Casting

Adding Negative Prices to Priced Timed Games

Highlights 2014, Paris

Results presented this week at CONCUR 2014

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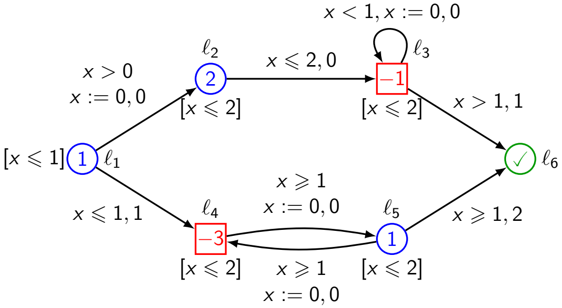
SEVENTH FRAMEWORK PROGRAMME

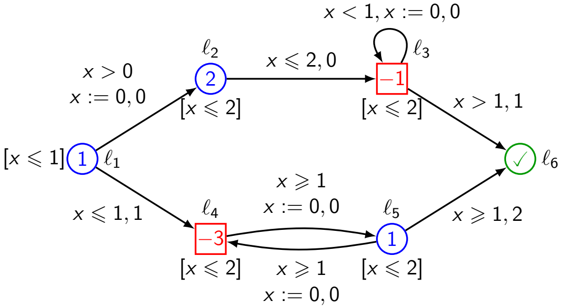
Priced Timed Games



Timed Automaton
with partition of states
between 2 players
+ reachability objective
+ rates in locations
+ costs over transitions

Semantics in terms of
infinite game with weights

Priced Timed Games



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$(\ell_1, 0)$

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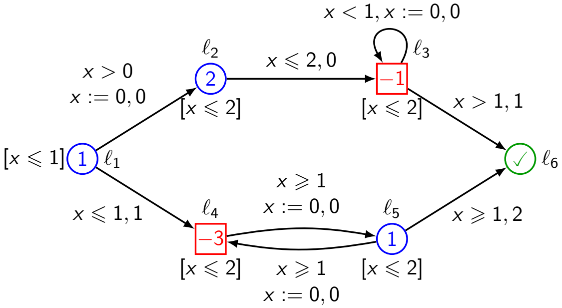

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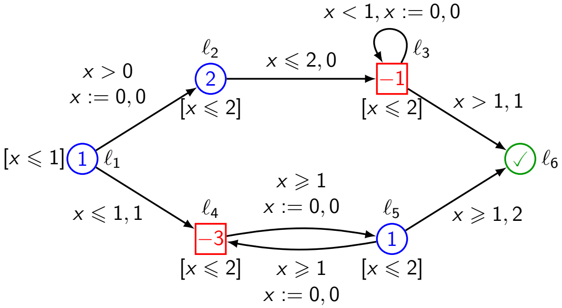
$(\ell_1, 0) \xrightarrow{0.4, \searrow} (\ell_4, 0.4)$

Priced Timed Games



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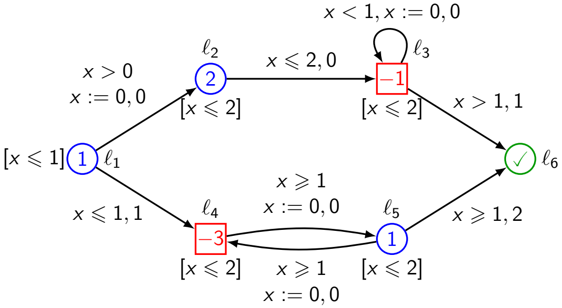
$$(\ell_1, 0) \xrightarrow{0.4, \searrow} (\ell_4, 0.4) \xrightarrow{0.6, \rightarrow} (\ell_5, 0)$$

Priced Timed Games



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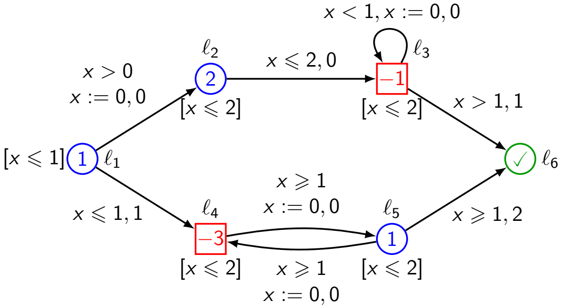
Priced Timed Games



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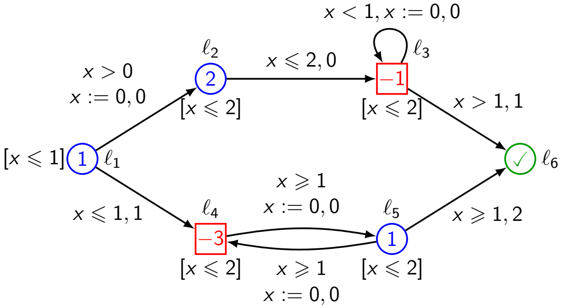
$$0.4 + 1 \quad -3 \times 0.6 \quad +1.5 \quad -3 \times 1.1 \quad +2 \times 2 + 2 \quad = 3.8$$

Priced Timed Games



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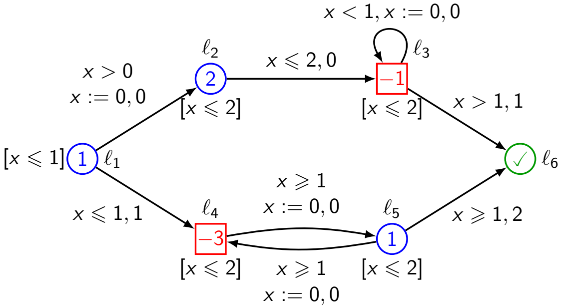
$$(\ell_1, 0) \xrightarrow{0.2, \nearrow} (\ell_2, 0) \xrightarrow{0.9, \rightarrow} (\ell_3, 0.9) \xrightarrow{0.2, \circlearrowleft} (\ell_3, 0) \xrightarrow{0.9, \circlearrowleft} (\ell_3, 0) \quad \cdots$$

$$0.2 \quad +0.9 \quad -0.2 \quad -0.9 \quad \cdots \quad = +\infty \ (\checkmark \text{ not reached})$$

Priced Timed Games



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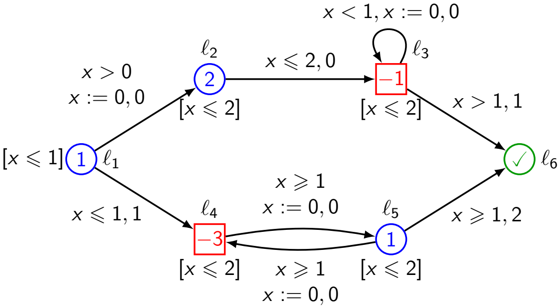
$$(\ell_1,0)\xrightarrow[0.4+1]{0.4,\searrow}(\ell_4,0.4)\xrightarrow[-3\times 0.6]{0.6,\rightarrow}(\ell_5,0)\xrightarrow[+1.5]{1.5,\leftarrow}(\ell_4,0)\xrightarrow[-3\times 1.1]{1.1,\rightarrow}(\ell_5,0)\xrightarrow[+2\times 2+2]{2,\nearrow}(\checkmark,2) \qquad = 3.8$$

$$(\ell_1,0)\xrightarrow[0.2]{0.2,\nearrow}(\ell_2,0)\xrightarrow[+0.9]{0.9,\rightarrow}(\ell_3,0.9)\xrightarrow[-0.2]{0.2,\circlearrowleft}(\ell_3,0)\xrightarrow[-0.9]{0.9,\circlearrowleft}(\ell_3,0)\ \cdots \qquad = +\infty \text{ (}\checkmark\text{ not reached)}$$

Cost of a play: $\begin{cases} +\infty & \text{if } \checkmark \text{ not reached} \\ \text{total payoff up to } \checkmark & \text{otherwise} \end{cases}$

Strategies and objectives



Strategy for each player: mapping of finite runs to a delay and an action

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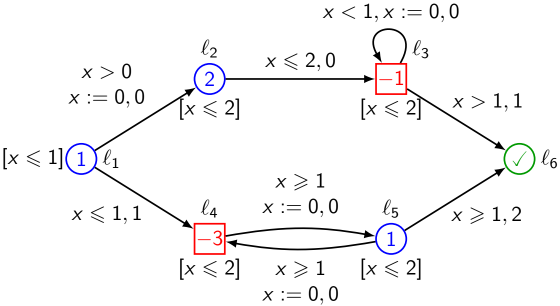

Strategy for each player: mapping of finite runs to a delay and an action

Goal of player ○: reach ✓ **and** minimize the cost

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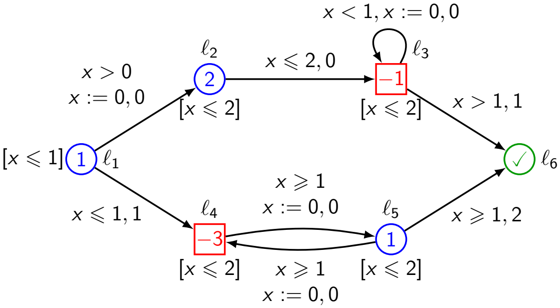

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Main object of interest:

$$\overline{\mathrm{Val}}(\ell, v) = \inf_{\sigma_\bigcirc \in \mathrm{Strat}_\bigcirc} \sup_{\sigma_\square \in \mathrm{Strat}_\square} \mathrm{Wt}(\mathrm{Play}((\ell, v), \sigma_\bigcirc, \sigma_\square)) \in \mathbf{R} \cup \{-\infty, +\infty\}$$

What player ○ can guarantee as a payoff? and design *good* strategies

State of the art

$\mathsf{F}_{\leqslant K}$✓: $\exists$ a strategy in the PTG (priced timed game) for player ○ reaching ✓ with a cost $\leqslant K$?

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$\mathsf{F}_{\leqslant K}\checkmark$: $\exists$ a strategy in the PTG (priced timed game) for player $\bigcirc$ reaching $\checkmark$ with a cost $\leqslant K$?

- One-player case (**Priced timed automata**): optimal reachability problem is PSPACE-complete
  - Algorithm based on regions [Bouyer, Brihaye, Bruyère, and Raskin, 2007];
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- PTGs with **non-negative costs and strictly non-Zeno cost cycles**: exponential algorithm [Bouyer, Cassez, Fleury, and Larsen, 2004, Alur, Bernadsky, and Madhusudan, 2004]

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This talk: **PTGs with negative costs**

Undecidability Results: Constrained-Price Reachability

- Known: $F_{\leqslant K}$✓ undecidable for 3 or more clocks

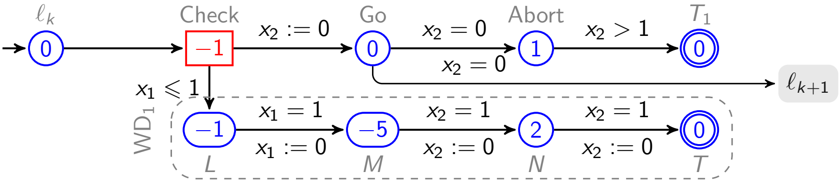
Proof by reduction of 2-counter machines: $x_1 = \frac{1}{2^{c_1}}$, $x_2 = \frac{1}{3^{c_2}}$, x_3 for work

Theorem:

$F_{\leqslant K}$✓ undecidable for PTGs with 2 or more clocks
idem for $F_{\geqslant K}$✓, $F_{> K}$✓, $F_{= K}$✓, $F_{< K}$✓

New encoding: $x_1 = \frac{1}{5^{c_1} 7^{c_2}}$, x_2 for work

Simulation of "ℓ_k : `decrement` c_1; `goto` ℓ_{k+1}" for Reach$(= 1)$



Other Undecidability Results

Theorem: Time-bounded Reachability

The following problem is undecidable for PTGs with 6 or more clocks:

Input: $K, T \in \mathbf{N}$
Question: $\mathsf{F}^{\leqslant T}_{\leqslant K}\checkmark$: $\exists$ strategy for $\bigcirc$ that reaches $\checkmark$ with cost $\leqslant K$ within time T?

Theorem: Repeated Reachability

The following problem is undecidable for PTGs with 3 or more clocks:

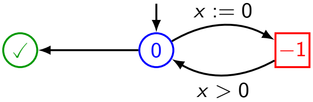
Input: $\eta \geqslant 0$
Question: $\mathsf{GF}_{[-\eta,\eta]}\checkmark$: $\exists$ strategy for $\bigcirc$ that visits $\checkmark$ infinitely often with a cost in $[-\eta, \eta]$?

Regain decidability?

More complex when negative costs

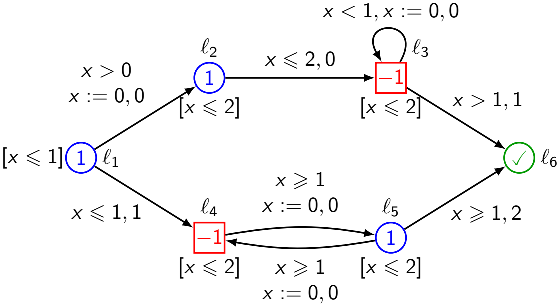
- Value $-\infty$: detection is as hard as mean-payoff. No hope for complexity better than **NP** $\cap$ **co-NP**, or pseudo-polynomial

- Memory complexity
  - Player ○ needs memory, even in the untimed setting: as seen in Axel's talk
  - Player □ may require infinite memory



One-clock Bi-Valued PTGs (1BPTGs)

Assumption: rates of locations $\{p^-, p^+\}$ included in $\{0, +d, -d\}$ ($d \in \mathbf{N}$) (no assumption on costs of transitions)



- Techniques of [Bouyer, Cassez, Fleury, and Larsen, 2004, Alur, Bernadsky, and Madhusudan, 2004] not applicable, e.g., because of Zeno costs cycles
- Exponential algorithms of [Bouyer, Larsen, Markey, and Rasmussen, 2006b, Rutkowski, 2011, Hansen, Ibsen-Jensen, and Miltersen, 2013] not working because of presence of negative costs

Results

Intuition: it is sufficient for both players to play arbitrarily close to borders of regions, so that corner-point abstraction [Bouyer, Brinksma, and Larsen, 2008] can be adapted to this game setting...

Theorem:

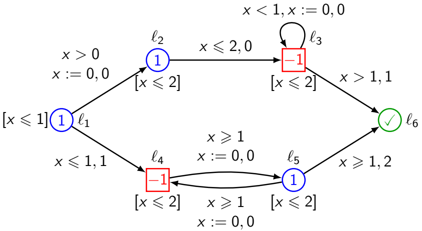
- Computation of the value $\overline{\mathsf{Val}}(\ell, v)$ of states of a 1BPTG in pseudo-polynomial time
- Synthesis of ε-optimal strategies for player $\bigcirc$ in pseudo-polynomial time

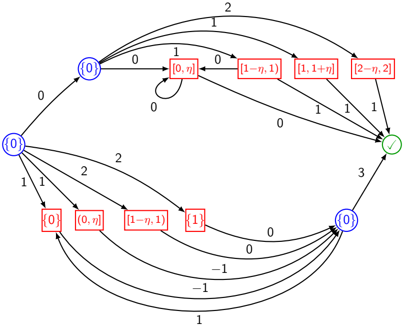
Theorem: Non-negative case

In case of a 1BPTG with only non-negative costs, all complexities drop down to polynomial.

Sketch of proof

1. **Reduce the space of strategies in the 1BPTG**
   - restrict to uniform strategies w.r.t. timed behaviors
2. **Build a finite priced game $\mathcal{G}$**
   - based on corner-point abstraction
3. **Study $\mathcal{G}$**
   - thanks to the results presented in Axel's talk
4. **Lift results of $\mathcal{G}$ to the original 1BPTG**





Summary and Future Work

Complete article published in the proceedings of CONCUR 2014[1]

Results

- More undecidability results due to the presence of negative costs
- 1BPTGs are determined: $\underline{\mathsf{Val}}(\ell, v) = \overline{\mathsf{Val}}(\ell, v)$
- Computation of the values, and synthesis of ε-optimal strategies for both players, in pseudo-polynomial time
- Strategy complexity: finite memory for $\bigcirc$, infinite memory for $\square$
- In case of $\geqslant 0$ prices, non-trivial class of 1-clock PTGs in PTIME
- Lifting of corner point abstraction to quantitative game setting

[1] See also `http://arxiv.org/abs/1404.5894` for a complete version

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- In case of $\geqslant 0$ prices, non-trivial class of 1-clock PTGs in PTIME
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- Implementation and test of this algorithm for real instances
- Decidability results for a bigger subset of PTGs with negative weights? careful since players may need to play far from boundaries in case of 2 clocks, or 1 clock and 3 distinct rates...

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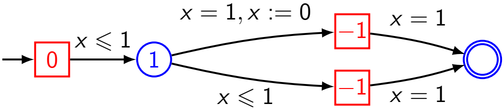

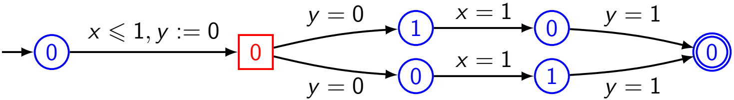

Summary and Future Work

Thank you for your attention

Questions?

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