Metric Interval Temporal Logic Revisited

MOVE seminar

Benjamin Monmege (LIF, Aix-Marseille Université)

Based on joint works with
Thomas Brihaye, Hsi-Ming Ho, Morgane Estiévenart (UMONS),
Gilles Geeraerts (ULB), and Nathalie Sznajder (LIP6)

30/03/2017

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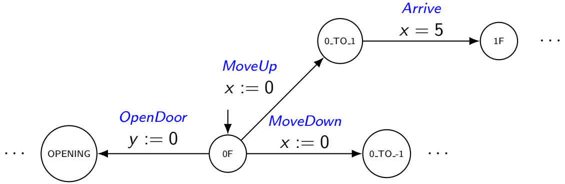

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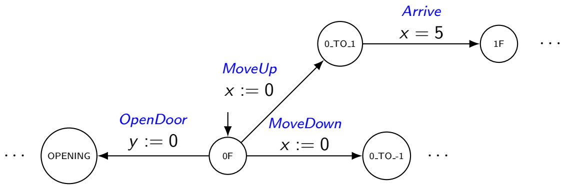

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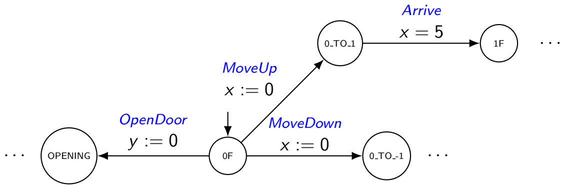
What we consider as a *behaviour* of the system?



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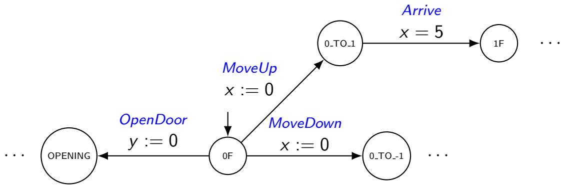

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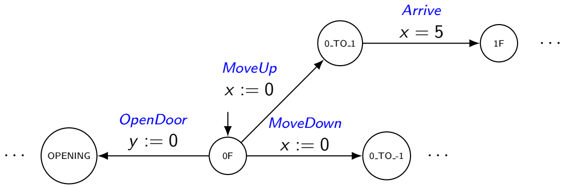

- **Pointwise** (event-based) view: **timed word**

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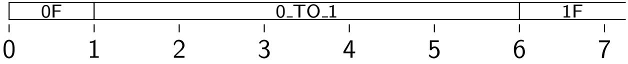
- **Continuous** (state-based) view: **signal** from $\mathbb{R}_{\geqslant 0}$ to states



Metric Temporal Logic (MTL)

$$\varphi ::= \top \mid a \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \, \mathsf{U}_I \, \varphi$$

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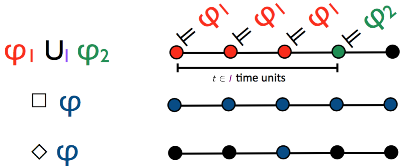

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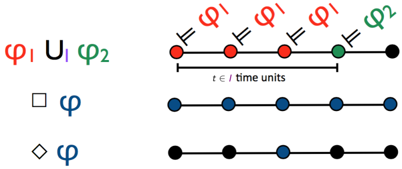

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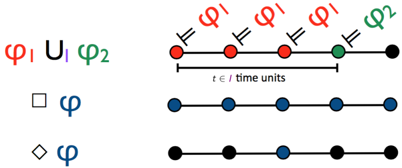

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   - check whether a specification is consistent
2. Model-check a timed model against an MTL formula
   - verification of the system
3. Synthesise a valid system from an MTL specification, under certain restrictions on the environment
   - reactive synthesis task

Part 1: satisfiability and model-checking

Based on a joint work with Thomas Brihaye (UMONS),
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Submitted at CAV 2017 @ Heidelberg

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Can we find a *fully decidable* subclass?

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ECL with projection (i.e. outermost second-order quantification) is equally expressive as timed automata.

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- The *de facto* standard tool for timed automata

UPPAAL in a nutshell
KG Larsen, P Pettersson, W Yi - International journal on software tools for ..., 1997 - Springer
Abstract. This paper presents the overal structure, the design criteria, and the main features of the tool box **Uppaal**. It gives a detailed user guide which describes how to use the various tools of **Uppaal** version 2.02 to construct abstract models of a real-time system, to simulate
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A tutorial on **uppaal**
G Behrmann, A David, KG Larsen - Formal methods for the design of real- ..., 2004 - Springer
Abstract This is a tutorial paper on the tool **Uppaal**. Its goal is to be a short introduction on the flavor of timed automata implemented in the tool, to present its interface, and to explain how to use the tool. The contribution of the paper is to provide reference examples and
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Other direction of research: usage of SMT solvers [Bersani, Rossi, and San Pietro, 2015, Kindermann, Junttila, and Niemelä, 2013, Woźna-Szcześniak, Szcześniak, M. Zbrzezny, and Zbrzezny, 2014]

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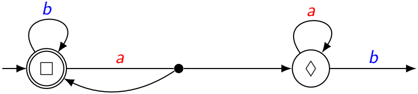

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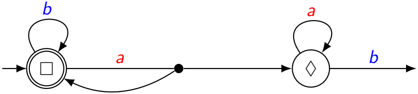
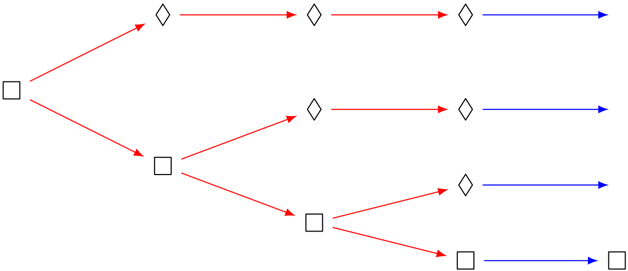


A run on *aaab*:



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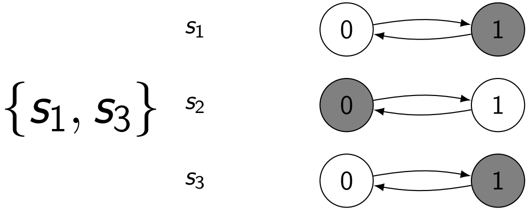

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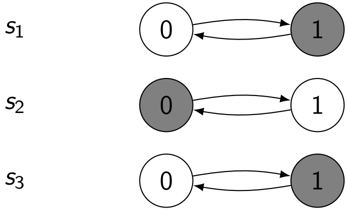

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How to synchronise these components?

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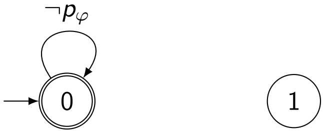

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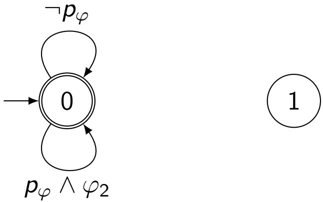

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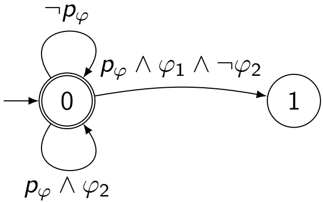

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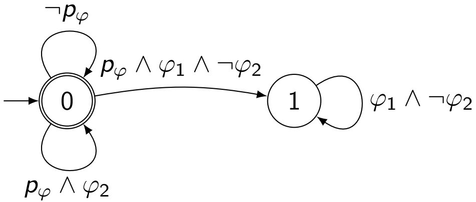

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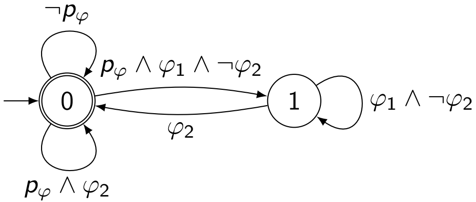

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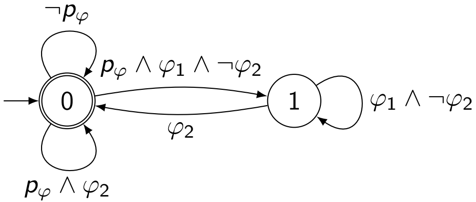

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If $\mathcal{C}_{\varphi_1 \mathcal{U} \varphi_2}$ accepts a (timed) word ρ then $\rho \models \Box(p_\varphi \Rightarrow \varphi_1 \,\mathcal{U}\, \varphi_2)$.

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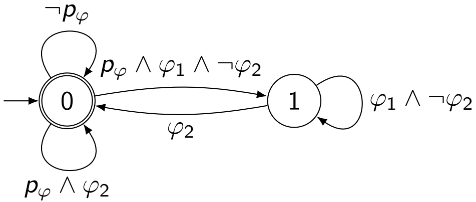

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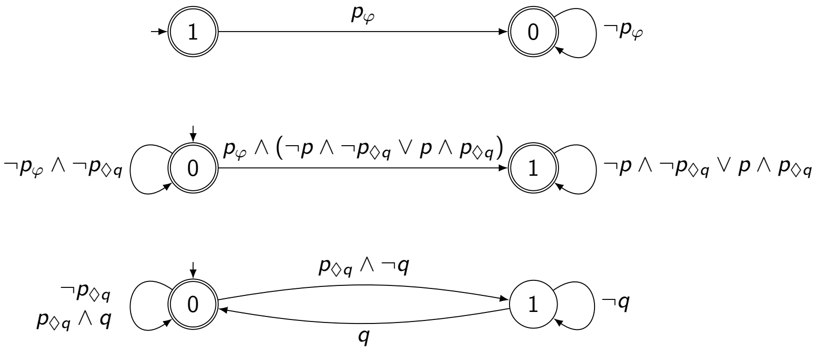
Proposition:

For each LTL formula φ over AP, we can construct a Büchi automaton $\mathcal{A}_\varphi = \mathcal{C}_{\psi_1} \times \cdots \times \mathcal{C}_{\psi_n}$ over AP $\cup$ AP' such that $\mathcal{L}(\varphi) = \mathcal{L}(\mathrm{proj}_{\mathrm{AP}}(\mathcal{A}_\varphi))$.

Compositional Gastin-Oddoux: full example
$\varphi = \Box(p \Rightarrow \Diamond q) \equiv \bot\,\mathcal{R}\,(\neg p \vee \top\,\mathcal{U}\,q)$



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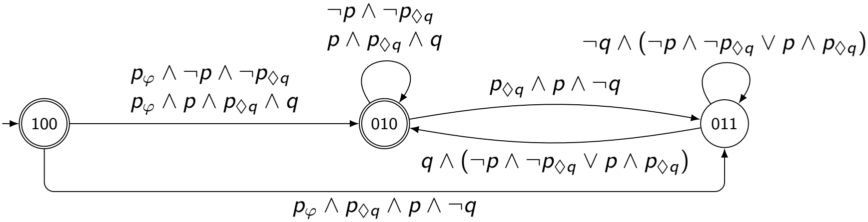

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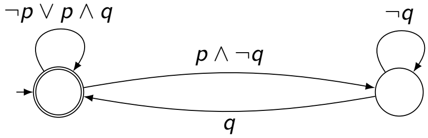

From MITL to one-clock alternating timed automata (OCATA) [Ouaknine and Worrell, 2005]

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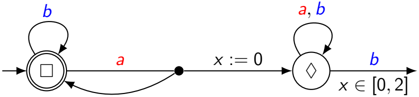

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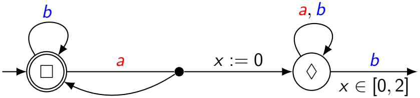
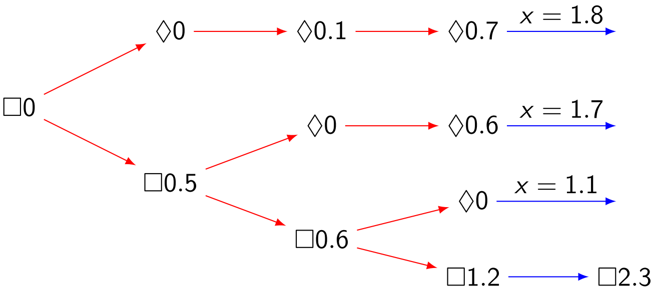


A run on $(a, 0.5)(a, 0.6)(a, 1.2)(b, 2.3)$:



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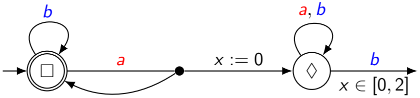

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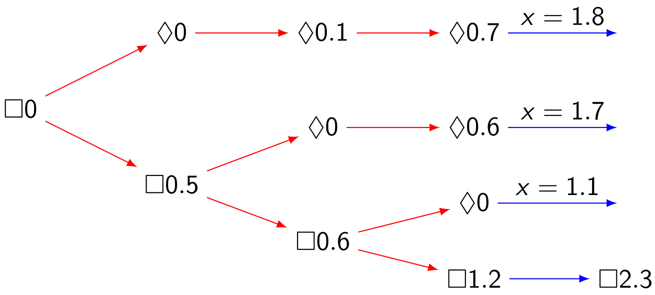

In this case we can simply keep the 'oldest' $\Diamond$.

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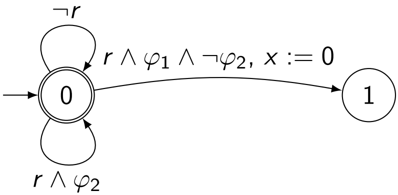

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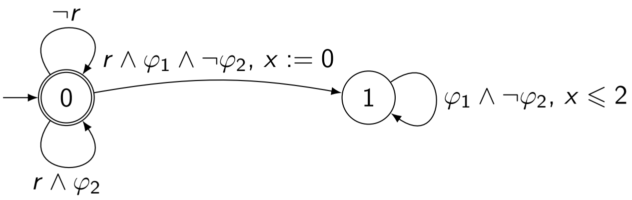

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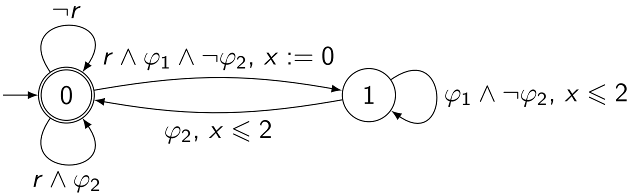

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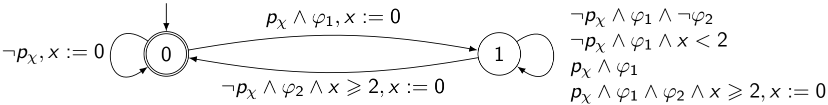

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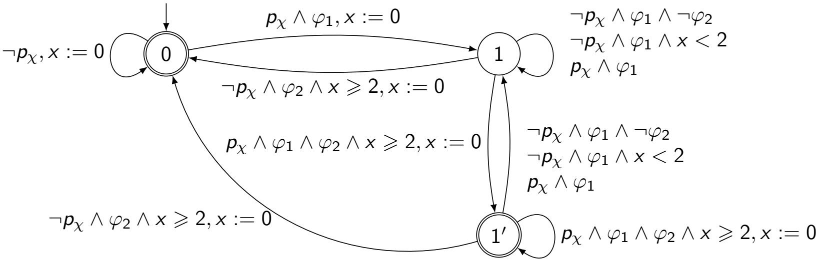

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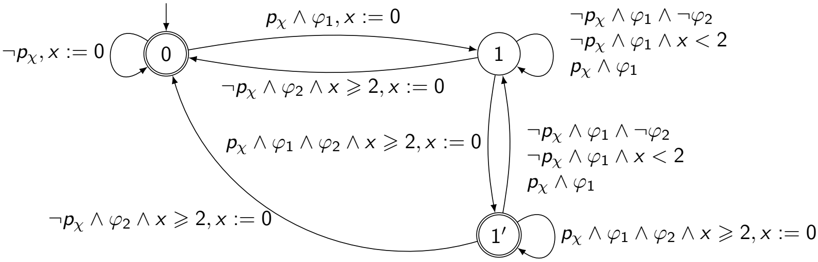

Proposition:

For each $\mathsf{MITL}_{0,\infty}$ formula φ with n timed subformulas, we can construct a projection-equivalent timed automaton $\mathcal{A}_\varphi$ that uses n clocks.

Full MITL: inspired by interval semantics for OCATA

New semantics for OCATA [Brihaye, Estiévenart, and Geeraerts, 2013]:

- allows one to bound the number of clock copies
- sufficiently expressive for MITL

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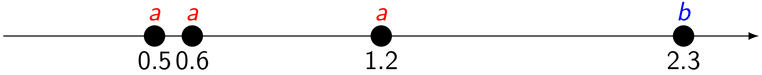

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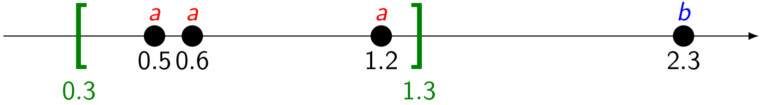

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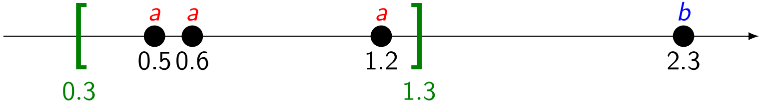
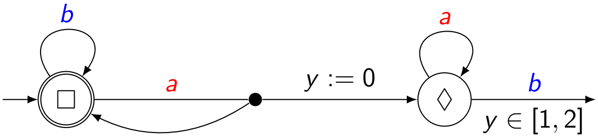
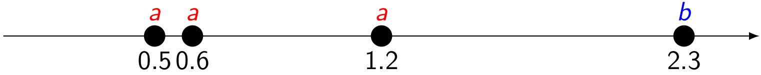


To check that this timed word satisfies φ, we **do not need to remember** the exact timestamp of each a

Example run with the interval semantics





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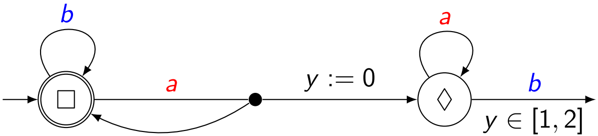
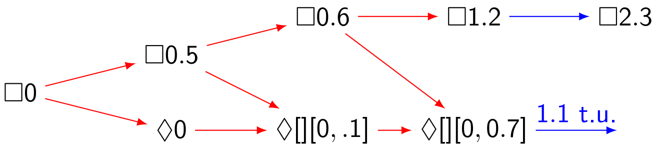
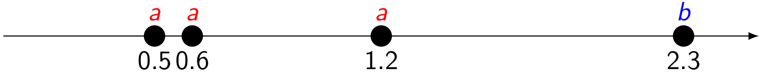
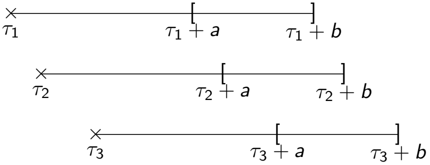






$\varphi = \varphi_1 \, \mathcal{U}_{[a,b]} \, \varphi_2$, with $0 < a < b < +\infty$: improvement on the interval semantics



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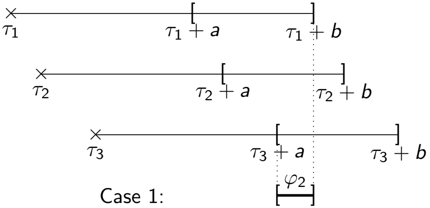

Case 1: $\{1, 2, 3\}$

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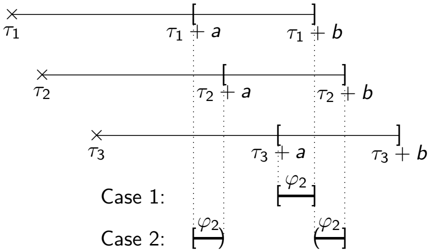

Case 1: $\{1, 2, 3\}$
Case 2: $\{1\}$, $\{2, 3\}$

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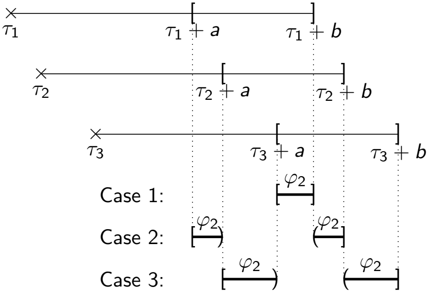

Case 1: $\{1, 2, 3\}$
Case 2: $\{1\}$, $\{2, 3\}$
Case 3: $\{1, 2\}$, $\{3\}$

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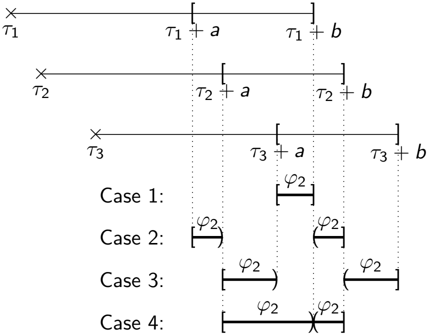

Case 1: $\{1, 2, 3\}$
Case 2: $\{1\}$, $\{2, 3\}$
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Case 4: $\{1, 2\}$, $\{2, 3\}$ or $\{1, 2, 3\}$, $\{2, 3\}$

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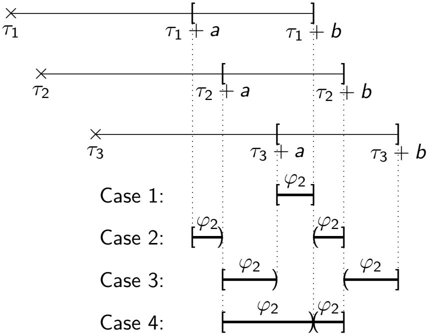

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In each case we only need to keep track of two clock values.

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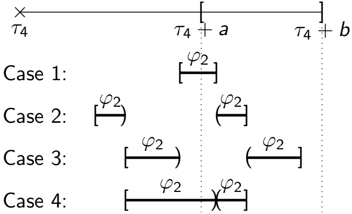

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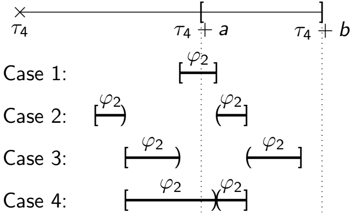

Case 1: Another branching into Case 1, Case 3 and Case 4
Case 2: Done
Case 3: Done
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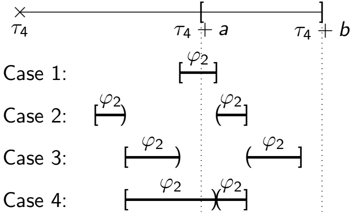

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Proposition:

For each MITL formula $\varphi = \varphi_1 \, \mathcal{U}_I \, \varphi_2$, $\mathcal{C}_\varphi$ uses $2 \cdot \lceil \frac{\sup I}{|I|} \rceil + 2$ clocks.

Up to half the number of clocks obtained in [Brihaye, Estiévenart, and Geeraerts, 2014]

Experiments

We have implemented the translation in the tool MightyL.

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$F(k,I) = \bigwedge_{i=1}^{k} \Diamond_I p_i$

$G(k,I) = \bigwedge_{i=1}^{k} \Box_I p_i$

$U(k,I) = (\cdots (p_1 \, \mathcal{U}_I \, p_2) \, \mathcal{U}_I \cdots) \, \mathcal{U}_I \, p_k$

$R(k,I) = (\cdots (p_1 \, \mathcal{R}_I \, p_2) \, \mathcal{R}_I \cdots) \, \mathcal{R}_I \, p_k$

$\theta(k,I) = \neg((\bigwedge_{i=1}^{k} \Box\Diamond p_i) \Rightarrow \Box(q \Rightarrow \Diamond_I r))$

$\mu(k) = \bigwedge_{i=1}^{k} \Diamond_{[3(i-1),3i]} t_i \wedge \Box\neg p$

Formula	MightyL	LTSmin	Uppaal
$F(5,[0,\infty))$	9ms	3.48s/2.18s/0.12s	0.75s
$F(5,[0,2])$	7ms	3.76s/2.23s/0.15s	0.84s
$F(5,[2,\infty))$	6ms	3.76s/2.26s/0.91s	1.64s
$F(3,[1,2])$	70ms	6m5.15s/38.01s/0.22s	9.00s
$F(5,[1,2])$	70ms	$>$15m	2m6s
$G(5,[0,\infty))$	10ms	3.83s/2.43s/0.05s	0.75s
$G(5,[0,2])$	10ms	4.01s/2.51s/0.10s	0.82s
$G(5,[2,\infty))$	9ms	4.06s/2.47s/0.04s	0.85s
$G(5,[1,2])$	15ms	7.81s/2.99s/0.09s	1.12s
$\mu(1)$	13ms	-	0.39s
$\mu(2)$	21ms	-	2.33s
$\mu(3)$	76ms	-	15.77s
$\mu(4)$	87ms	-	2m23s

Formula	MightyL	LTSmin	Uppaal
$U(5,[0,\infty))$	16ms	1.90s/1.44s/0.05s	0.41s
$U(5,[0,2])$	8ms	2.08s/1.54s/0.06s	0.42s
$U(5,[2,\infty))$	8ms	2.08s/1.53s/0.09s	0.52s
$U(3,[1,2])$	49ms	4m0.14s/23.54s/0.09s	4.92s
$U(5,[1,2])$	97ms	$>$15m	21.80s
$R(5,[0,\infty))$	7ms	1.86s/1.42s/0.03s	0.40s
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$R(5,[2,\infty))$	7ms	1.92s/1.42s/0.03s	0.42s
$R(5,[1,2])$	10ms	5.37s/2.16s/0.04s	0.62s
$\theta(1,[100,1000])$	9ms	1.88s/1.74s/0.04s	0.25s
$\theta(2,[100,1000])$	13ms	5.04s/3.17s/0.19s	0.86s
$\theta(3,[100,1000])$	14ms	36.57s/16.27s/3.20s	21.84s
$\theta(4,[100,1000])$	15ms	5m30s/4m18s/2m16s	18m39s

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$R(5,[2,\infty))$	7ms	1.92s/1.42s/0.03s	0.42s
$R(5,[1,2])$	10ms	5.37s/2.16s/0.04s	0.62s
$\theta(1,[100,1000])$	9ms	1.88s/1.74s/0.04s	0.25s
$\theta(2,[100,1000])$	13ms	5.04s/3.17s/0.19s	0.86s
$\theta(3,[100,1000])$	14ms	36.57s/16.27s/3.20s	21.84s
$\theta(4,[100,1000])$	15ms	5m30s/4m18s/2m16s	18m39s

Formula	MightyL	LTSmin	Uppaal	SMT-based approach
$\Diamond_{[0,30]}(p \Rightarrow \Box_{[0,20]} p)$ valid	7ms	0.98s	0.32s	7s
$\Box_{[0,30]}\neg p \vee \Diamond_{[0,20]} p$ valid	7ms	0.95s	0.14s	not considered
$\Diamond_{[0,30]} p \wedge \Diamond_{[0,20]} p$ redundant	13ms	1.99s	0.44s	14s
$\Box_{[0,20]}\Diamond_{[0,20]} p \wedge \Box_{[0,40]} p \wedge \Diamond_{[20,40]} \top$ redundant	22ms	1m26s	2.63s	not considered

Summary for the satisfiability/model-checking of MITL

Contributions:

- A compositional translation from MITL to timed automata
- An implementation that works with UPPAAL and the like

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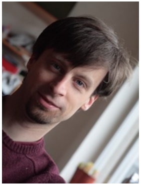
Possible future directions:

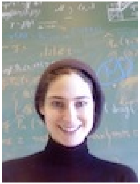
- Native support for ECL
- Past modalities, counting modalities
- Antichain-based optimisations
- ...

Part 2: reactive synthesis

Based on a joint work with Thomas Brihaye (UMONS), Morgane Estiévenart (UMONS), Gilles Geeraerts (ULB), Hsi-Ming Ho (UMONS) and Nathalie Sznajder (LIP6, UPMC)







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Reactive synthesis

$\Sigma = \Sigma_C \uplus \Sigma_E$

- controllable actions owned by controller C:
  $\{$*MoveUp*, *MoveDown*, *OpenDoor*, *Opened*, $\ldots\}$
- uncontrollable actions owned by environment E:
  $\{$*0F-Up*, *0F-Down*, $\ldots$, *-1F*, *0F*, $\ldots$ *Open*, *Close*, $\ldots\}$

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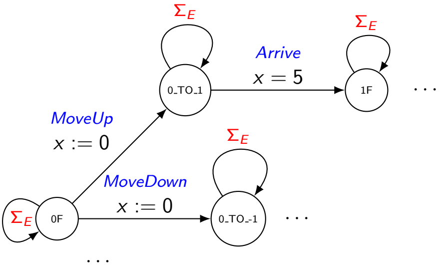

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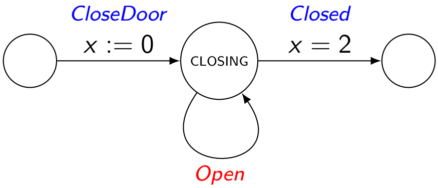

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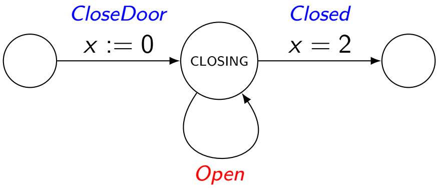

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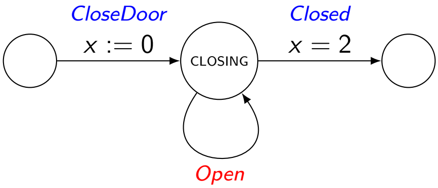

$(\Delta_C, \textit{Closed})$ $(\Delta_E, \textit{Open})$

Only action(s) with the shortest delay $\min(\Delta_C, \Delta_E)$ may be played.

Reactive synthesis

‘The lift responds to any call in less than 10 t.u.’

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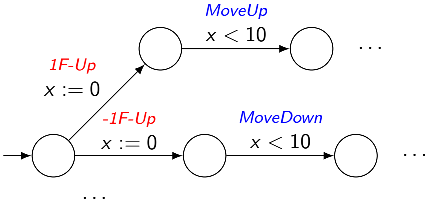

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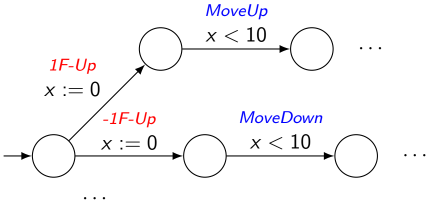

Specification $\mathcal{L}$: a set of timed words over Σ

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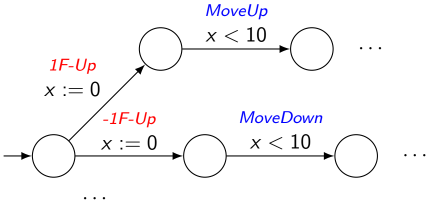

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Reactive synthesis problem (RS)

Given plant $\mathcal{P}$ and specification $\mathcal{L}$, find a strategy of Controller such that no matter what Environment does, every play satisfies the specification.

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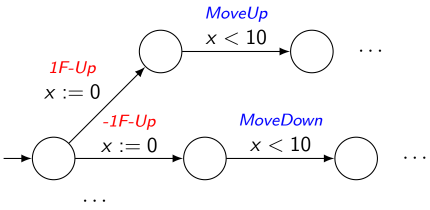

Specification $\mathcal{L}$: a set of timed words over Σ

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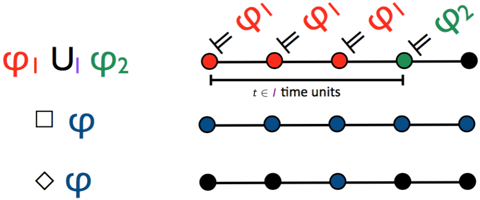
Realizability problem: the special case where all actions are always enabled, i.e., $\mathcal{P}$ is a universal DTA over Σ.

Metric Temporal Logic (MTL)

$$\varphi ::= \top \mid a \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \, \mathsf{U}_I \, \varphi$$

with $a \in \Sigma$, $I \subseteq [0, \infty)$ with bounds in $\mathbb{Q} \cup \{+\infty\}$

Models: finite (or infinite) timed words $\sigma = (a_1, t_1)(a_2, t_2) \cdots$



Theorem: [Doyen, Geeraerts, Raskin, and Reichert, 2009]

Reactive synthesis problem is undecidable for ECL (hence, MTL) specifications, even without plant.

A toy example

Let $\mathcal{P}$ be a universal plant, and the spec be

'Each a is followed exactly 1 t.u. later by a b.'

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The infinite-word realizability problem is undecidable for ECL specifications.

Implementable reactive synthesis (IRS)

C = deterministic symbolic transition system $\mathcal{T}$

- set of locations; if finite $\rightarrow \mathcal{T}$ is a DTA
- finite set of clocks X
- finite set of possible clock constraints *precision* (m, K):

  $$g ::= \top \mid g \wedge g \mid x < \alpha/m \mid x \leqslant \alpha/m \mid x = \alpha/m \mid x \geqslant \alpha/m \mid x > \alpha/m$$

  with $x \in X$, $m \in \mathbb{N}$ and $0 \leqslant \alpha \leqslant K$.

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Implementable reactive synthesis problem (IRS): *Given $\mathcal{P}$ and $\mathcal{L}$, find such a $\mathcal{T}$ that no matter what E does, every play satisfies the specification.*

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NOT CONTROLLABLE for IRS

- each $\mathcal{T}$ has a bounded set of clocks

Reactive synthesis for MTL

Reactive synthesis	Undec.	[Doyen, Geeraerts, Raskin, and Reichert, 2009]

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Controller = timed automaton

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Recovering decidability...

Clock constraints in $\mathcal{T}$:

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Fix X and $(m, K) \Longrightarrow$ the alphabet of $\mathcal{T}$ is given!

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Definition

Bounded-resources reactive synthesis problem (BResRS): *Given $\mathcal{P}$, $\mathcal{L}$, and* **a set of clocks** X **and precision** (m, K), *find such a resource-bounded $\mathcal{T}$ that no matter what E does, every play satisfies the specification.*

Reactive synthesis for MTL

Reactive synthesis	Undec. [Doyen, Geeraerts, Raskin, and Reichert, 2009]

$\downarrow$ Controller = timed automaton

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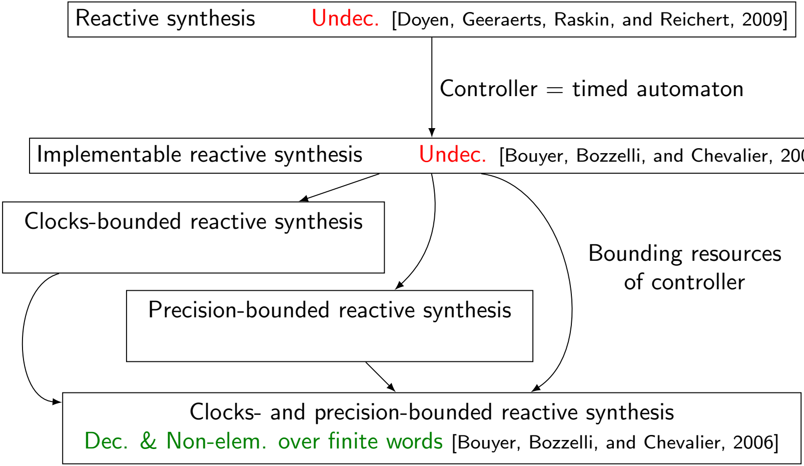
Bounding resources of controller

Clocks- and precision-bounded reactive synthesis
Dec. & Non-elem. over finite words [Bouyer, Bozzelli, and Chevalier, 2006]

Reactive synthesis for MTL



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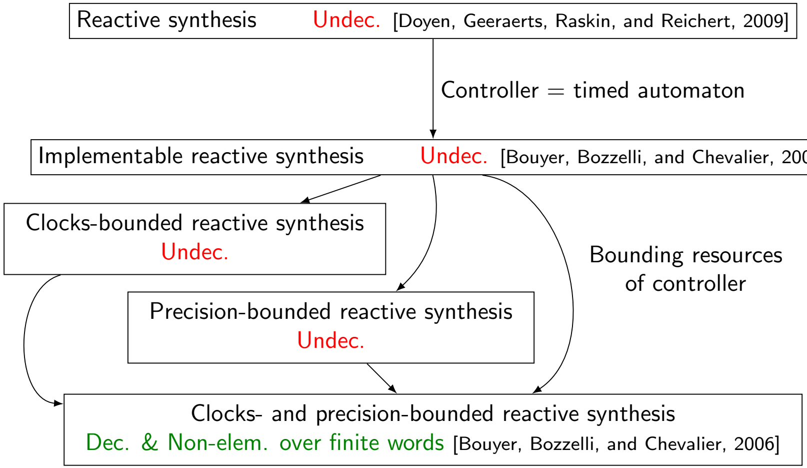


Regaining hope? Less expressive specifications

Undecidability proofs heavily use 'punctuality' of MTL:
$\mathit{request} \to \Diamond_{=1} \mathit{grant}$

$$\mathit{request} \to \Diamond_{[1,2]} \mathit{grant}$$

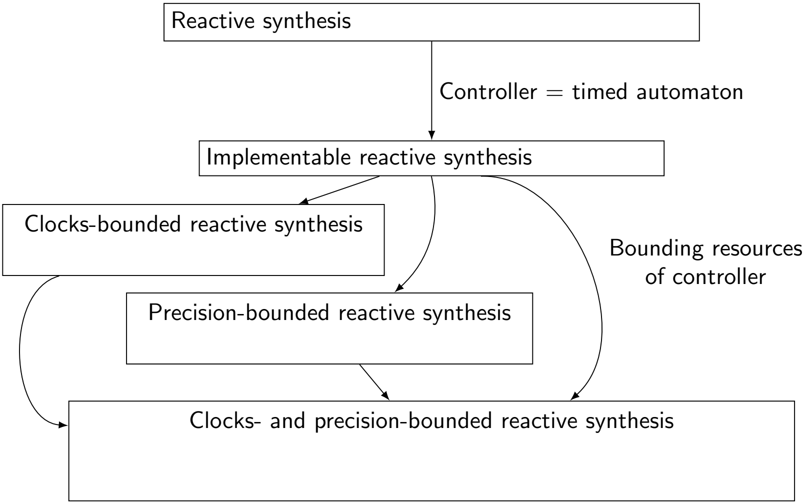
$$\mathit{request} \to \Diamond_{\leqslant 3} \mathit{grant}$$

MITL = non-punctual fragment of MTL:

$$\varphi ::= \top \mid a \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \, \mathsf{U}_I \, \varphi$$

with $a \in \Sigma$, $I \subseteq [0, \infty)$ is a **non-singular** with bounds in $\mathbb{Q} \cup \{+\infty\}$

Contribution: reactive synthesis for MITL



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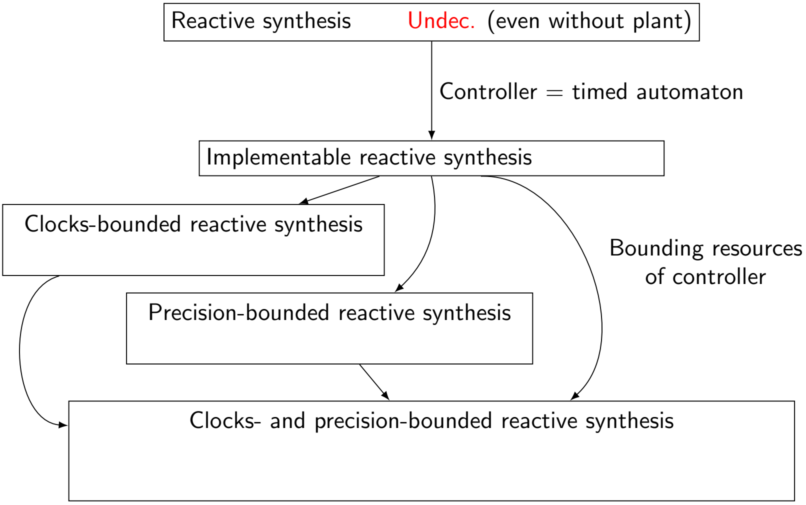

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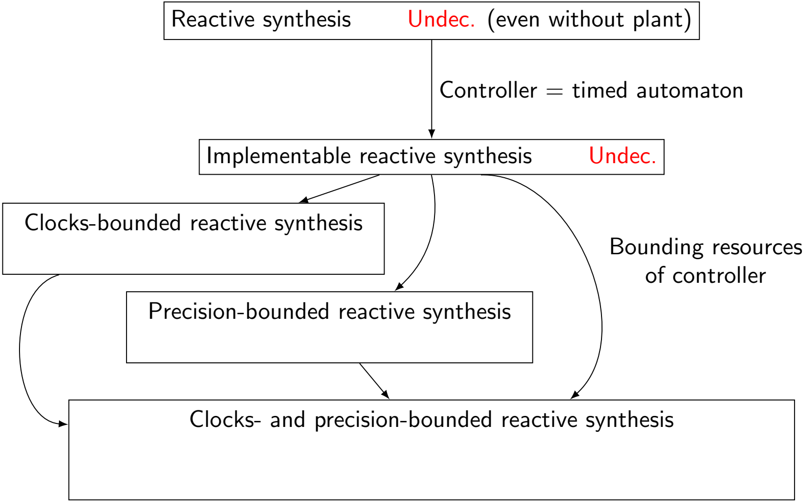

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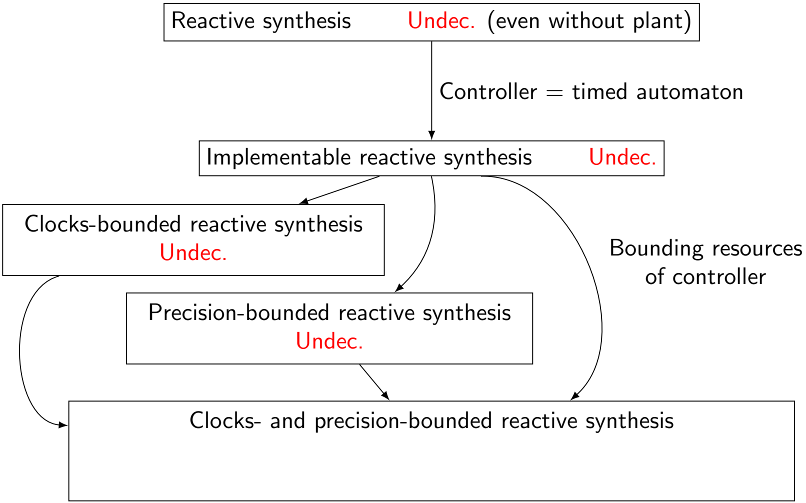

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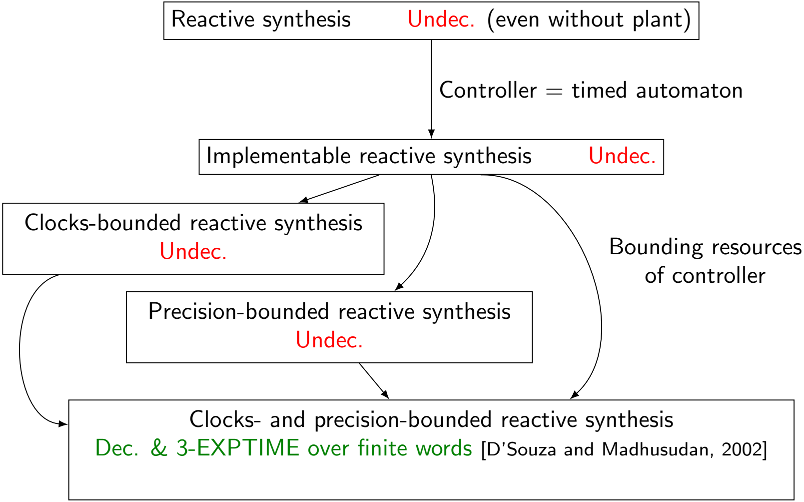

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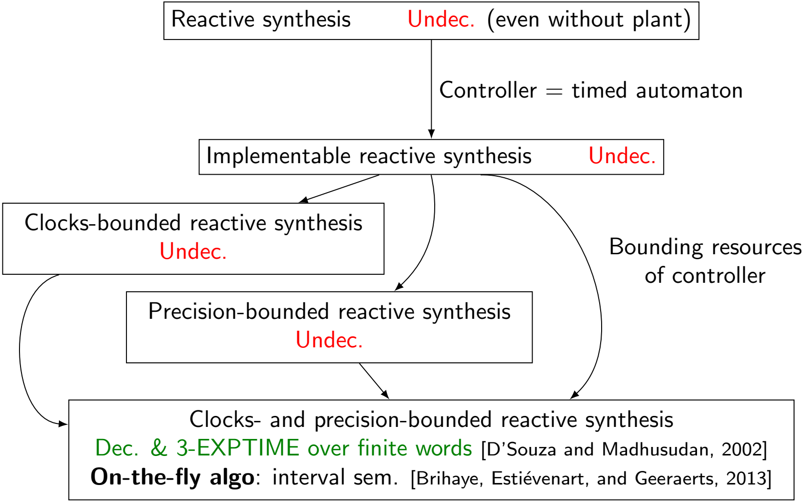
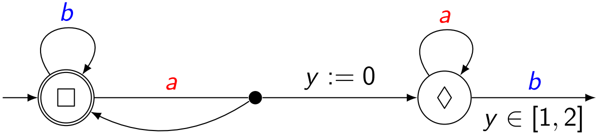


From MTL to OCATA

$$\varphi = \Box(a \Rightarrow \Diamond[[1,2]]b)$$



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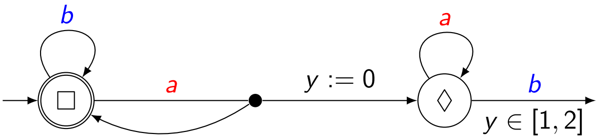
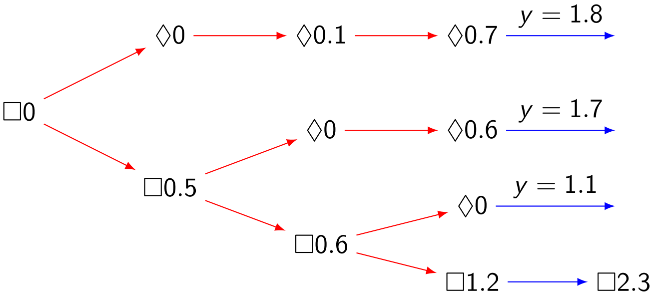


Execution on the timed word $(a, 0.5)(a, 0.6)(a, 1.2)(b, 2.3)$:



Bounded-resources reactive synthesis for MTL

- Action (a, g, R)
  - a: an action in $\Sigma_C \cup \Sigma_C$
  - g: guard over clocks of X and $X_{\mathcal{P}}$
  - R: resets of clocks of X

$$\boxed{(q, \{H_1, H_2, \ldots, H_n\})} \xrightarrow{a, g, R} \boxed{(q', \{H'_1, H'_2, \ldots, H'_{n'}\})}$$

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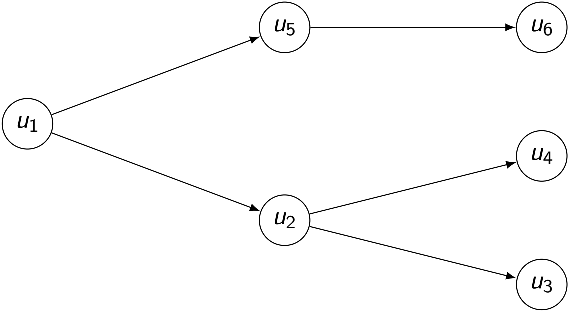
- If tree finite and winning strategy: we have a (finite) controller $\mathcal{T}$

Make the tree finite

For MTL specifications [Bouyer, Bozzelli, and Chevalier, 2006]: stop the computation with a well-quasi order $\sqsubseteq$ on the labels of the nodes



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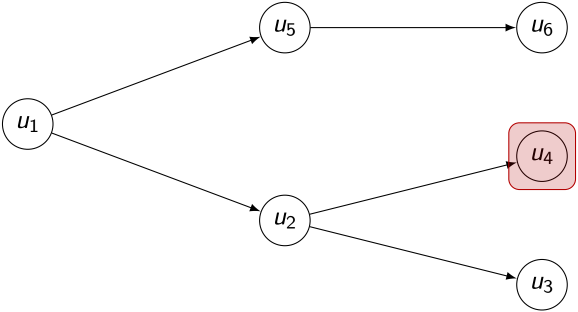

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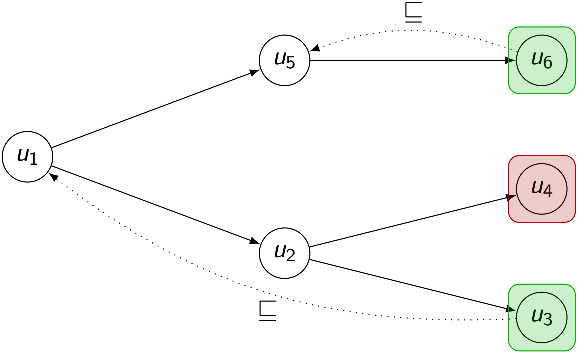

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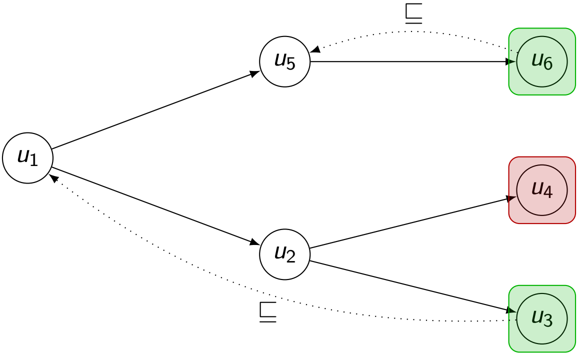

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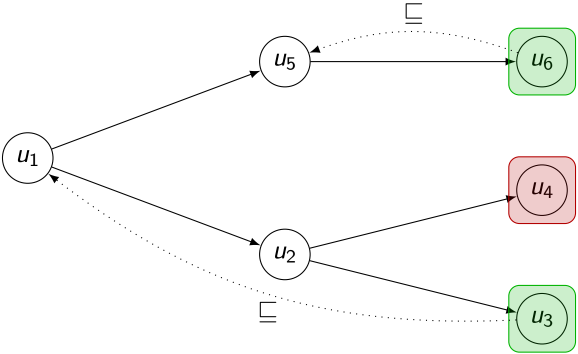

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- Complexity: **non-primitive recursive** due to well-quasi orderings

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Realisable instances

T	n	# clocks	exec. time (sec) / #nodes
1	1	0	46 / 52
1	1	1	199 / 147
1	1	2	4,599 / 1,343
2	2	1	2,632 / 645
2	2	2	18,453 / 2,358
3	3	1	182,524 / 2,297
3	3	2	$>$5min
4	4	0	54,893 / 667
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Unealisable instances

T	n	# clocks	exec. time (sec) / #nodes
2	1	0	77 / 84
2	1	1	824 / 311
2	1	2	3,079 / 1,116
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- This was before MIGHTYL, which could make things easier...

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Thank you for your attention! Questions?

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