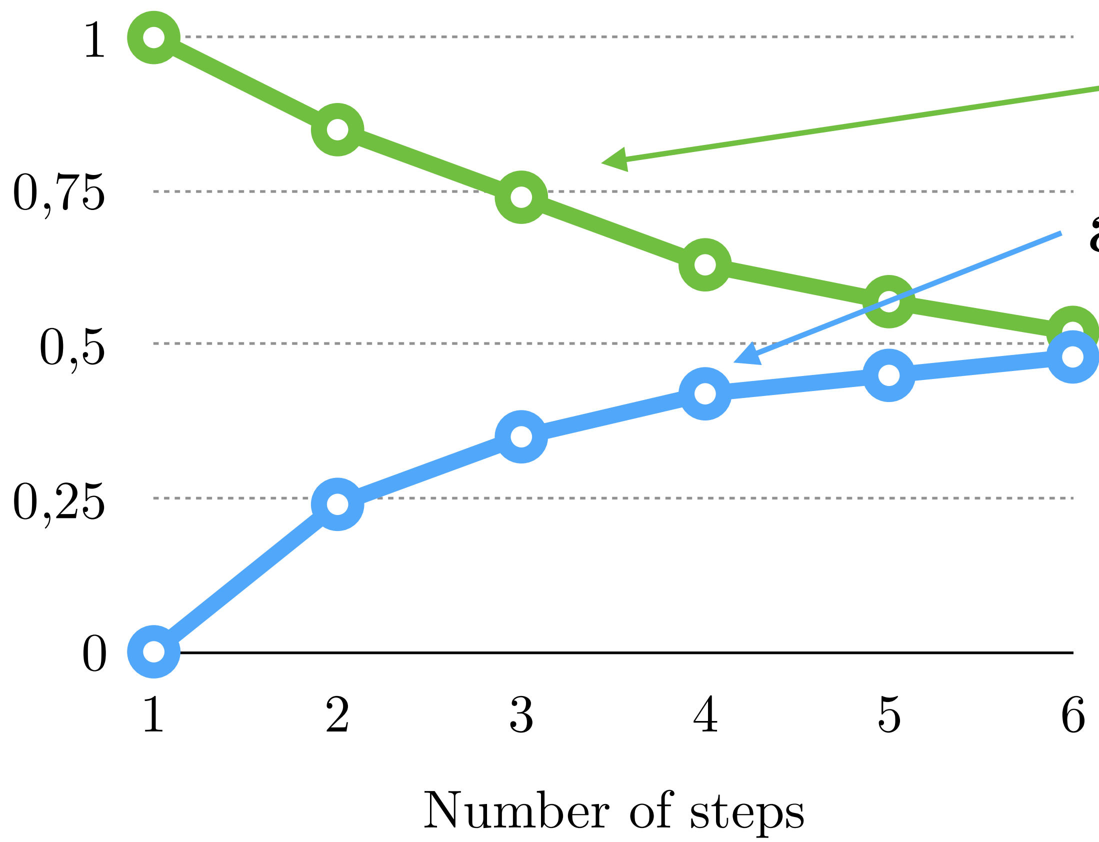

not always...!

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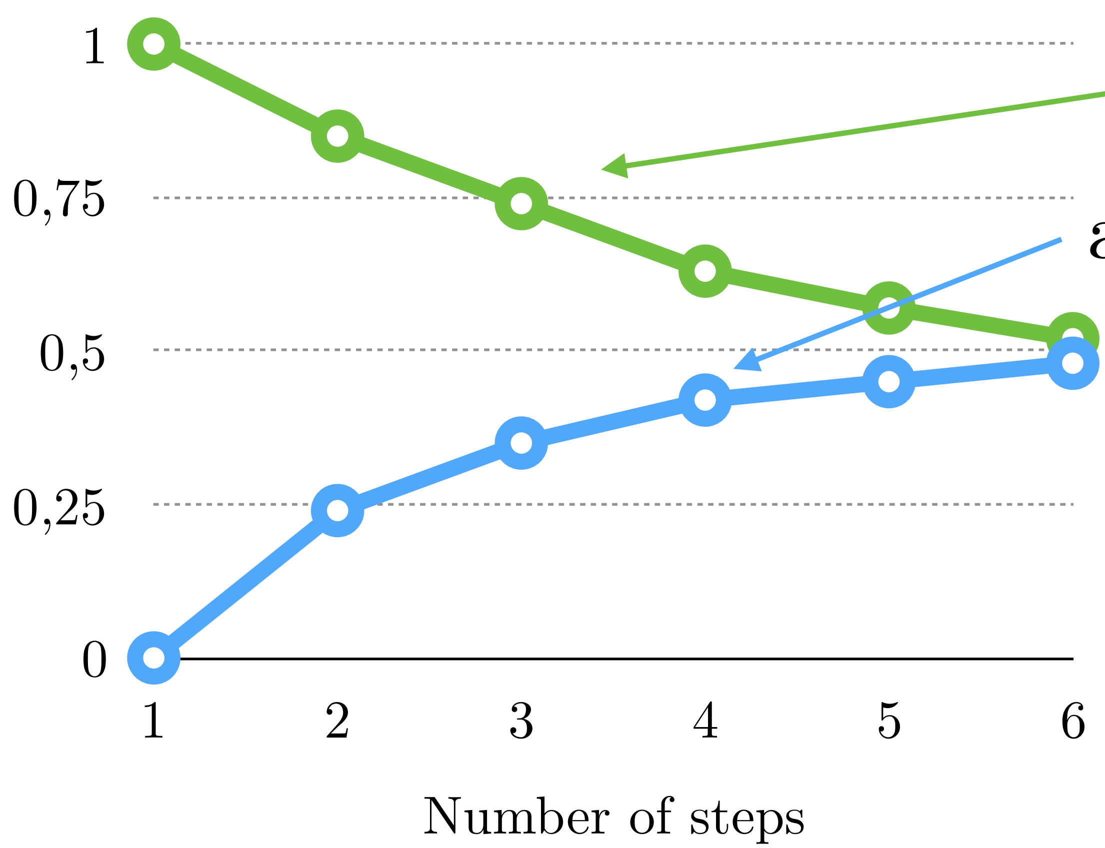

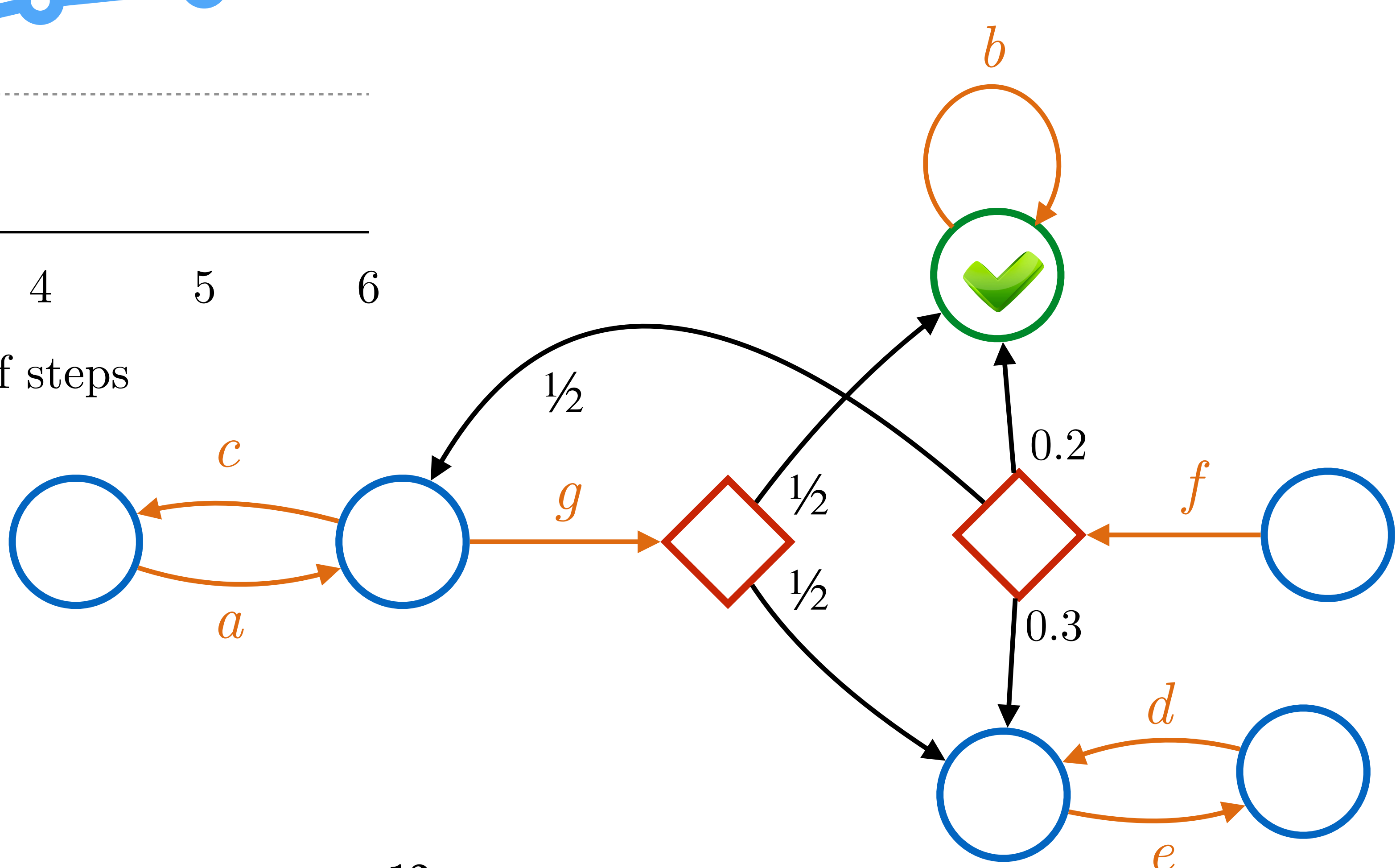

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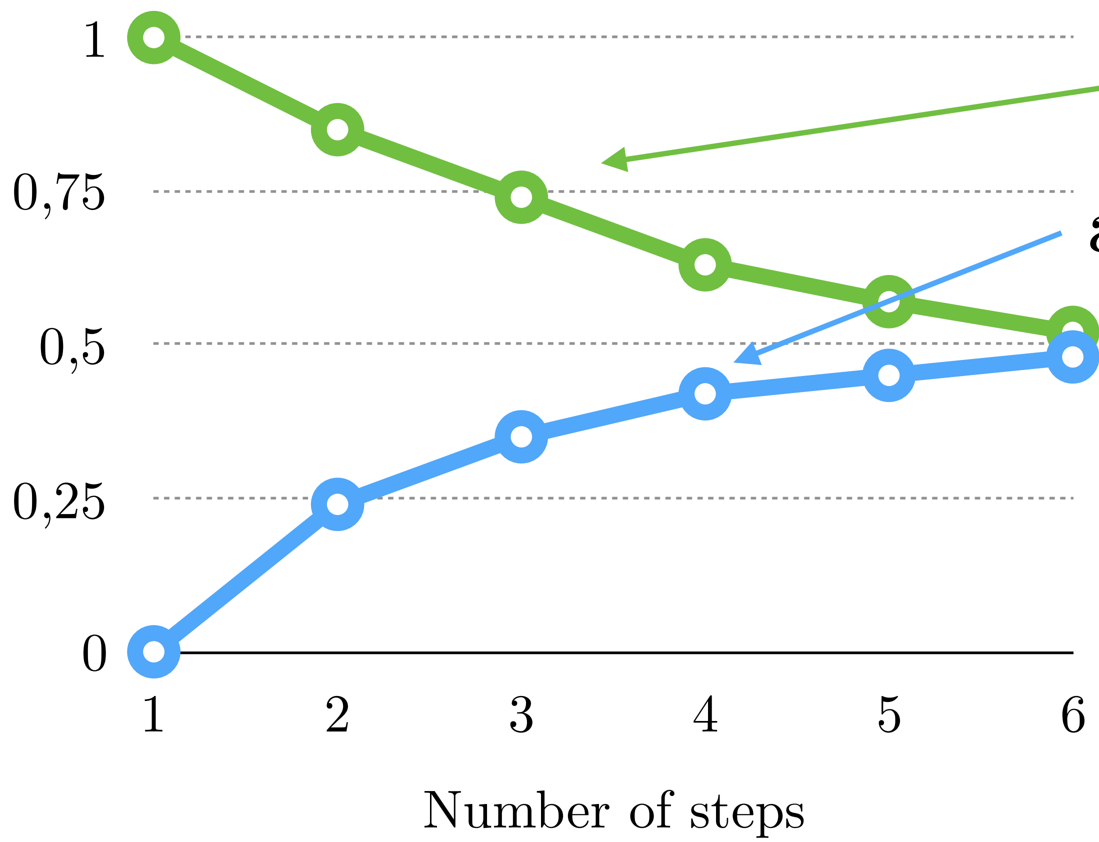

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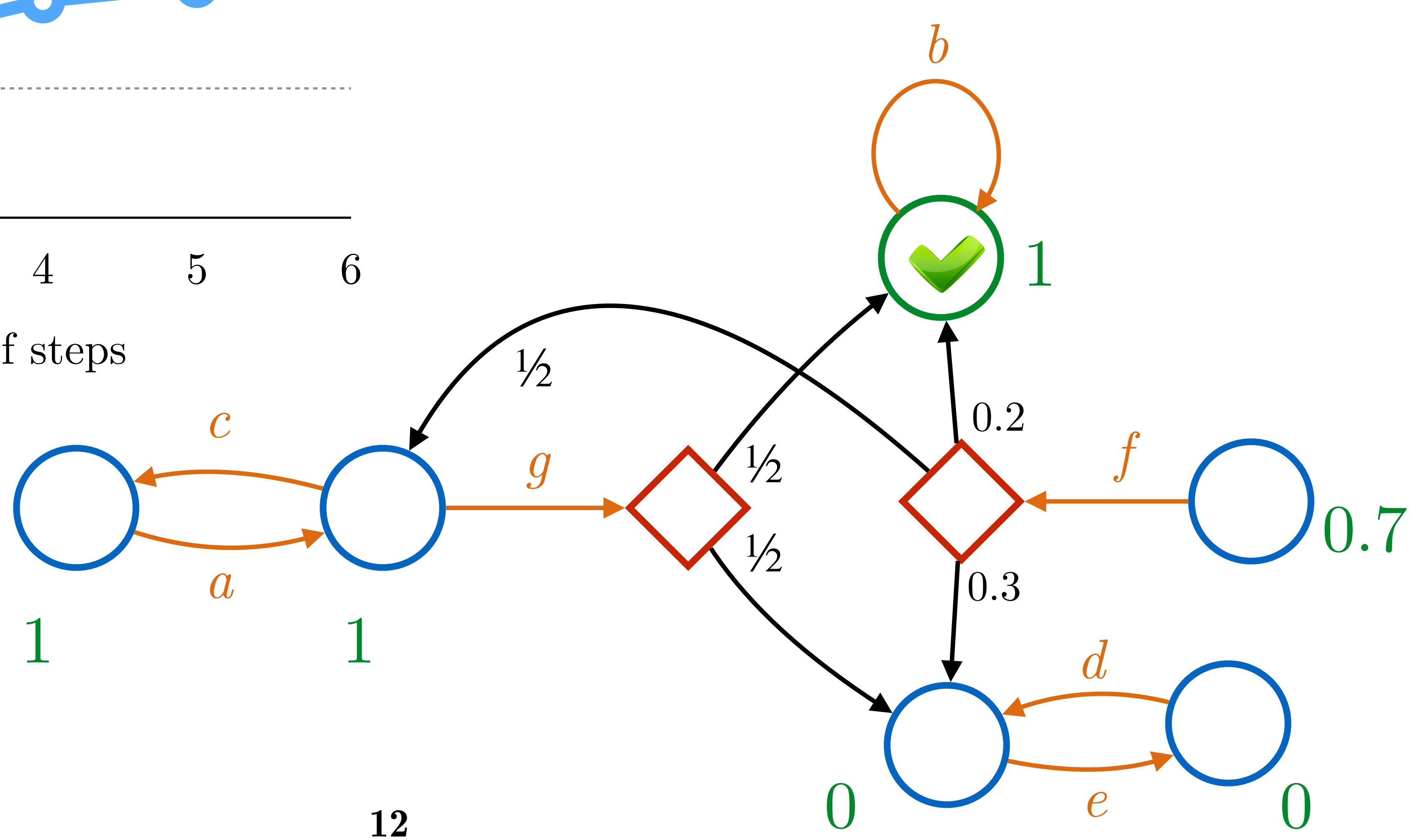

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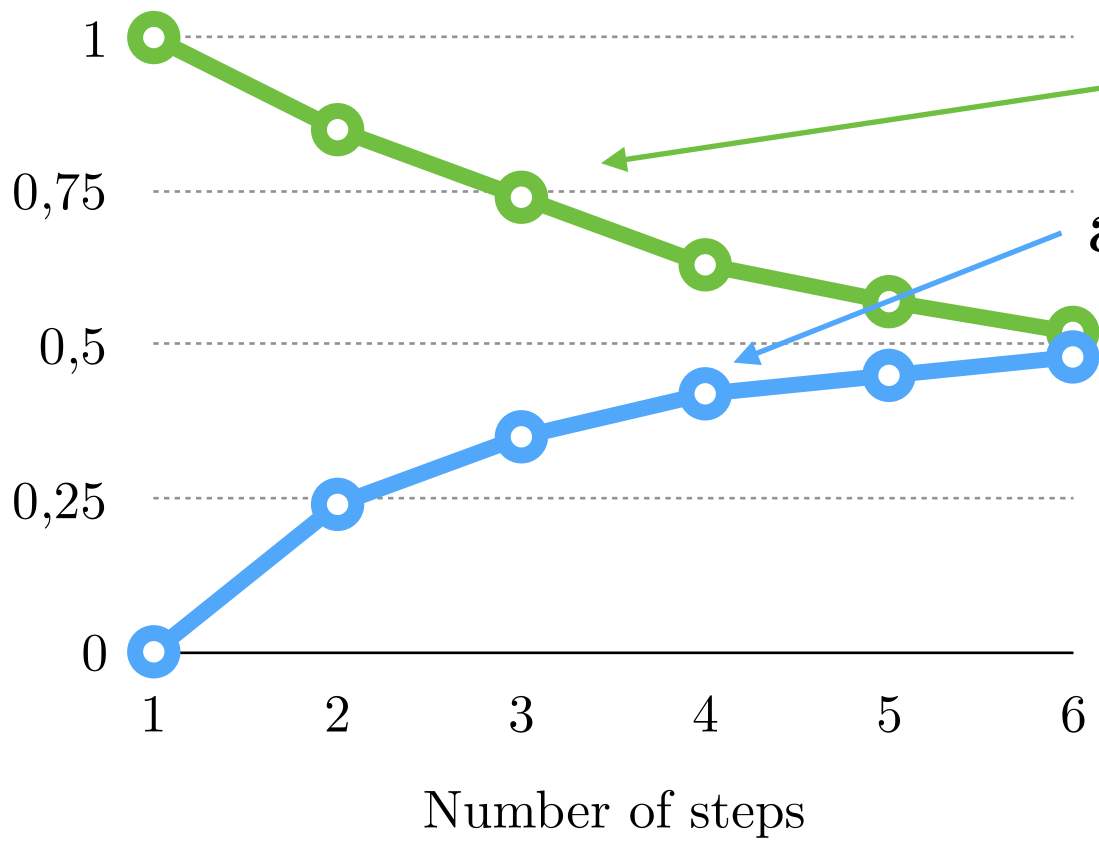

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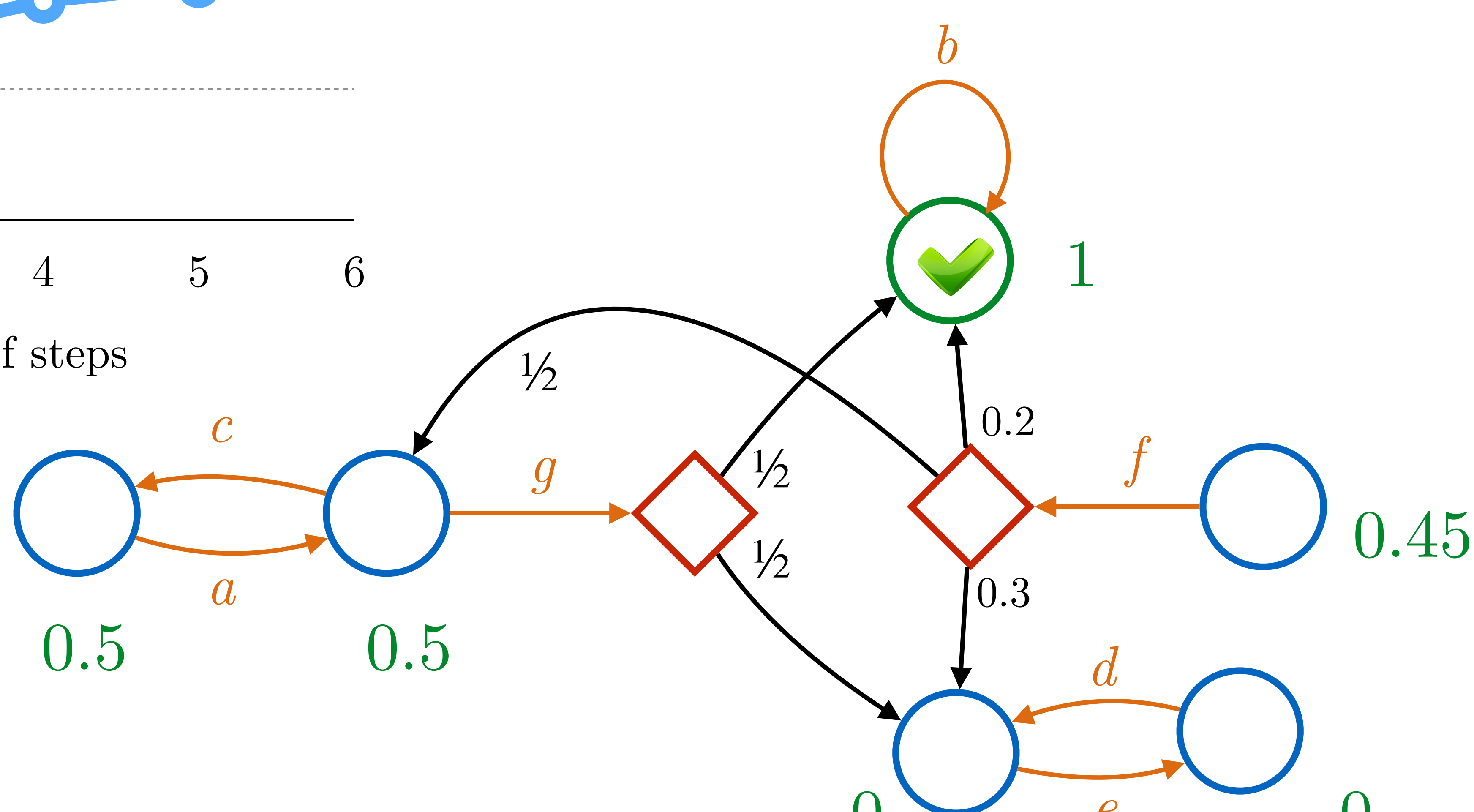
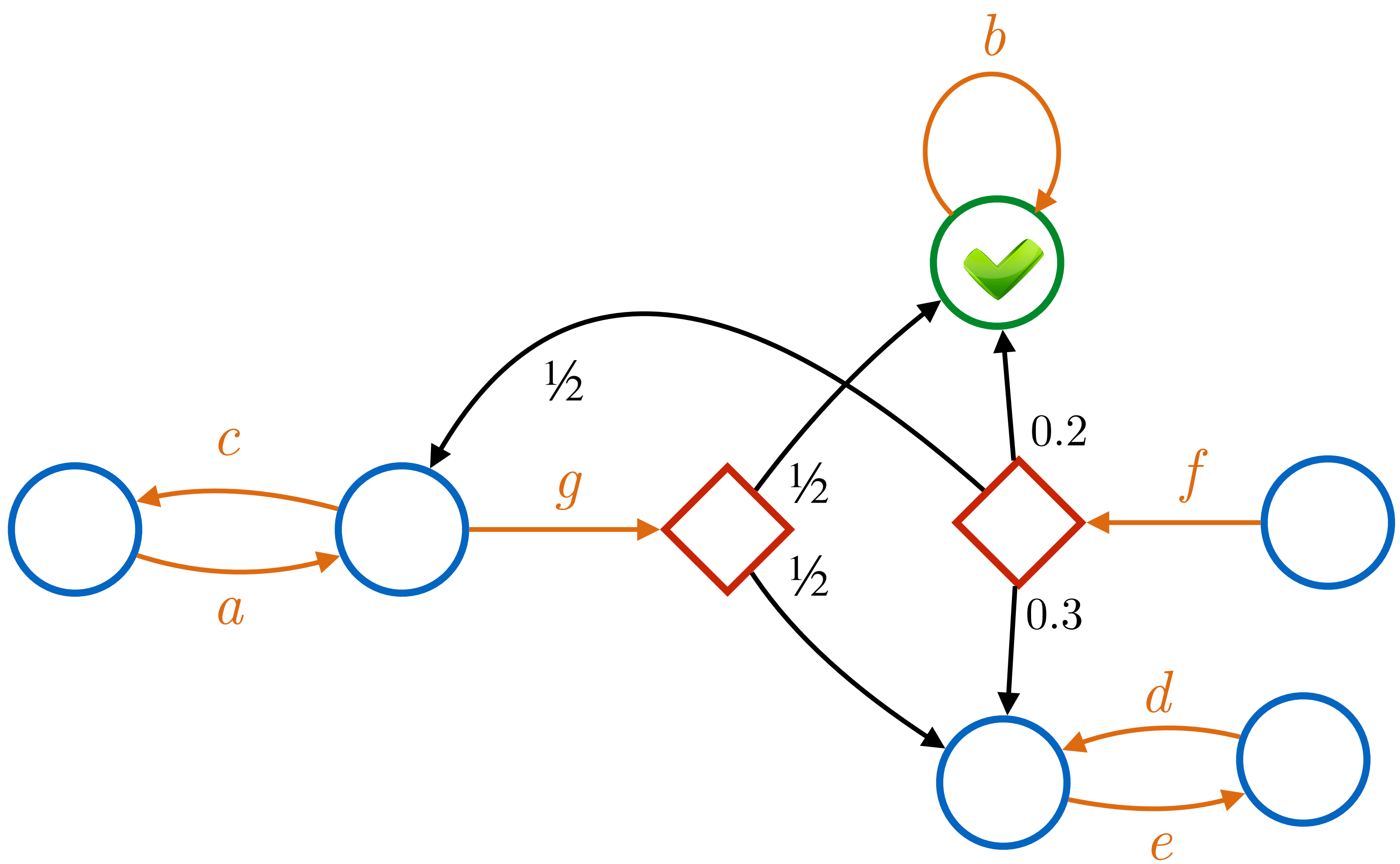


Solution: ensure uniqueness!

Usual techniques applied for MDPs do not apply…



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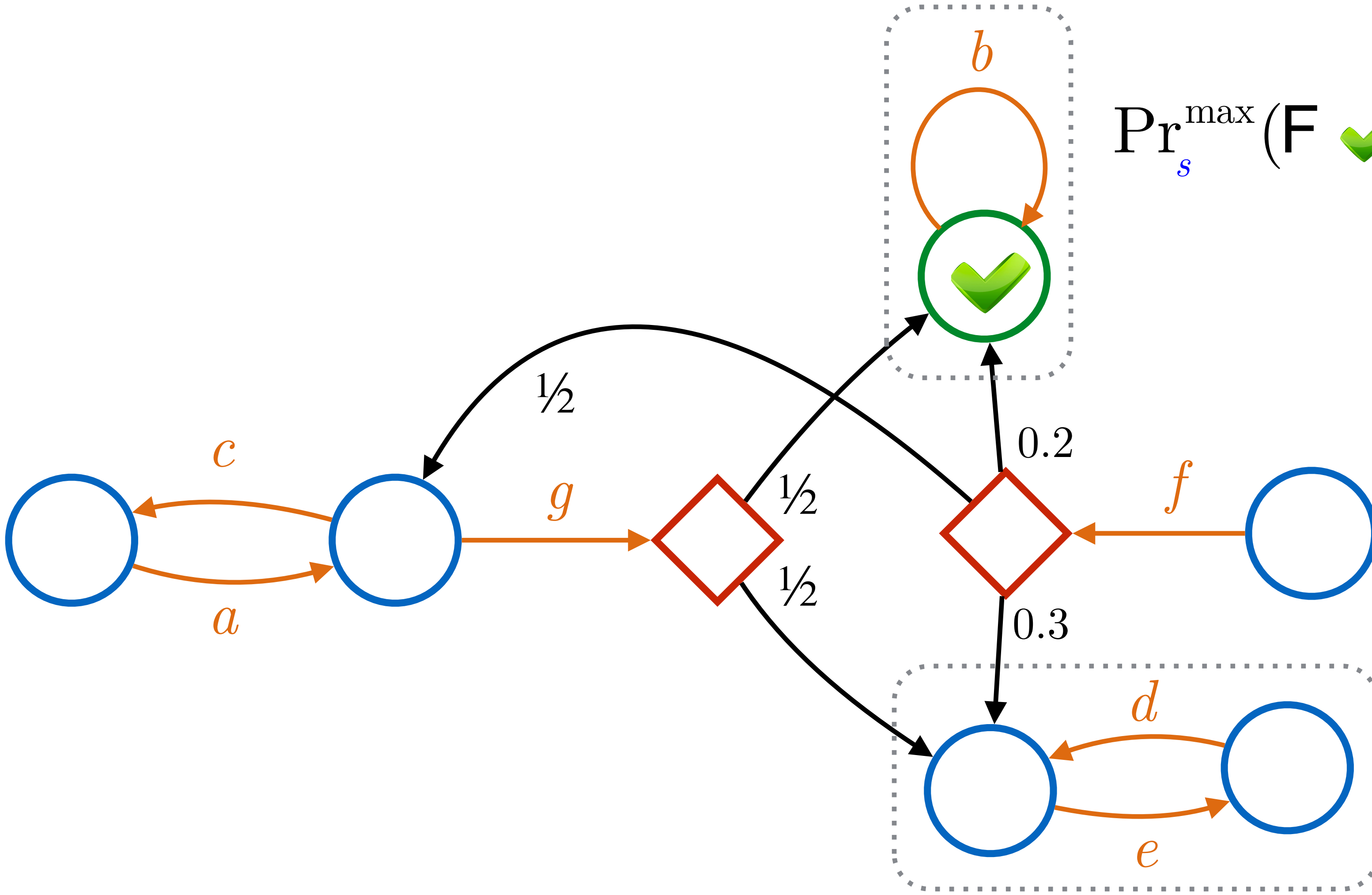

$\mathrm{Pr}_s^{\max}(\mathsf{F}\ ✓) = 1$

$\mathrm{Pr}_s^{\max}(\mathsf{F}\ ✓) = 0$

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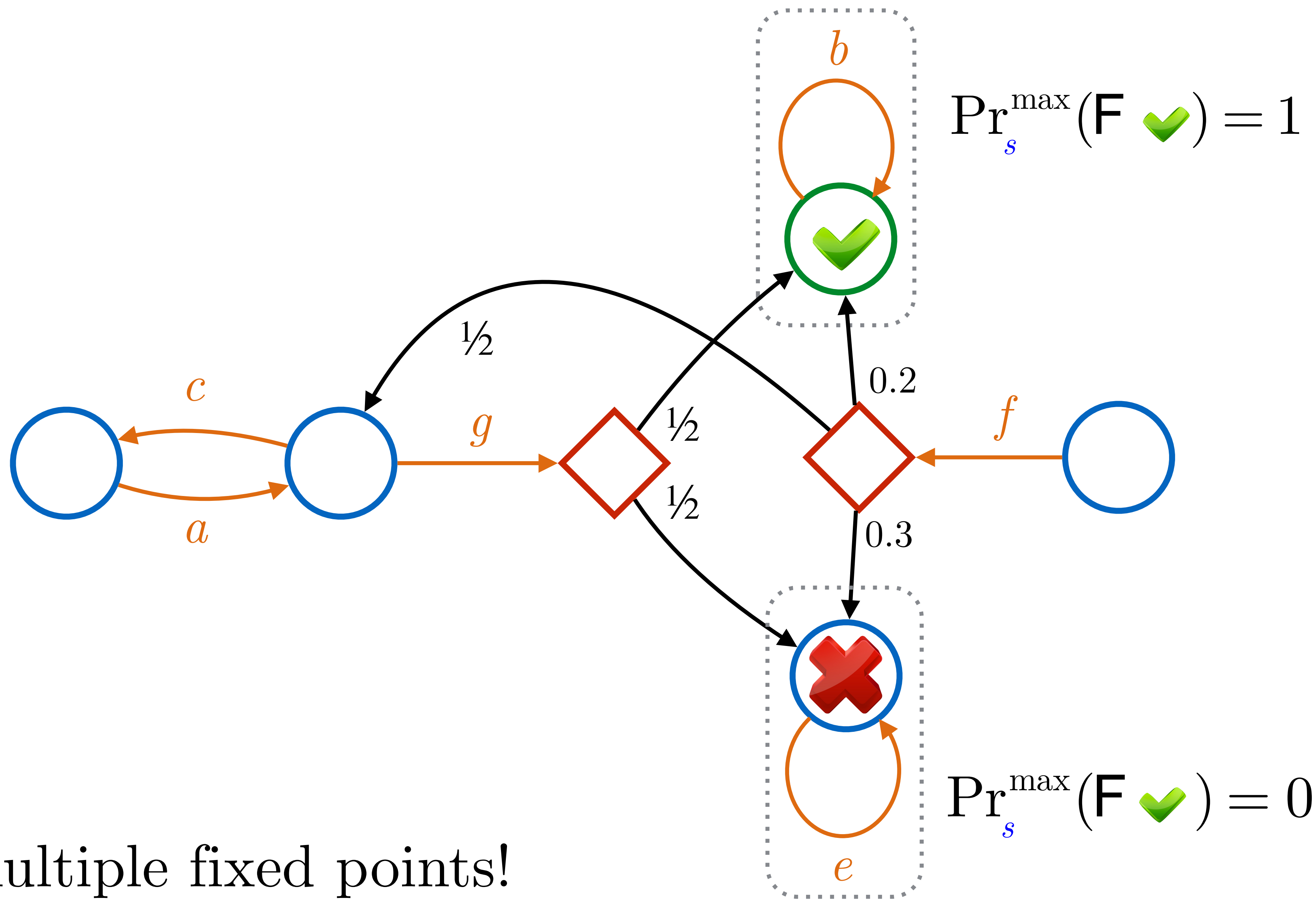

Still multiple fixed points!

Solution: ensure uniqueness!

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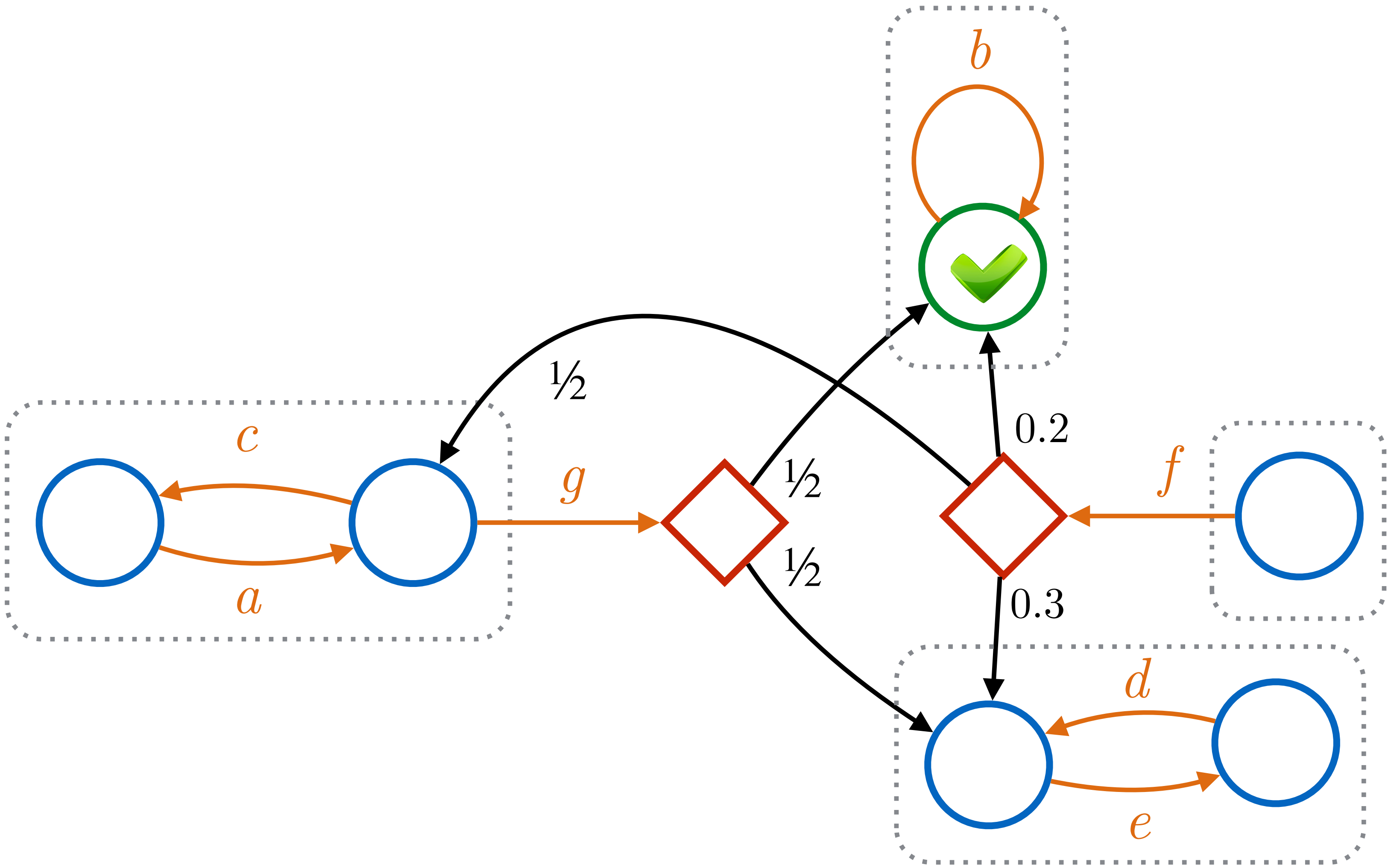

NEW! Use Maximal End Components… (computable in polynomial time)

[de Alfaro, 1997]

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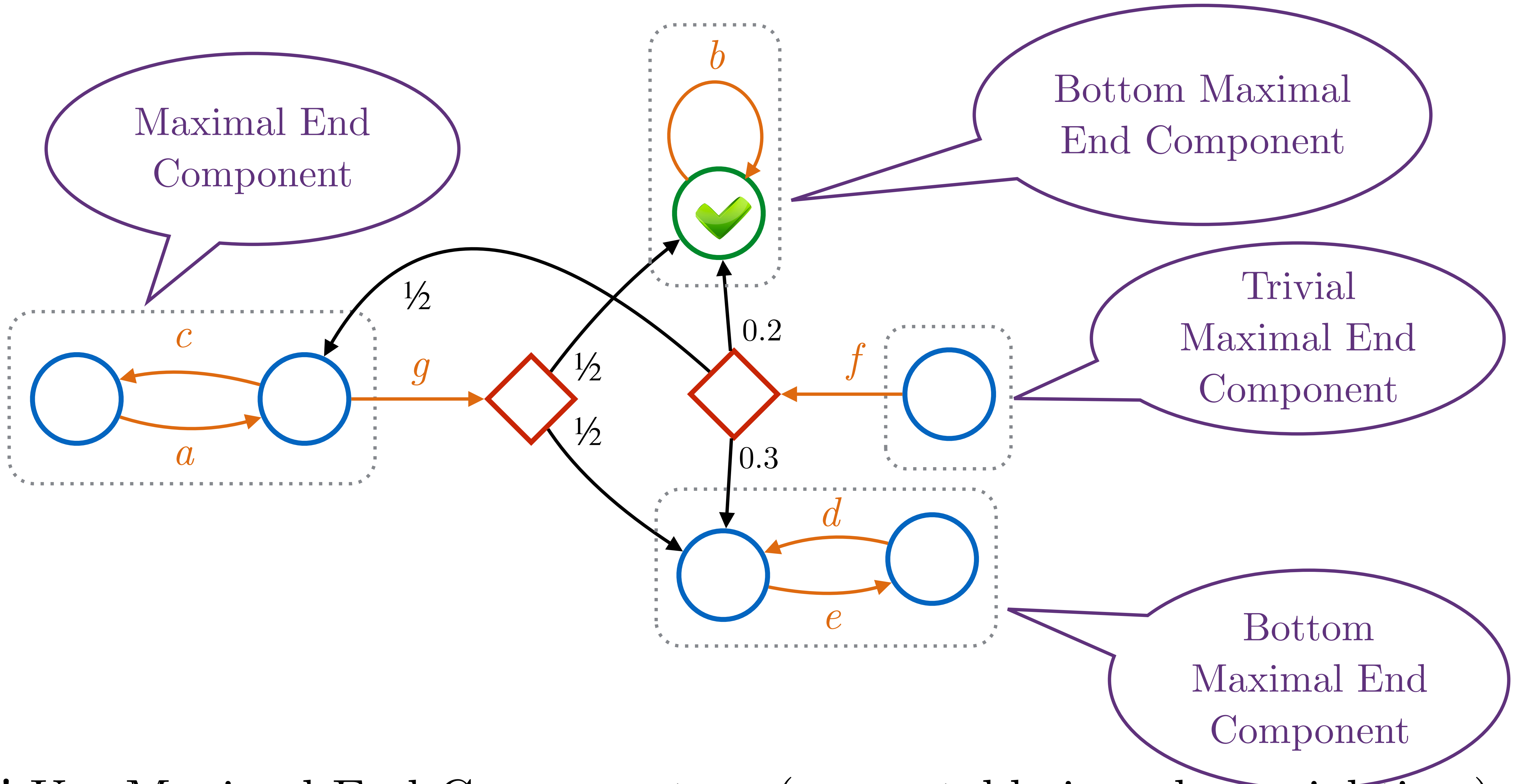

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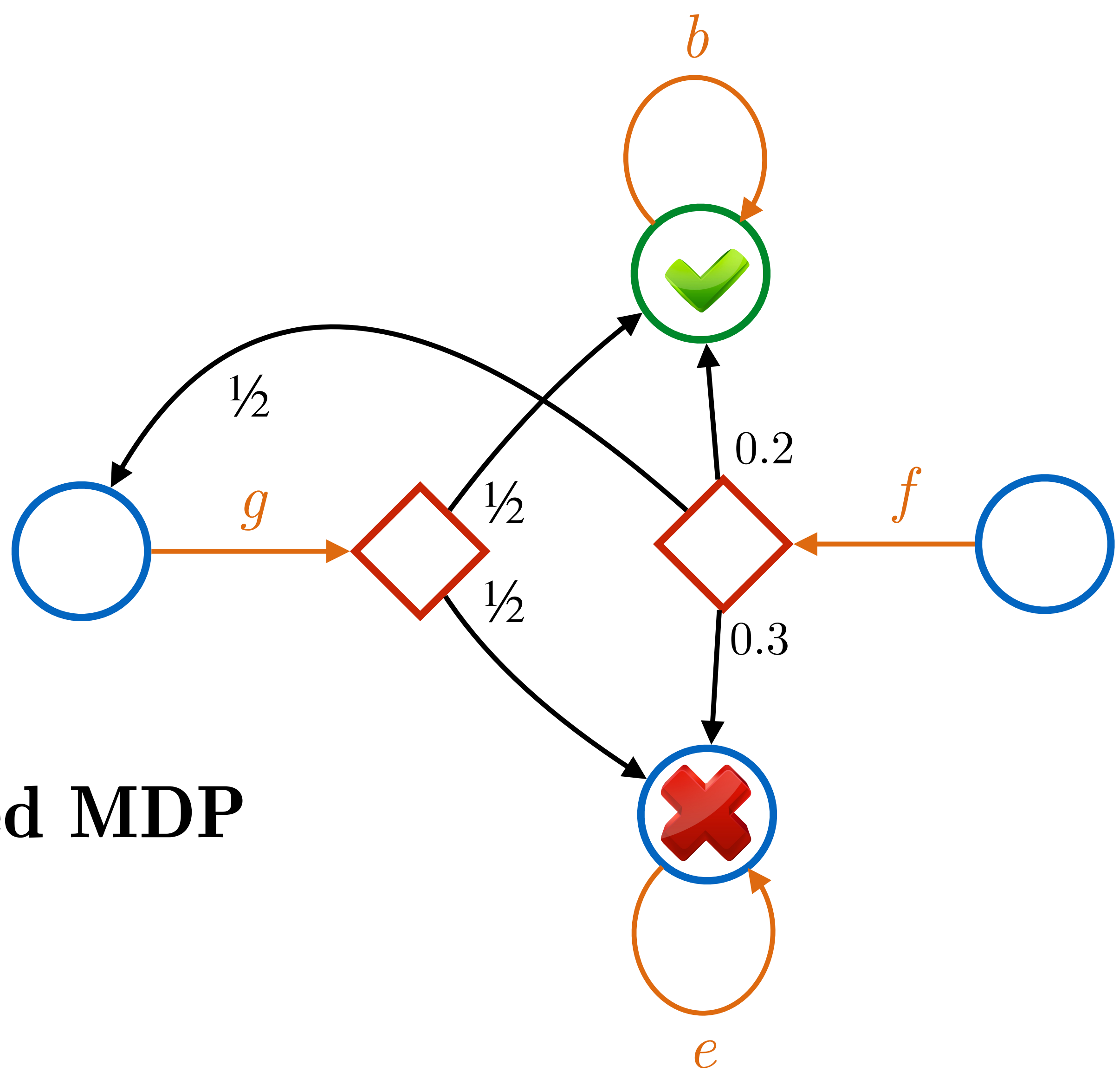
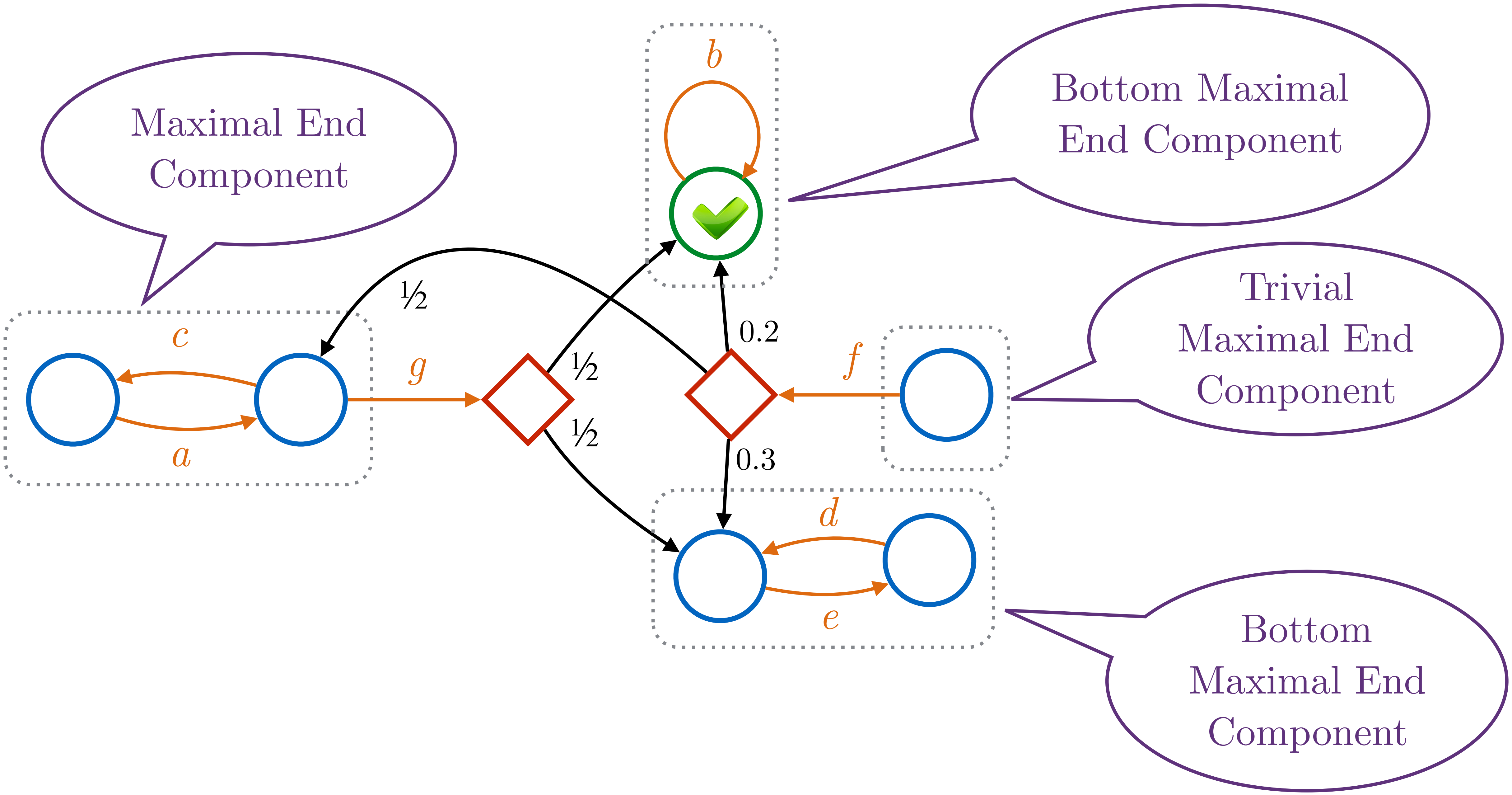


Max-reduced MDP

NEW! Use Maximal End Components… (computable in polynomial time)
and trivialize them! Now, unicity of the fixed point

[de Alfaro, 1997]

An even smaller MDP for minimal probabilities



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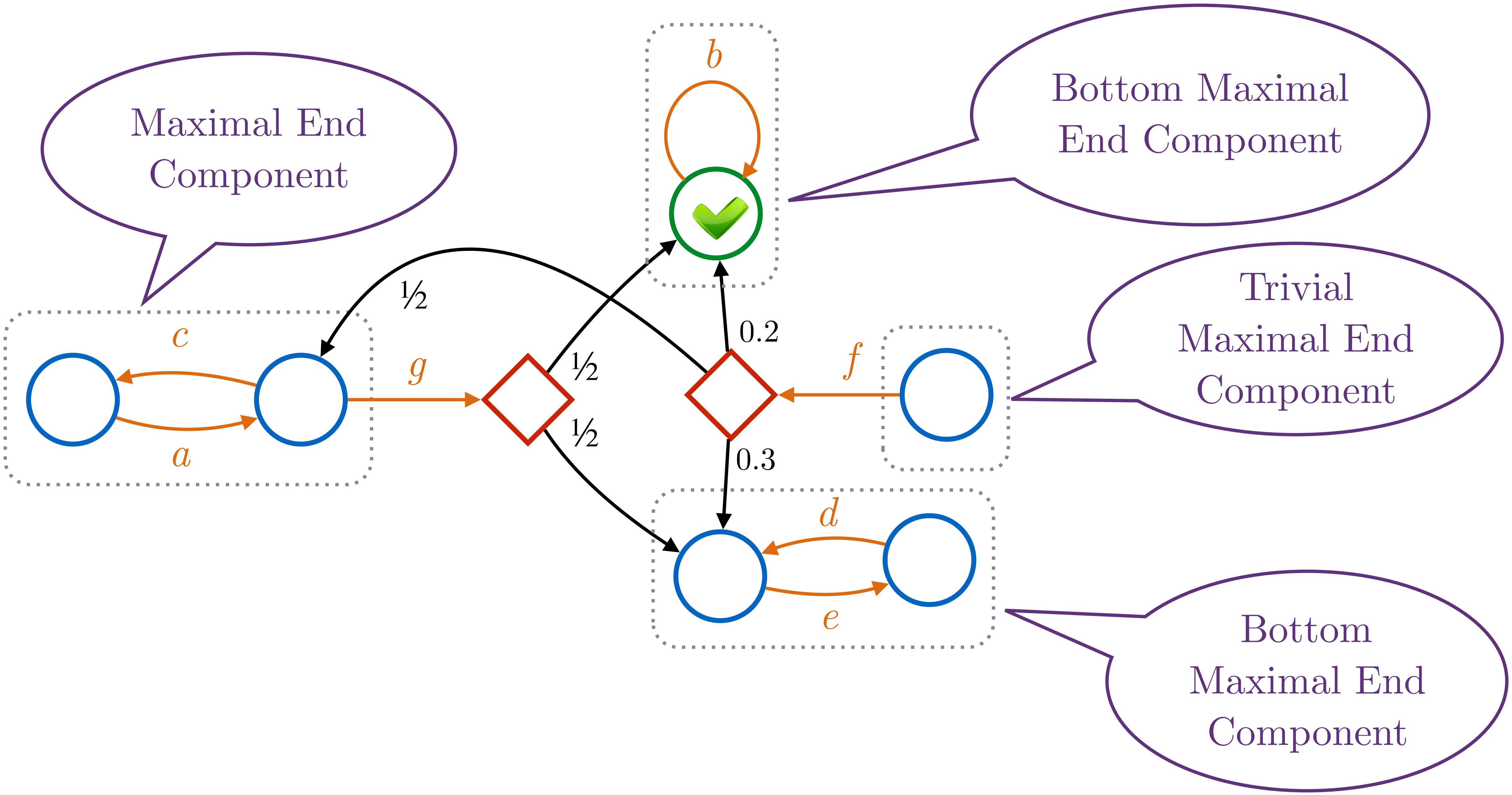
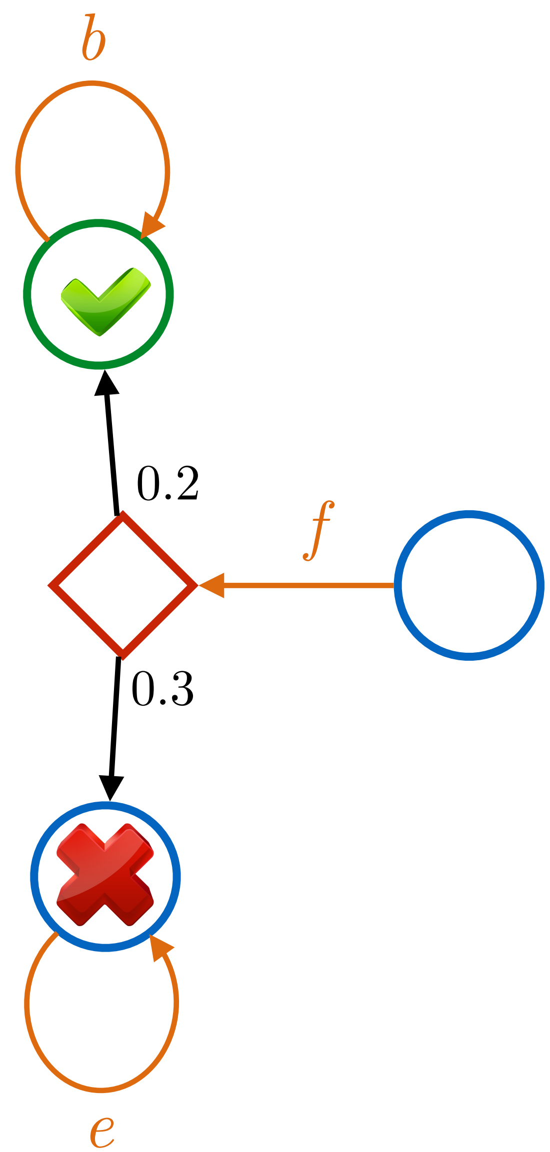


Non-trivial (and non accepting) MEC
have null minimal probability!

An even smaller MDP for minimal probabilities



Min-reduced MDP

Non-trivial (and non accepting) MEC
have null minimal probability!

Interval iteration algorithm in reduced MDPs

Input: Min-reduced MDP $\mathcal{M} = (S, \alpha_{\mathcal{M}}, \delta_{\mathcal{M}})$, convergence threshold ε
Output: Under- and over-approximation of $Pr^{\min}_{\mathcal{M}}(\mathsf{F}\,✓)$

$x_{✓} := 1$; $x_{✗} := 0$; $y_{✓} := 1$; $y_{✗} := 0$
foreach $s \in S \setminus \{✓, ✗\}$ **do** $x_s := 0$; $y_s := 1$
repeat
  foreach $s \in S \setminus \{✓, ✗\}$ **do**
    $x'_s := \min_{a \in A(s)} \sum_{s' \in S} \delta_{\mathcal{M}}(s,a)(s')\, x_{s'}$
    $y'_s := \min_{a \in A(s)} \sum_{s' \in S} \delta_{\mathcal{M}}(s,a)(s')\, y_{s'}$
  $\delta := \max_{s \in S}(y'_s - x'_s)$
  foreach $s \in S \setminus \{✓, ✗\}$ **do** $x'_s := x_s$; $y'_s := y_s$
until $\delta \leqslant \varepsilon$
return $(x_s)_{s \in S}, (y_s)_{s \in S}$

Interval iteration algorithm in reduced MDPs

Input: Min-reduced MDP $\mathcal{M} = (S, \alpha_{\mathcal{M}}, \delta_{\mathcal{M}})$, convergence threshold ε

Output: Under- and over-approximation of $Pr^{\min}_{\mathcal{M}}(\mathsf{F}\,✔)$

```
x_✔ := 1; x_✘ := 0; y_✔ := 1; y_✘ := 0
foreach s ∈ S \ {✔, ✘} do x_s := 0; y_s := 1
repeat
    foreach s ∈ S \ {✔, ✘} do
        x'_s := min_{a∈A(s)} Σ_{s'∈S} δ_M(s,a)(s') x_{s'}
        y'_s := min_{a∈A(s)} Σ_{s'∈S} δ_M(s,a)(s') y_{s'}
    δ := max_{s∈S}(y'_s − x'_s)
    foreach s ∈ S \ {✔, ✘} do x'_s := x_s; y'_s := y_s
until δ ⩽ ε
return (x_s)_{s∈S}, (y_s)_{s∈S}
```

Sequences x and y converge towards the minimal probability to reach ✔. Hence, the algorithm terminates by returning an interval of length at most ε for each state containing $\Pr^{\min}_{s}(\mathsf{F}\,✔)$.

Interval iteration algorithm in reduced MDPs

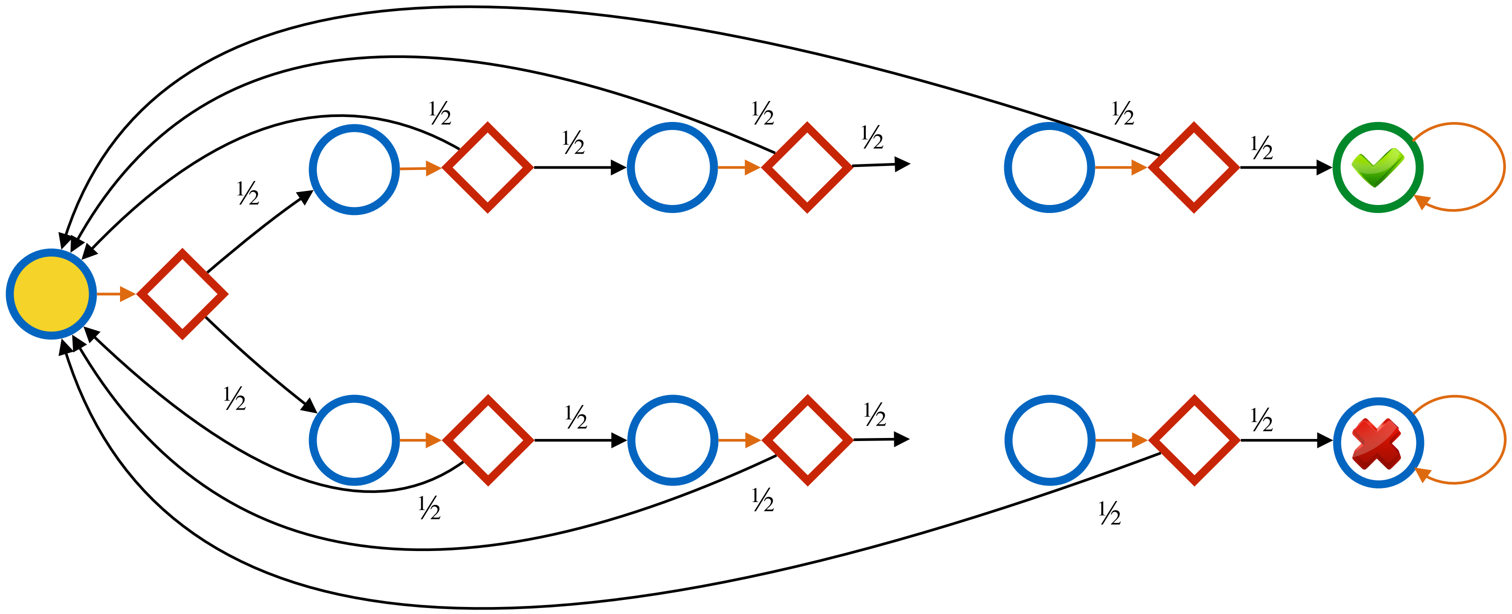
Input: Min-reduced MDP $\mathcal{M} = (S, \alpha_{\mathcal{M}}, \delta_{\mathcal{M}})$, convergence threshold ε
Output: Under- and over-approximation of $Pr^{\min}_{\mathcal{M}}(\mathsf{F}\checkmark)$

$x_{\checkmark} := 1;\ x_{\times} := 0;\ y_{\checkmark} := 1;\ y_{\times} := 0$
foreach $s \in S \setminus \{\checkmark, \times\}$ **do** $x_s := 0;\ y_s := 1$
repeat
  foreach $s \in S \setminus \{\checkmark, \times\}$ **do**
    $x'_s := \min_{a \in A(s)} \sum_{s' \in S} \delta_{\mathcal{M}}(s,a)(s')\, x_{s'}$
    $y'_s := \min_{a \in A(s)} \sum_{s' \in S} \delta_{\mathcal{M}}(s,a)(s')\, y_{s'}$
  $\delta := \max_{s \in S}(y'_s - x'_s)$
  foreach $s \in S \setminus \{\checkmark, \times\}$ **do** $x'_s := x_s;\ y'_s := y_s$
until $\delta \leqslant \varepsilon$
return $(x_s)_{s \in S}, (y_s)_{s \in S}$

Sequences x and y converge towards the minimal probability to reach $\checkmark$. Hence, the algorithm terminates by returning an interval of length at most ε for each state containing $\mathrm{Pr}^{\min}_{s}(\mathsf{F}\checkmark)$.

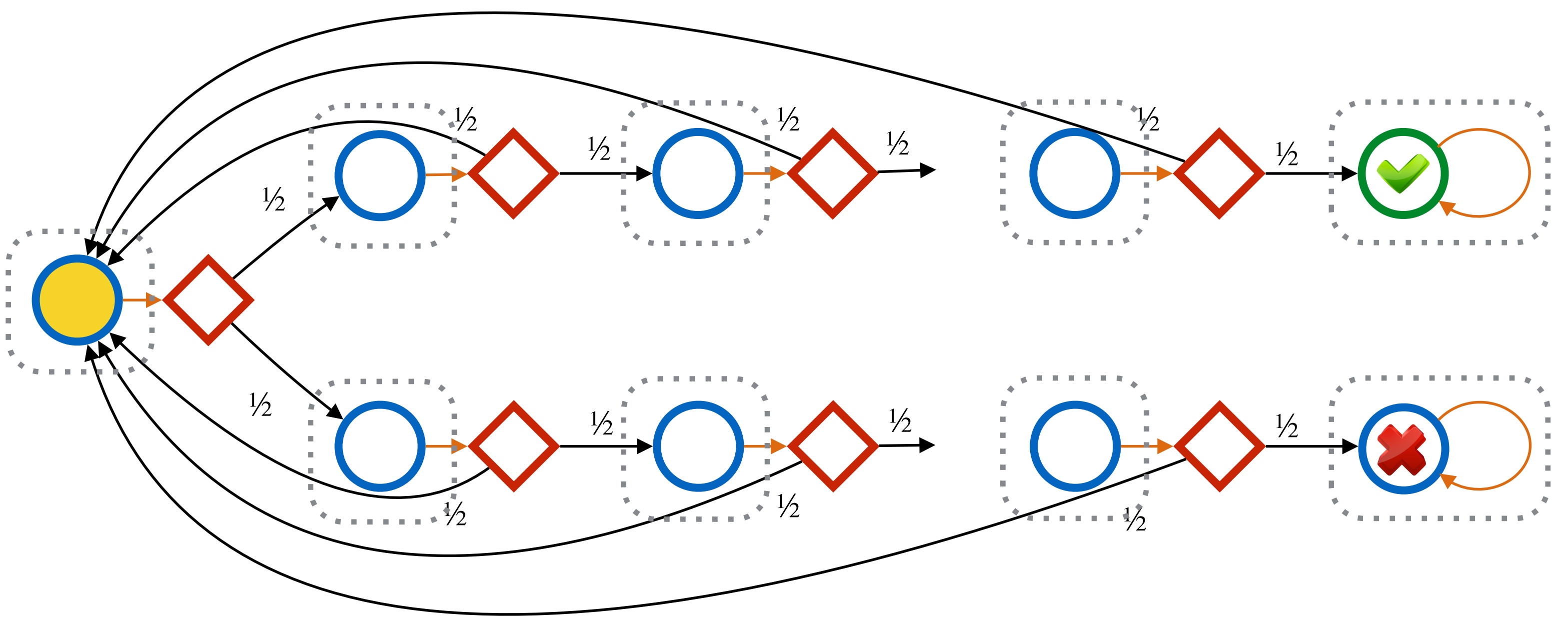
Possible speed-up: only check size of interval for a given state...

Rate of convergence



x stores reachability probabilities, y stores safety probabilities, i.e., after n iterations: $x_s = \Pr_s^{\min}(\mathsf{F}^{\leq n} ✓)$ $y_s = \Pr_s^{\min}(\mathsf{G}^{\leq n}(\neg ✗))$

Rate of convergence



2 BMECs and only trivial MECs

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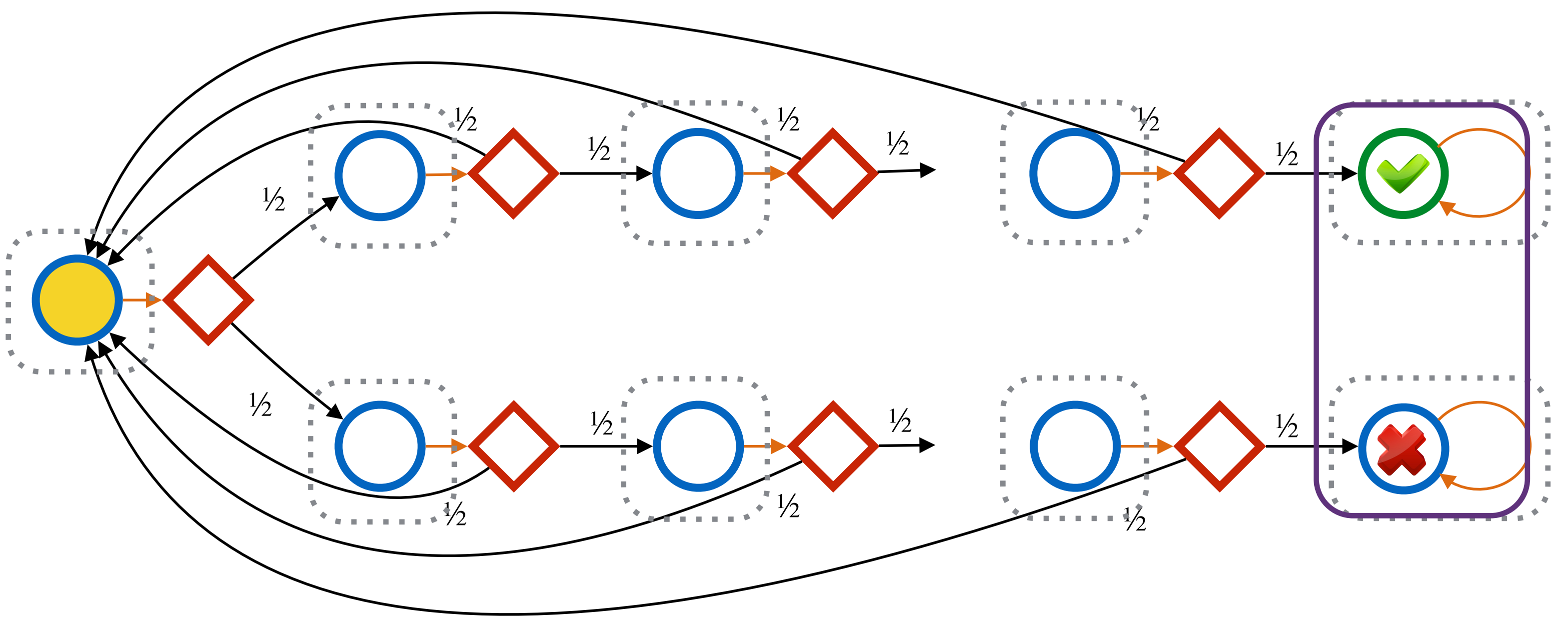

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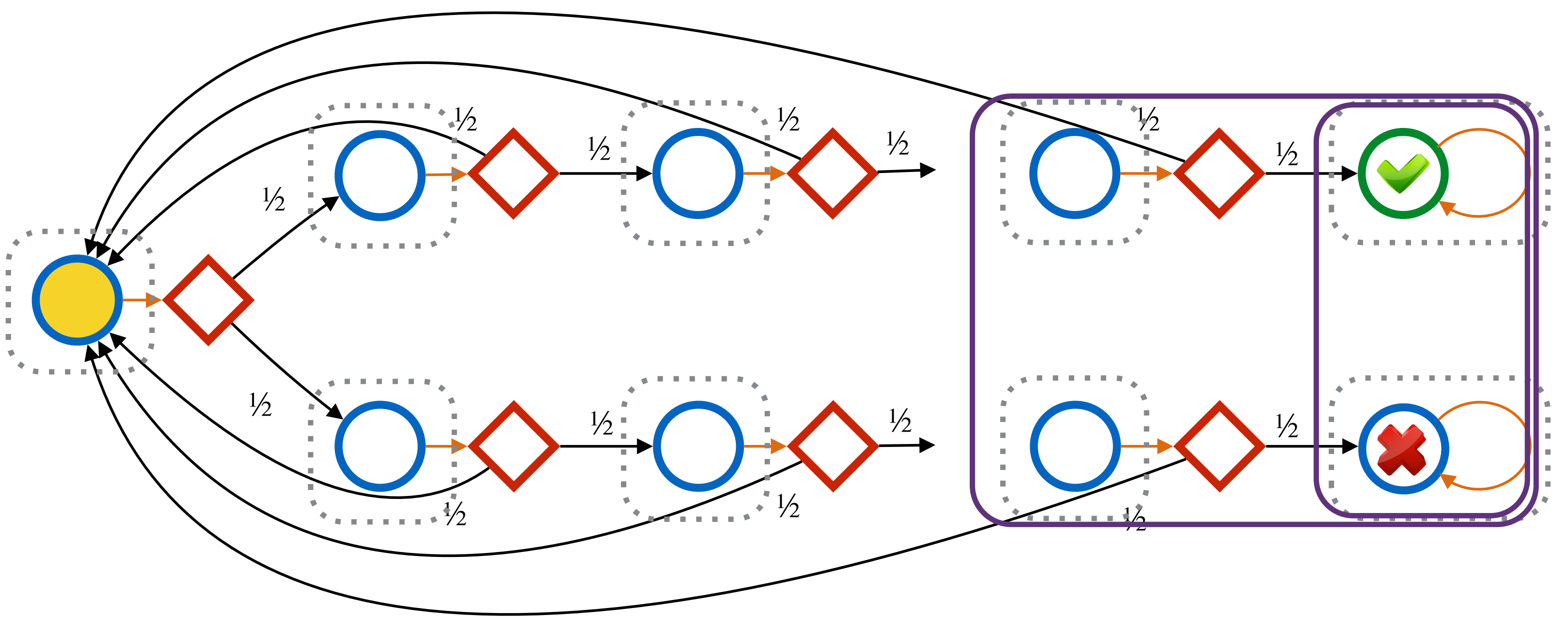

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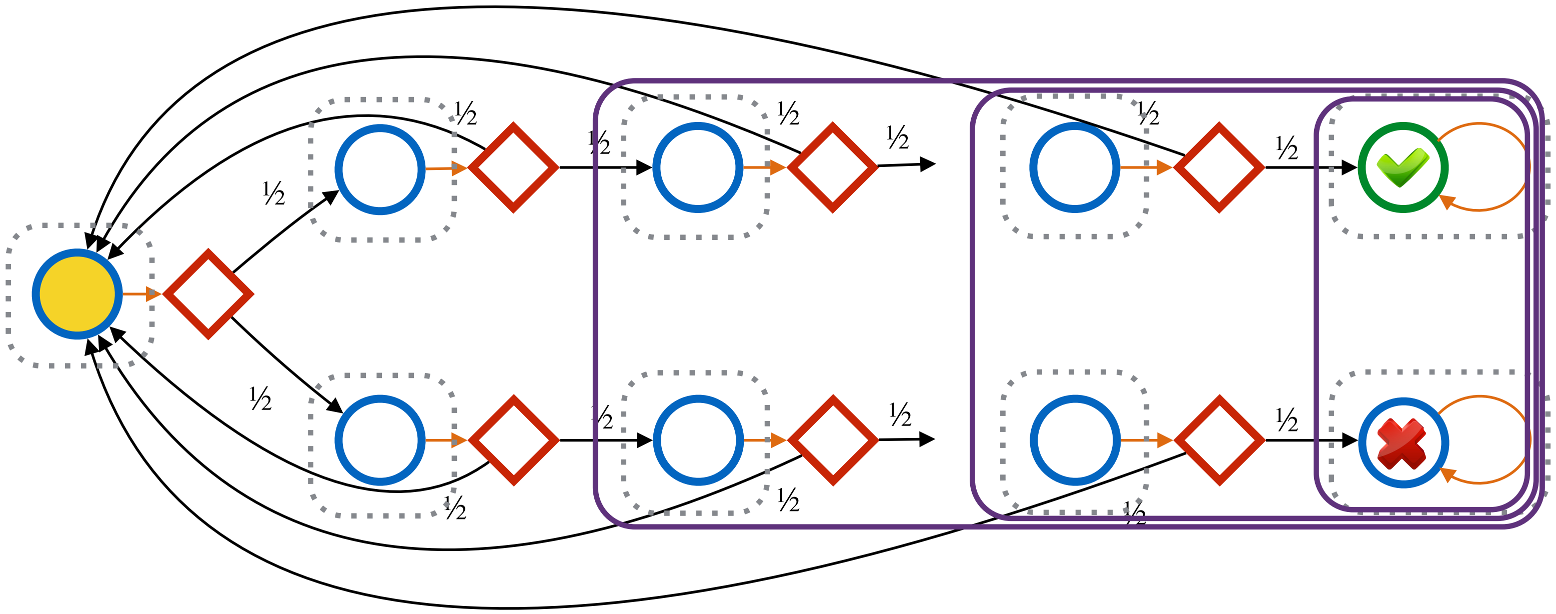

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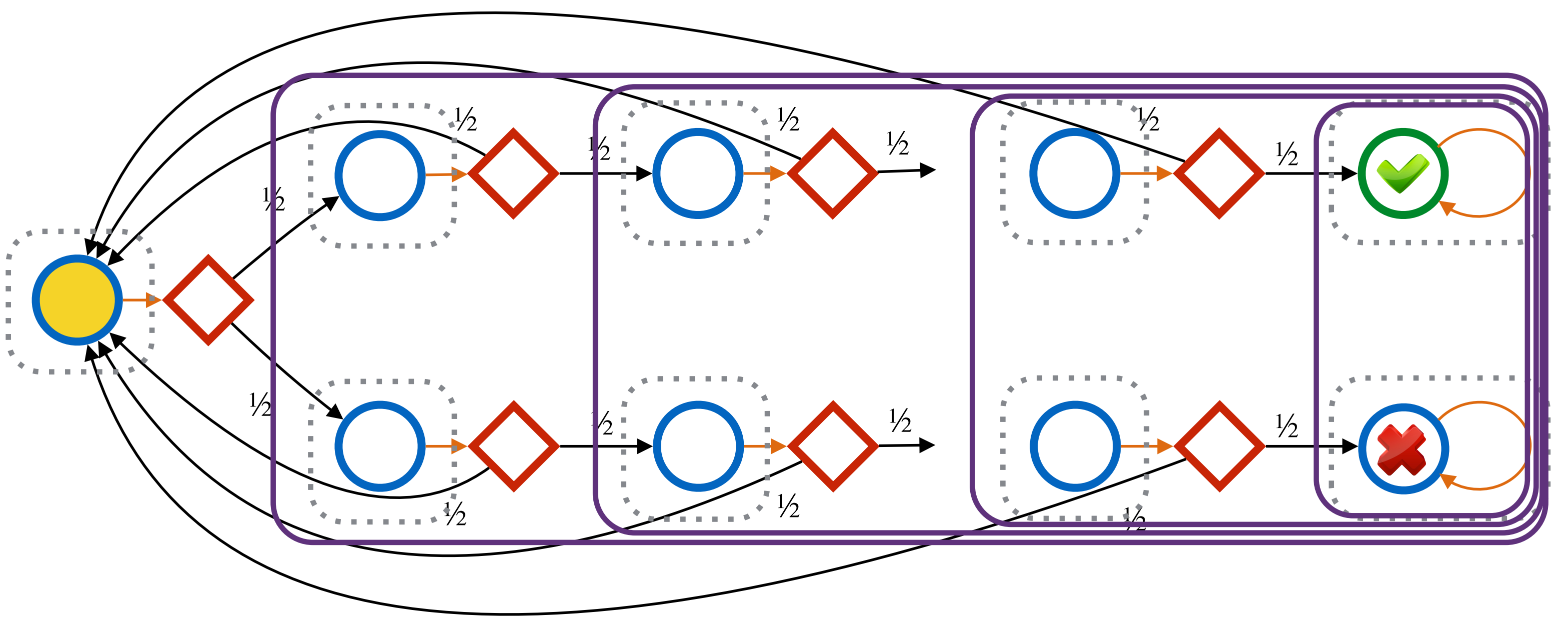

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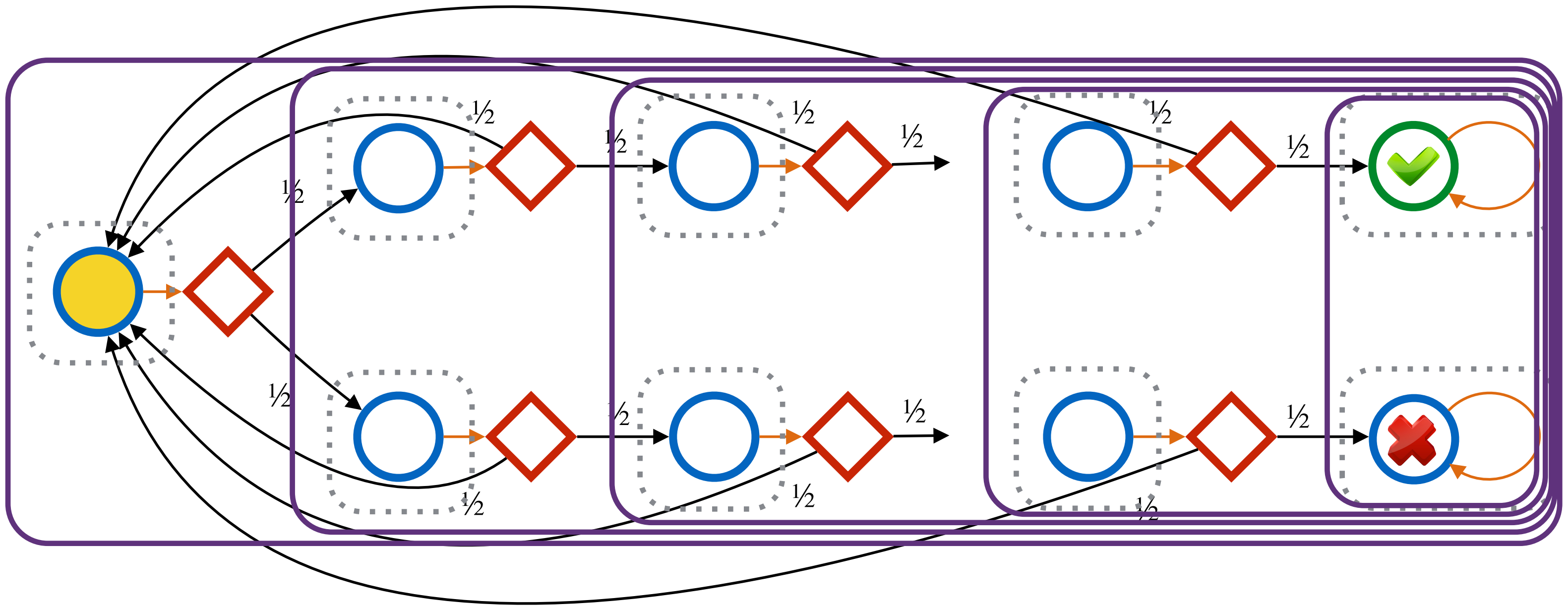

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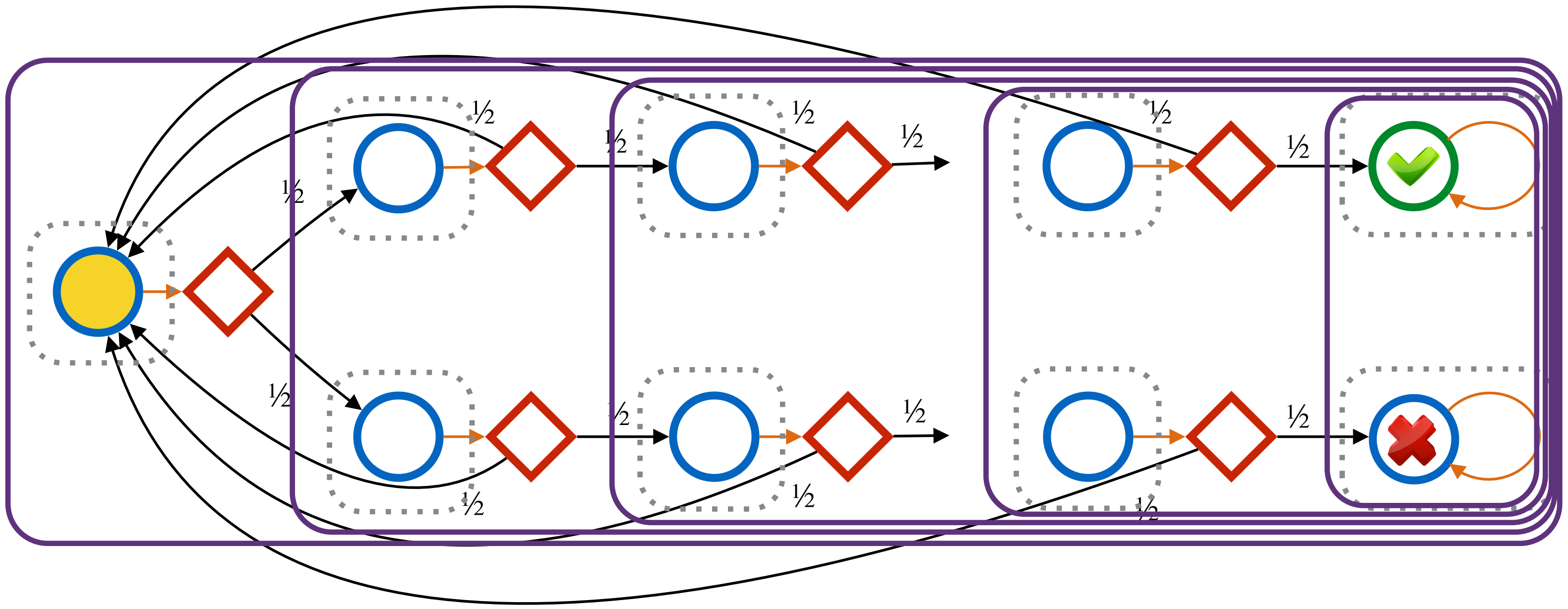

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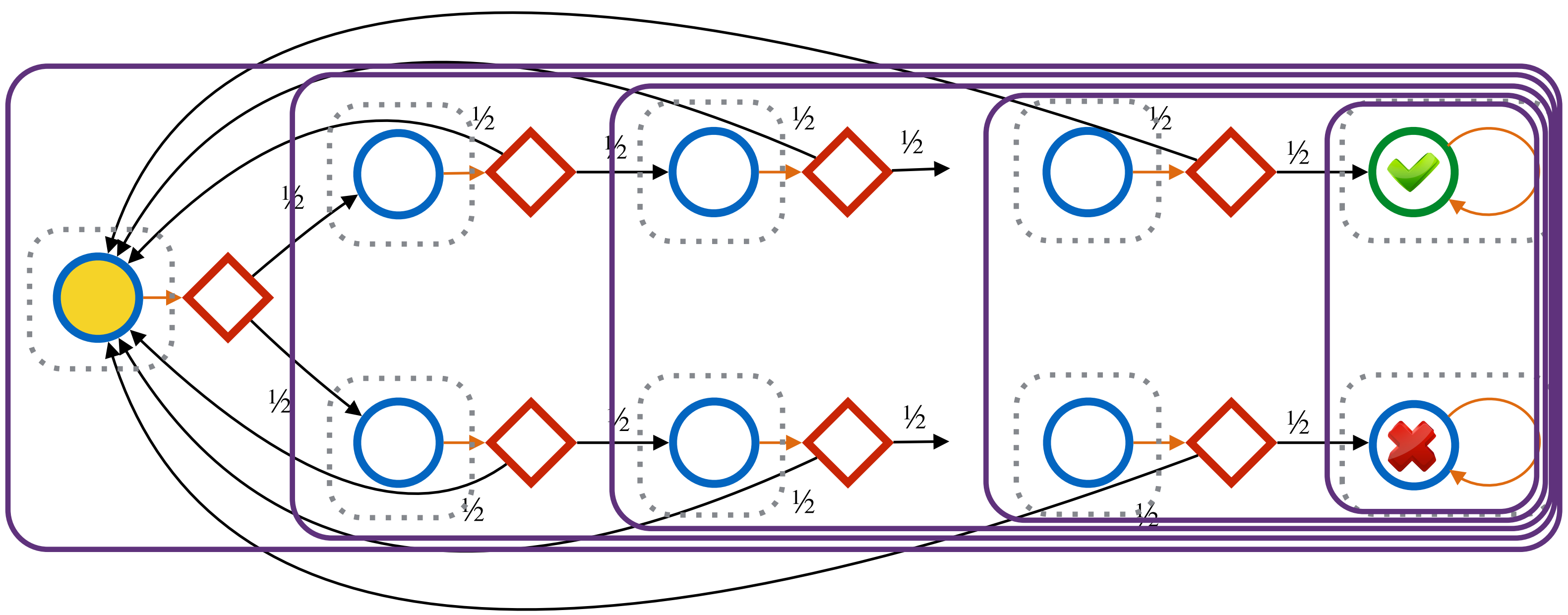

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Leaking property: $\forall n \in \mathbb{N} \quad \Pr_s^{\max}(\mathsf{G}^{\leq nI} \neg(\checkmark \vee \times)) \leq (1-\eta^I)^n$

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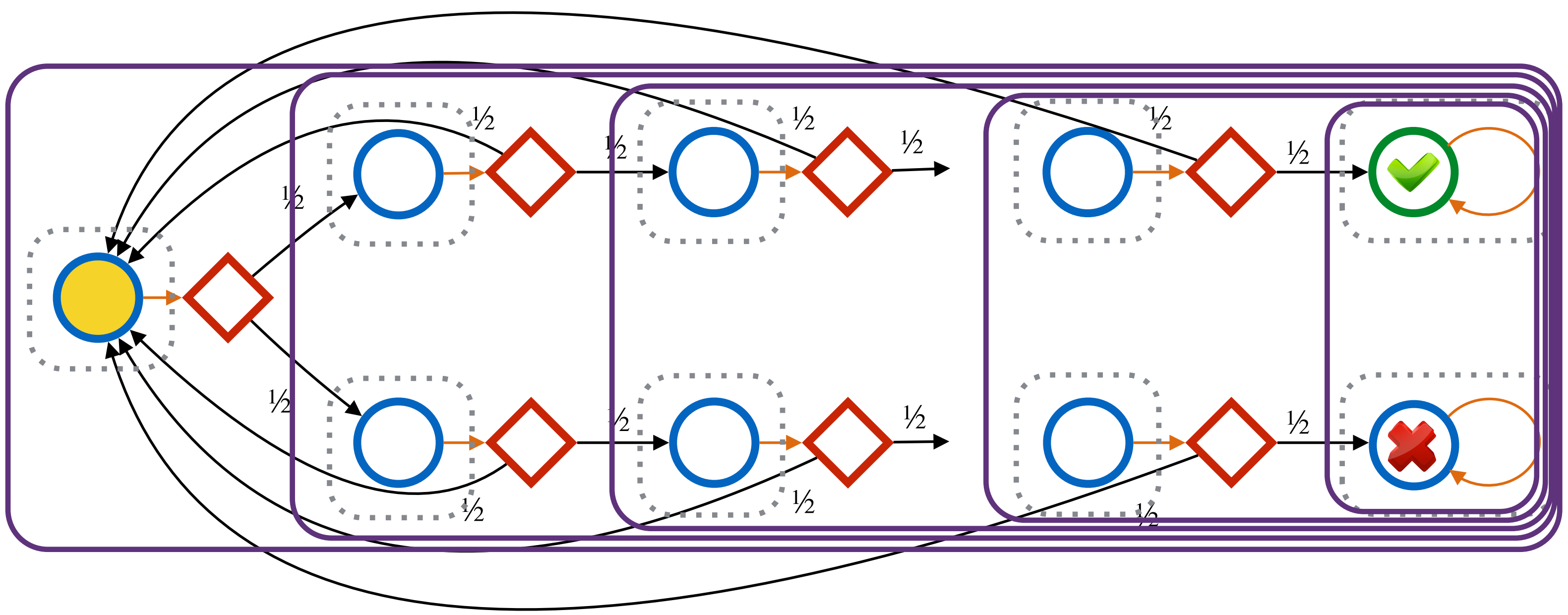

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$y_s^{(nI)} - x_s^{(nI)} = \Pr_s^{\sigma}(\mathsf{G}^{\leq nI}(\neg$ ✖$)) - \Pr_s^{\sigma'}(\mathsf{F}^{\leq nI}$ ✔$) \leq \Pr_s^{\sigma'}(\mathsf{G}^{\leq nI}(\neg$ ✖$)) - \Pr_s^{\sigma'}(\mathsf{F}^{\leq nI}$ ✔$)$

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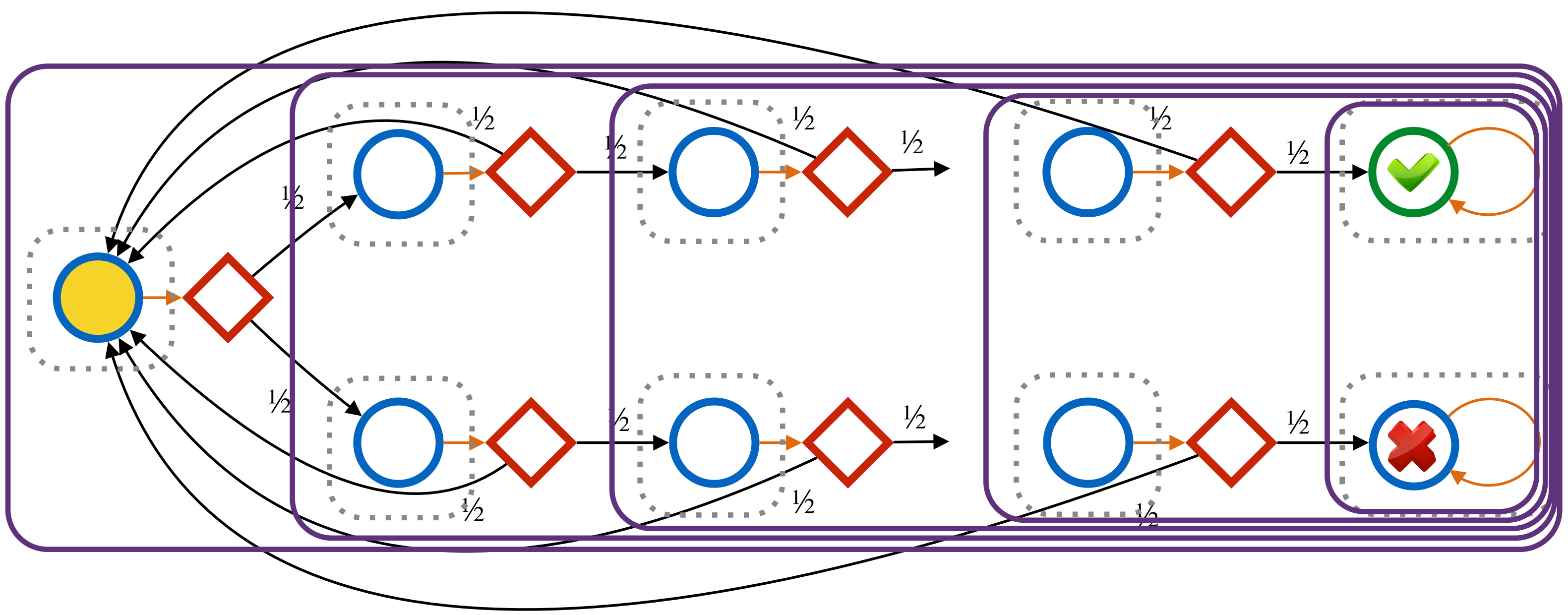

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$$= \Pr_s^{\sigma'}(\mathsf{G}^{\leq nI} \neg(\checkmark \vee \times)) \leq (1-\eta^I)^n$$

since $\mathsf{G}^{\leq n}(\neg \times) \equiv \mathsf{G}^{\leq n} \neg(\checkmark \vee \times) \oplus \mathsf{F}^{\leq n} \checkmark$

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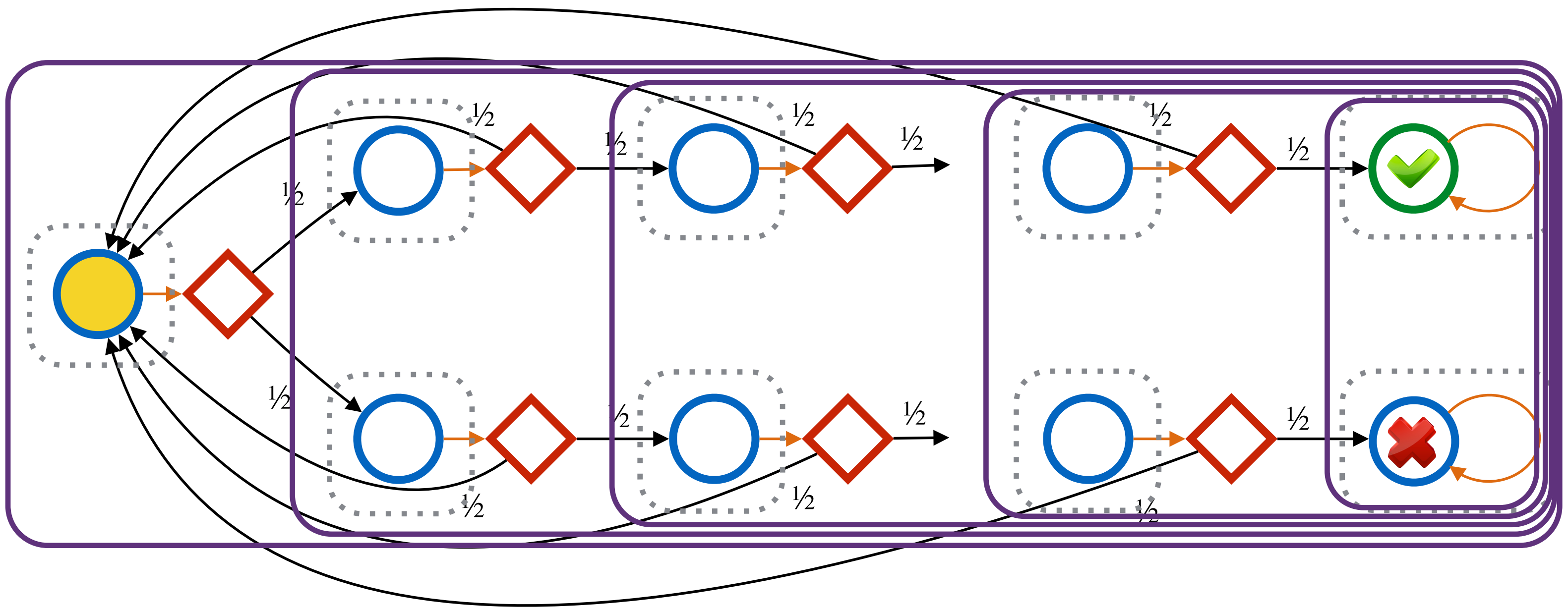

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The interval iteration algorithm converges in at most $I\left\lceil \frac{\log \varepsilon}{\log(1-\eta^I)} \right\rceil$ steps.

Stopping criterion for exact computation

MDPs with rational probabilities:

- d the largest denominator of transition probabilities
- N the number of states
- M the number of transitions with non-zero probabilities

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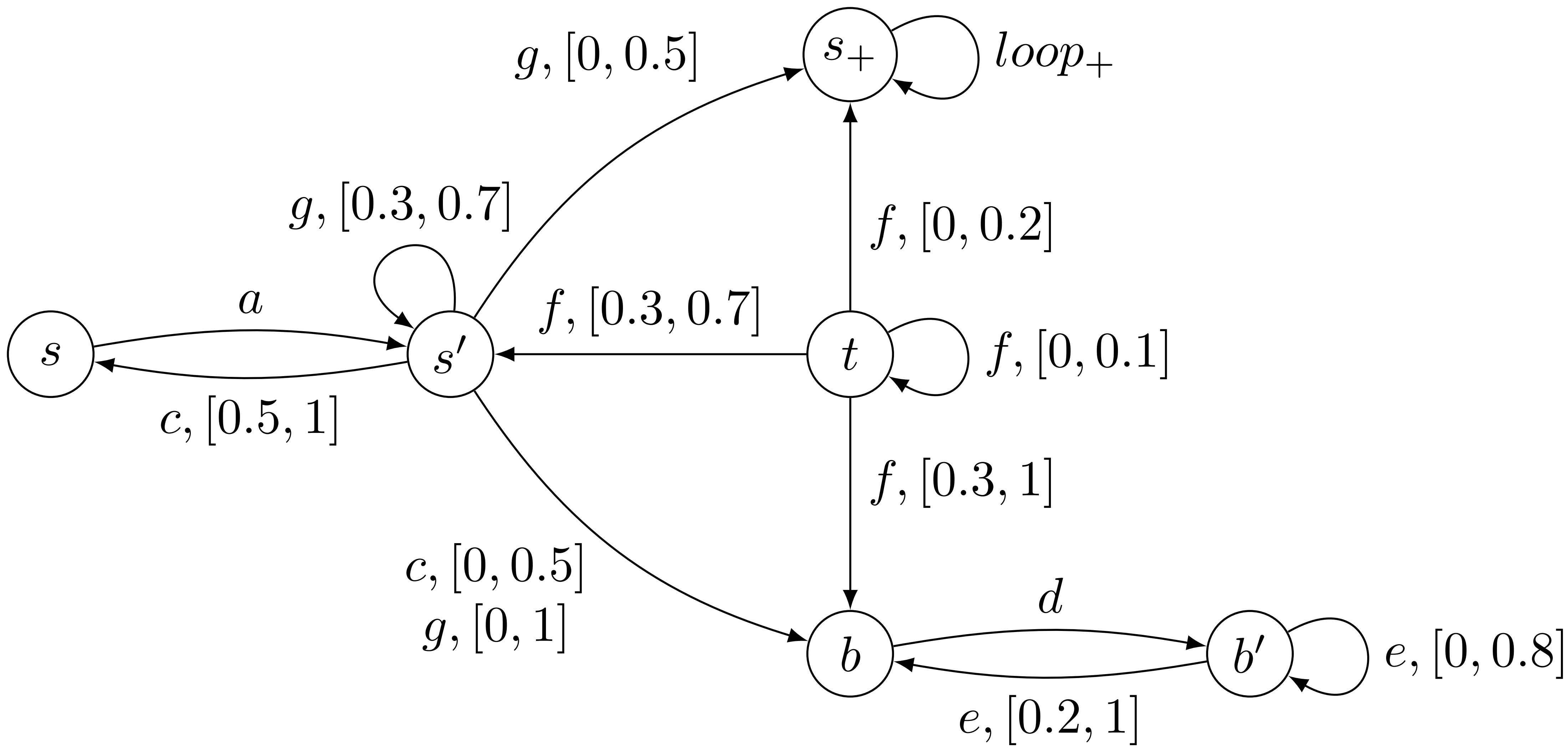
Sketch of proof:

- use $\varepsilon = 1/2\alpha$ as threshold (with α gcd of optimal probabilities)
- upper bound on α based on matrix properties of Markov chains: $\alpha = \mathcal{O}(N^N d^{2N^2})$

Interval MDPs



$\mathcal{M} = (S, \alpha, \breve{\delta}, \widehat{\delta})$

$\delta : S \times \alpha \to [0,1]^S$

Policy $\sigma : (S \cdot \alpha)^\star \cdot S \to Dist(\alpha) \times (Dist(S))^\alpha$

IMDP vs MDP

- IMDPs = extension of MDPs with an infinite (uncountable) set of actions
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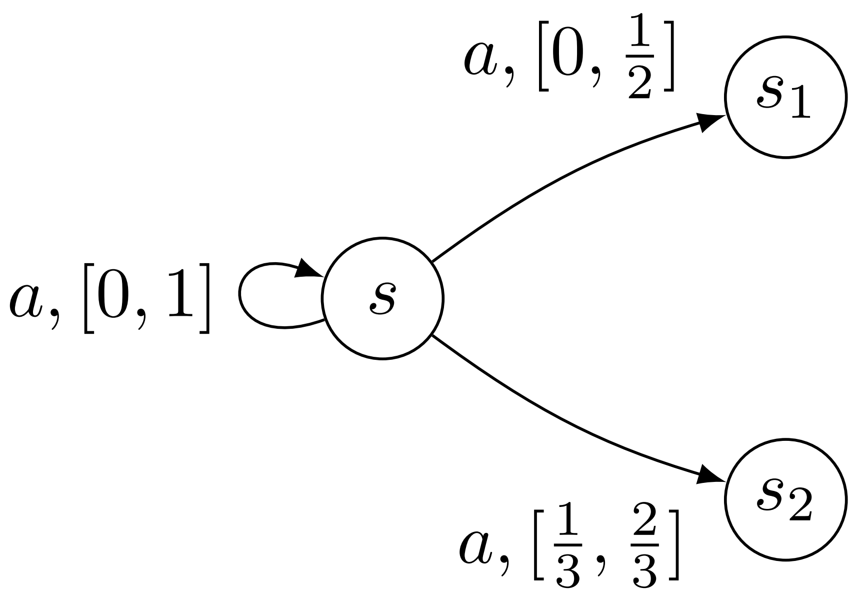

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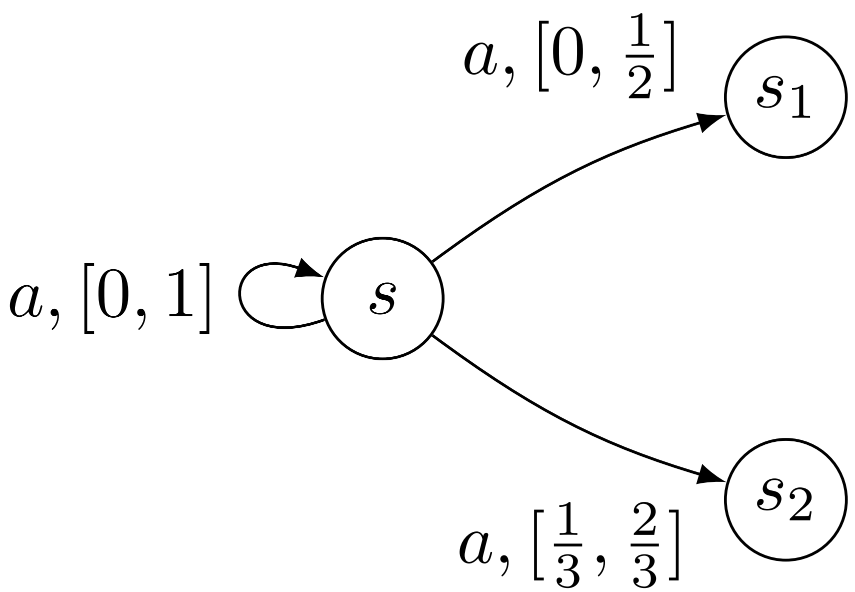

Possible distributions:

$$p \in Dist(S) \text{ such that } \sum_{s' \in S} p(s') = 1$$

$$\text{and } \forall s' \; \breve{\delta}(s' \mid s, a) \leq p(s') \leq \widehat{\delta}(s' \mid s, a)$$

Solutions of a (bounded) linear program!

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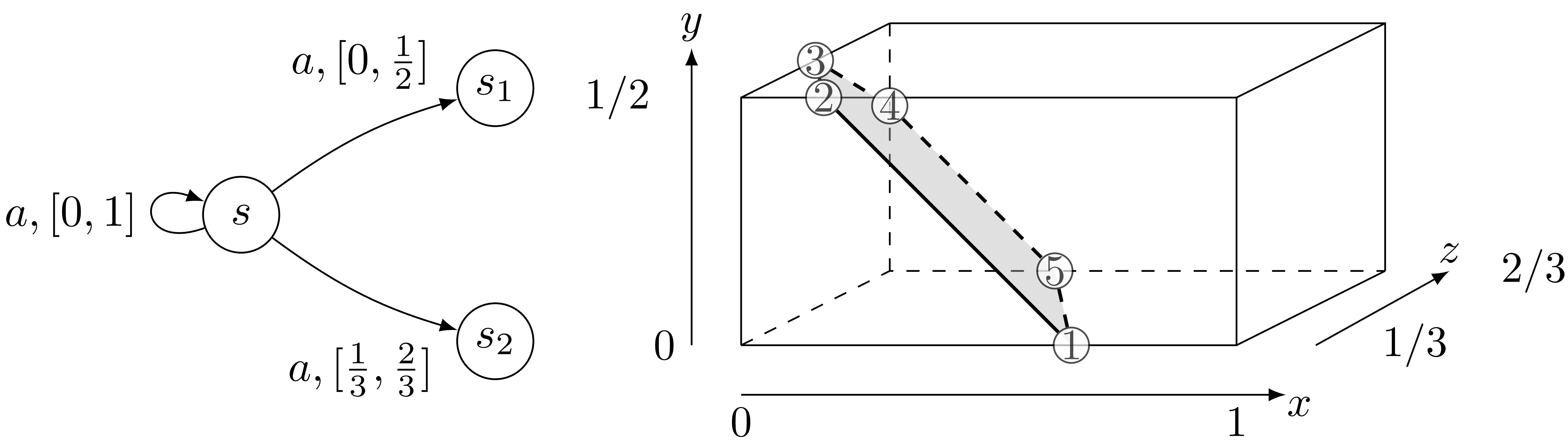

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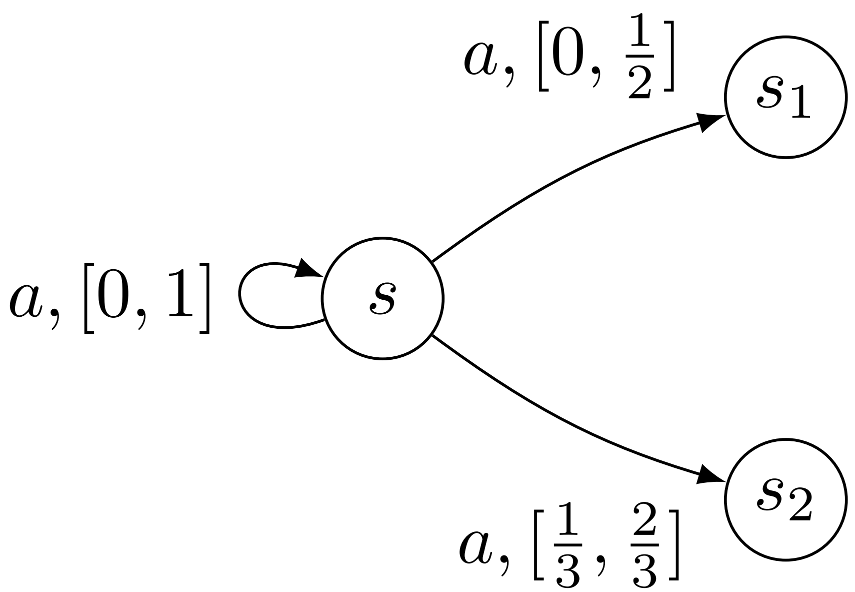

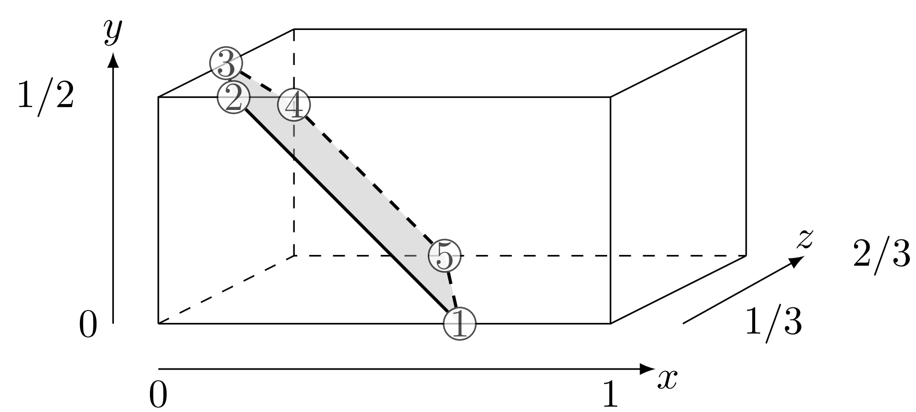

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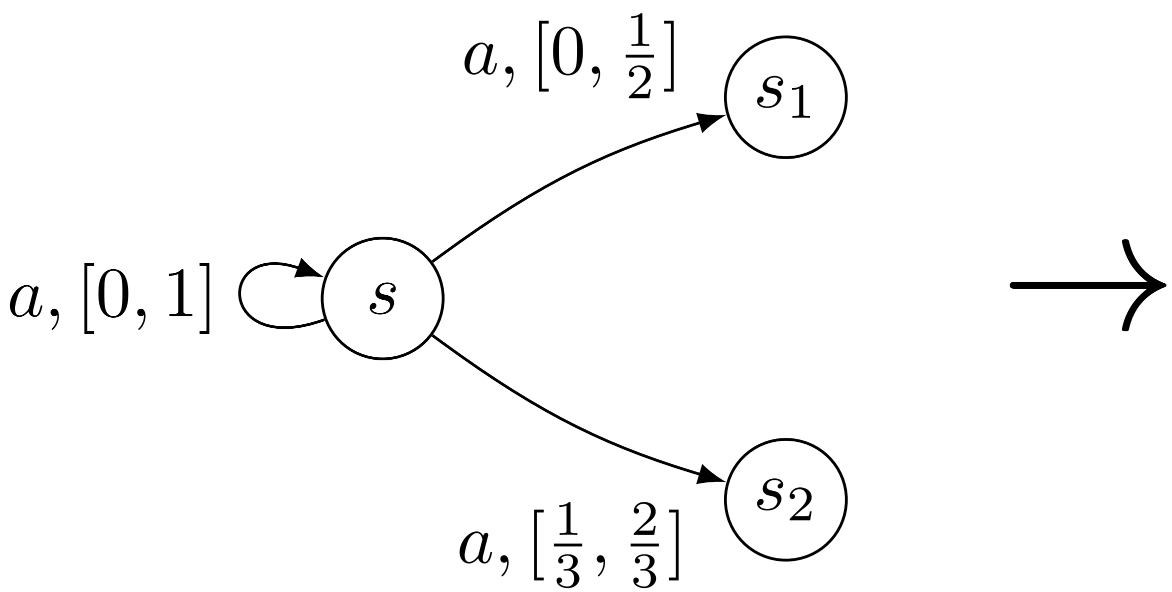

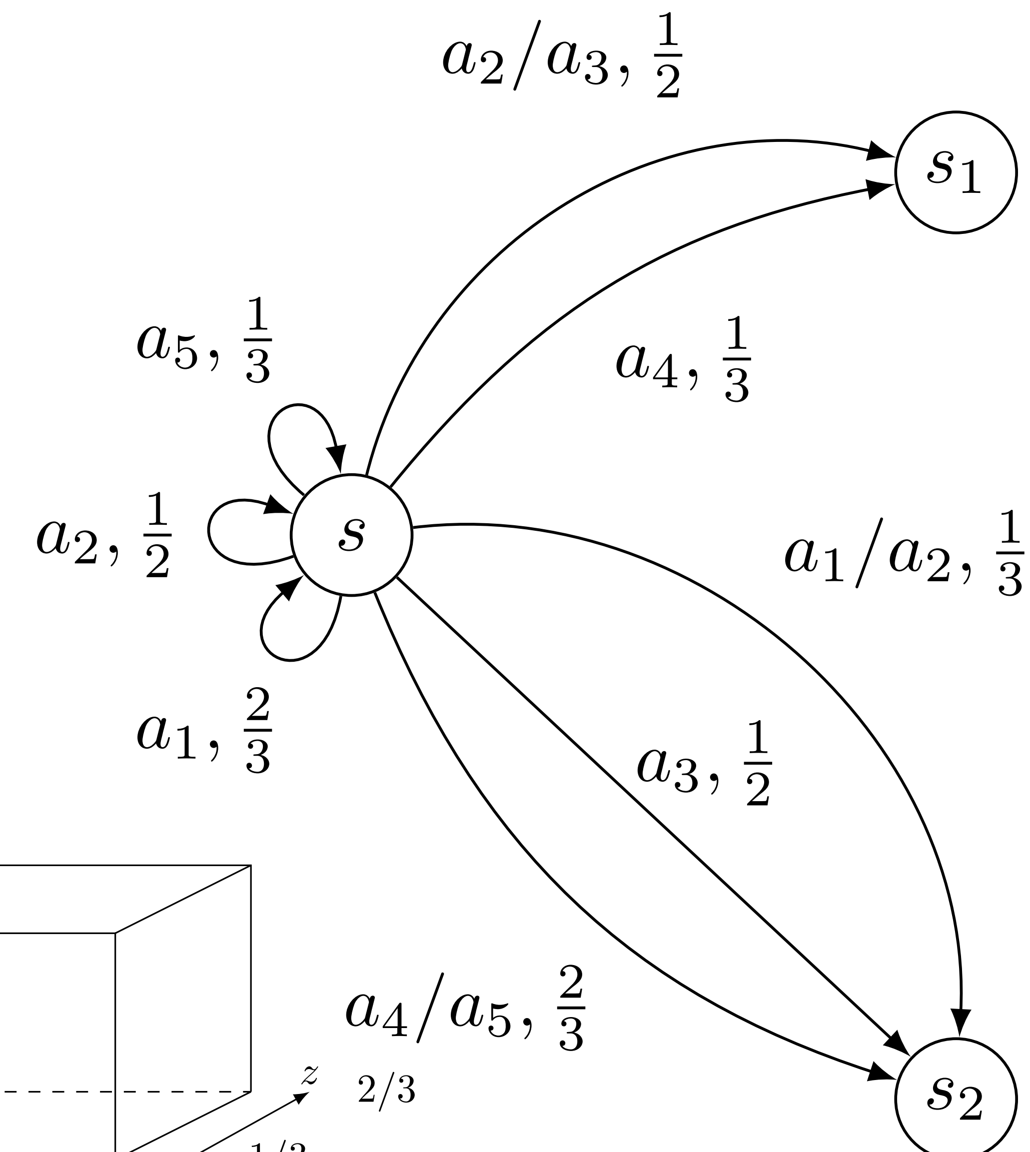

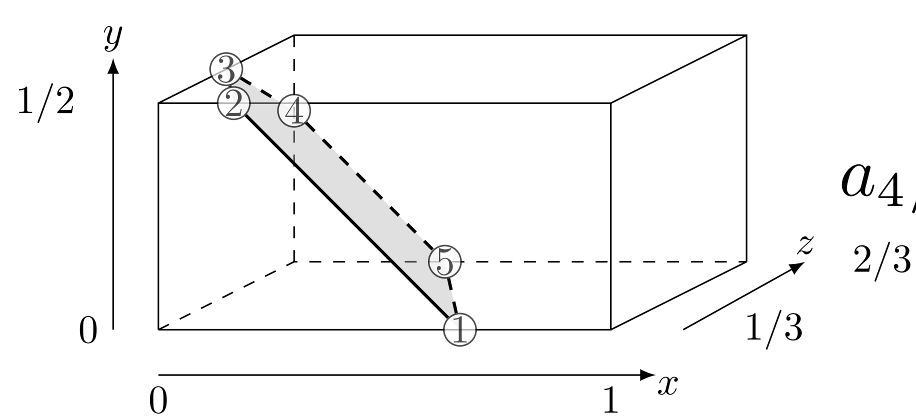

Value iteration for IMDPs

- Simulate on the IMDP the value iteration on its MDP...
- One step is the application of
- Achievable in polynomial time by sorting x...

[Sen, Viswanathan, Agha, 2006]

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$$f_{\max}(x)_s = \max_{a \in A(s)} \max_{p \in \mathrm{BFS}(a)} \sum_{s' \in S} p(s') \times x_{s'}$$

- Achievable in polynomial time by sorting x…

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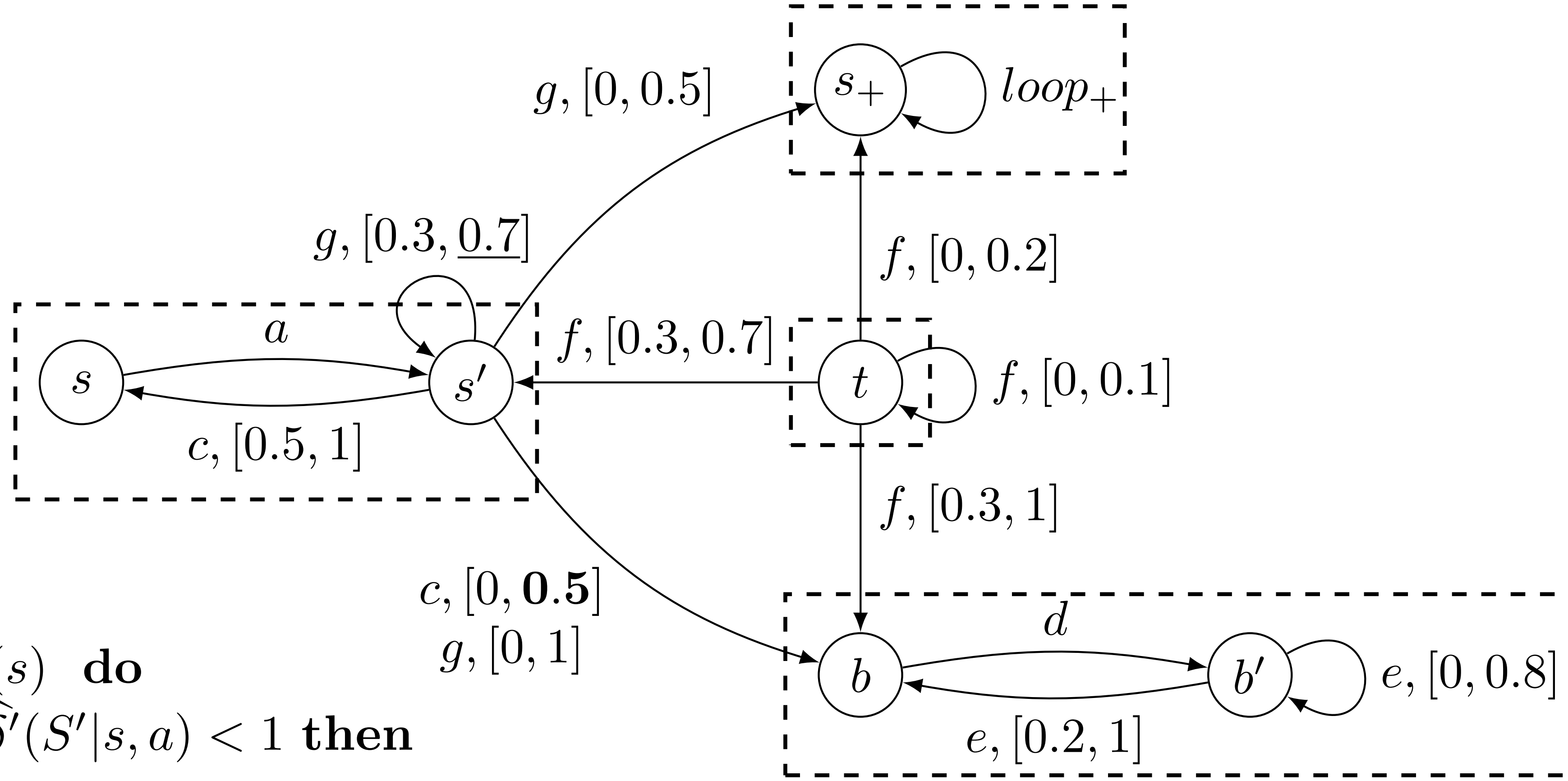
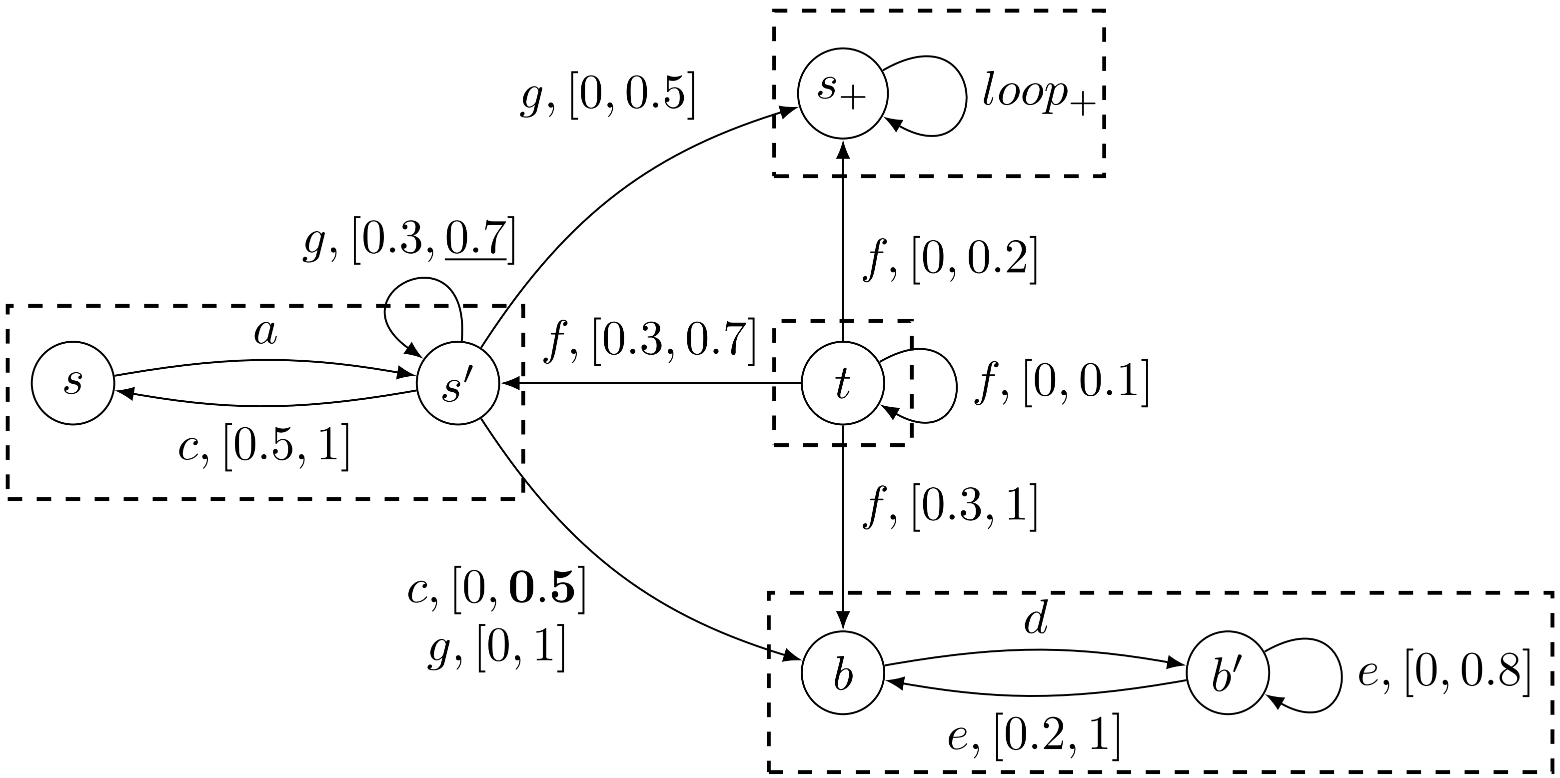
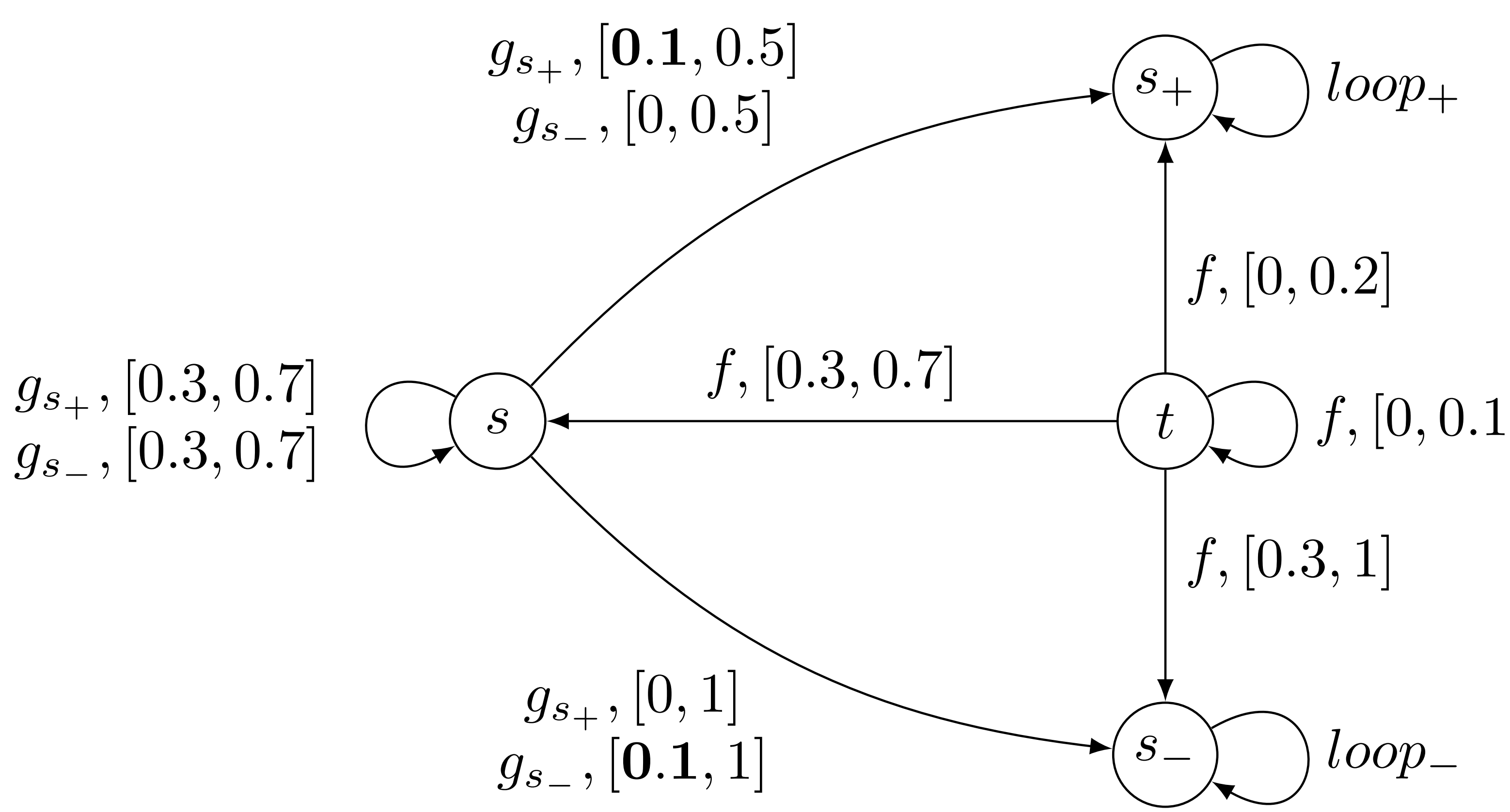
MEC decomposition



Push$(stack, \mathcal{M})$; $\mathcal{SM} \leftarrow \emptyset$
while not Empty$(stack)$ **do**
  $(S', \alpha', \check{\delta}', \widehat{\delta}') \leftarrow$ Pop$(stack)$
  for $s \in S'$ **and** $a \in \alpha' \cap A(s)$ **do**
    if $\check{\delta}'(S \setminus S'|s,a) > 0 \vee \widehat{\delta}'(S'|s,a) < 1$ **then**
      $\alpha' \leftarrow \alpha' \setminus \{a\}$
    else
      for $s' \notin S'$ **do** $\widehat{\delta}'(s'|s,a) \leftarrow 0$
  $E \leftarrow \emptyset$
  for $s, s' \in S'$ **and** $a \in \alpha' \cap A(s)$ **do**
    if $\widehat{\delta}'(s'|s,a) > 0 \wedge \check{\delta}'(S \setminus \{s'\}|s,a) < 1$ **then** $E \leftarrow E \cup \{(s,s')\}$
  compute the strongly connected components of (S', E): $S_1, \ldots, S_K$
  if $K > 1$ **then**
    for $i = 1$ **to** K **do** Push$(stack, (S_i, \alpha' \cap \bigcup_{s \in S_i} A(s), \check{\delta}'|_{S_i}, \widehat{\delta}'|_{S_i}))$
  else $\mathcal{SM} \leftarrow \mathcal{SM} \cup \{(S', \alpha', \check{\delta}', \widehat{\delta}')\}$
return $\mathcal{SM}$

Max-reduction





Conclusion and related work

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- General results on **convergence rate**
- Criterion for computation of **exact value**
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