Why Negatively-Priced Timed Games are Hard?

Dagstuhl Seminar on *Non-Zero-Sum Games and Control*

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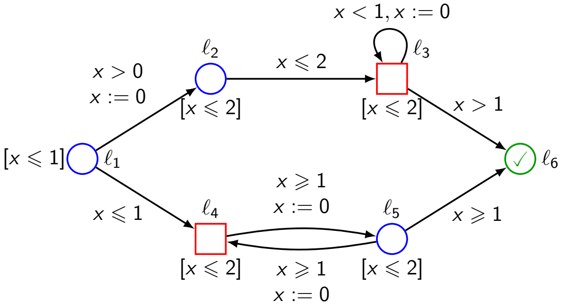
February 5, 2014

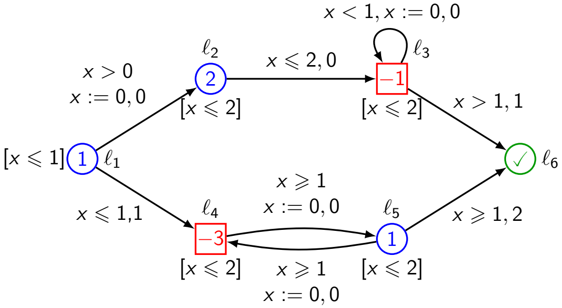
Priced Timed Games



Timed Automaton
with partition of states
between 2 players
+ reachability objective
+ rates in locations
+ costs over transitions

Semantics in terms of
infinite game with weights

Priced Timed Games



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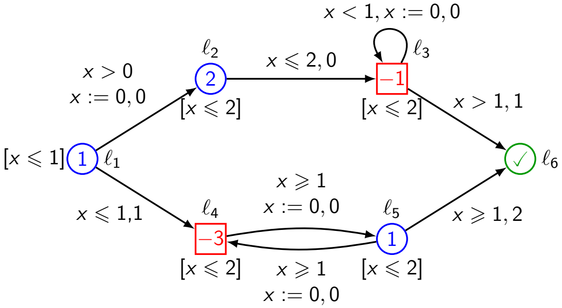
$(\ell_1, 0)$

Priced Timed Games



$(\ell_1, 0) \xrightarrow{0.4, \searrow} (\ell_4, 0.4)$

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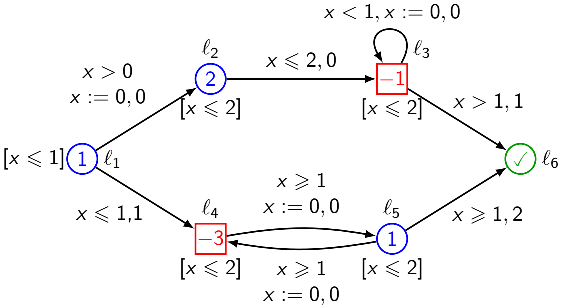

$(\ell_1, 0) \xrightarrow{0.4, \searrow} (\ell_4, 0.4) \xrightarrow{0.6, \rightarrow} (\ell_5, 0)$

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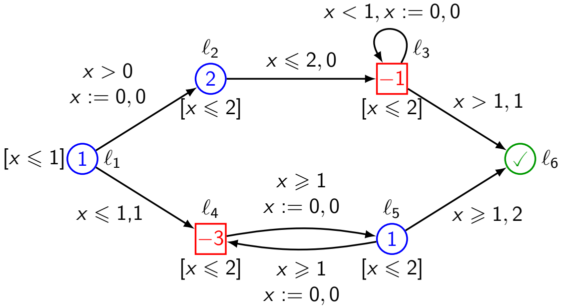

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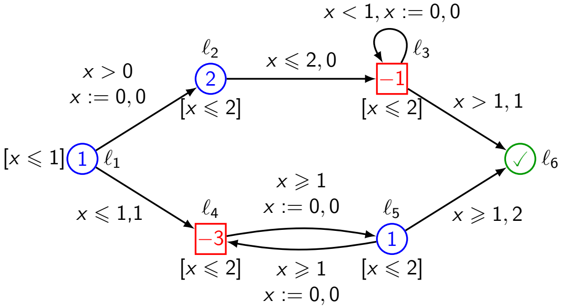
Priced Timed Games



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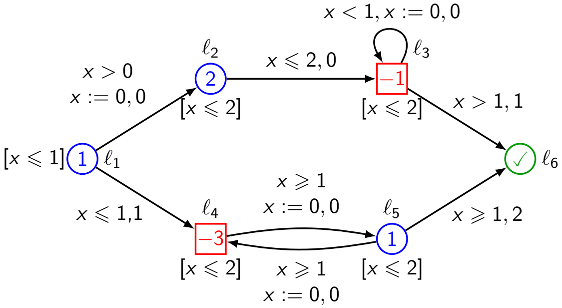
$$0.4 + 1 \quad -3 \times 0.6 \quad +1.5 \quad -3 \times 1.1 \quad +2 \times 2 + 2 \quad = 3.8$$

Priced Timed Games



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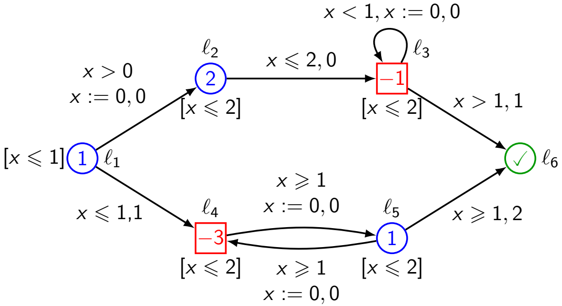
$$(\ell_1,0)\xrightarrow[0.2]{0.2,\nearrow}(\ell_2,0)\xrightarrow[+0.9]{0.9,\rightarrow}(\ell_3,0.9)\xrightarrow[-0.2]{0.2,\circlearrowleft}(\ell_3,0)\xrightarrow[-0.9]{0.9,\circlearrowleft}(\ell_3,0)\quad \cdots \quad = +\infty \ (\checkmark \text{ not reached})$$

Priced Timed Games



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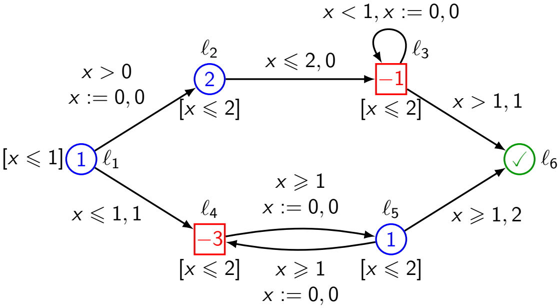
Cost of a play: $\begin{cases} +\infty & \text{if } \checkmark \text{ not reached} \\ \text{total payoff up to } \checkmark & \text{otherwise} \end{cases}$

Strategies and objectives



Strategy for each player: mapping of finite runs to a delay and an action

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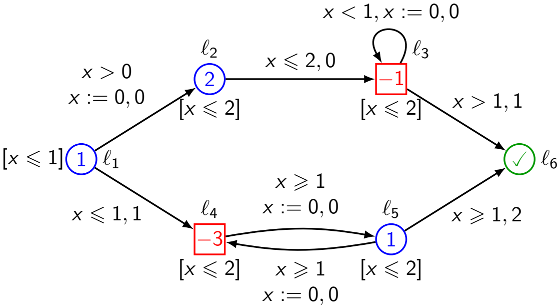

Strategy for each player: mapping of finite runs to a delay and an action

Goal of player ○: reach ✓ **and** minimize the cost

Goal of player □: avoid ✓ **or, if not possible,** maximize the cost

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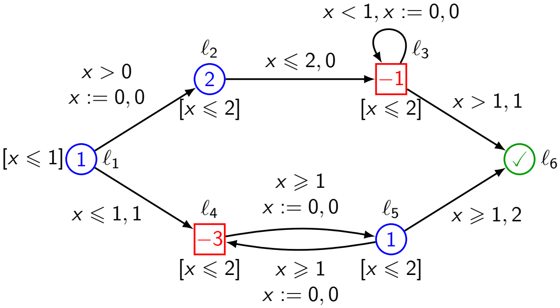

Strategy for each player: mapping of finite runs to a delay and an action

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Main object of interest:

$$\overline{\mathsf{Val}}(\ell, v) = \inf_{\sigma_\bigcirc \in \mathsf{Strat}_\bigcirc} \sup_{\sigma_\square \in \mathsf{Strat}_\square} \mathsf{Wt}(\mathsf{Play}((\ell, v), \sigma_\bigcirc, \sigma_\square)) \in \mathbf{R} \cup \{-\infty, +\infty\}$$

What player ○ can guarantee as a payoff? and design *good* strategies

State of the art

$\mathsf{F}_{\leqslant K}$✓: $\exists$ a strategy in the PTG (priced timed game) for player ○ reaching ✓ with a cost $\leqslant K$?

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- One-player case (**Priced timed automata**): optimal reachability problem is PSPACE-complete
  - Algorithm based on regions [Bouyer, Brihaye, Bruyère, and Raskin, 2007];
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This talk: **PTGs with negative costs**

Undecidability results: less clocks...

- Known: $F_{\leqslant K}$✓ undecidable for 3 or more clocks

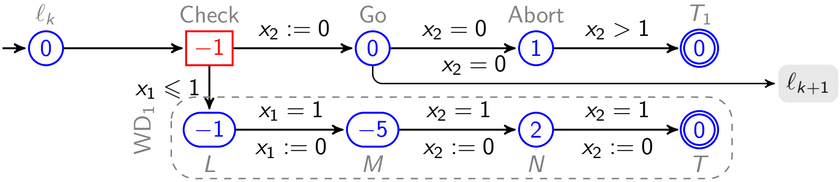
Proof by reduction of 2-counter machines: $x_1 = \frac{1}{2^{c_1}}$, $x_2 = \frac{1}{3^{c_2}}$, x_3 for work

Theorem:

$F_{\leqslant K}$✓ undecidable for PTGs with 2 or more clocks
idem for $F_{\geqslant K}$✓, $F_{>K}$✓, $F_{=K}$✓, $F_{<K}$✓

New encoding: $x_1 = \frac{1}{5^{c_1} 7^{c_2}}$, x_2 for work

Simulation of "ℓ_k : `decrement` c_1; `goto` ℓ_{k+1}" for Reach(= 1)



Regain decidability
in the presence of negative prices?

More complex when negative costs

- Value $-\infty$: detection is as hard as mean-payoff. No hope for complexity better than $\mathbf{NP} \cap \mathbf{co\text{-}NP}$, or pseudo-polynomial

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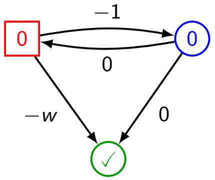

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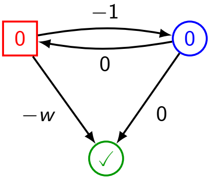
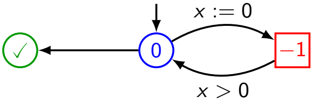


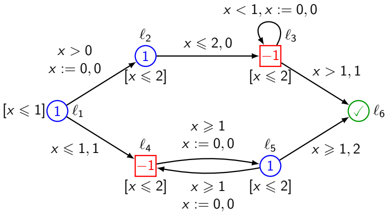
  - Player □ may require infinite memory



Building over the corner-point abstraction

Only known decidable fragment with negative weights: 1-player case (priced timed automata)

- Main tool: refinement of regions similar to corner point abstraction



regions: $\{0\}, (0, 1), \{1\}, (1, 2), \{2\}, (2, +\infty)$
regions refined with corner information:
$\{0\}, (0, \eta), (1 - \eta, 1), \{1\}, (1, 1 + \eta), (2 - \eta, 2), \{2\}, (2, +\infty)$

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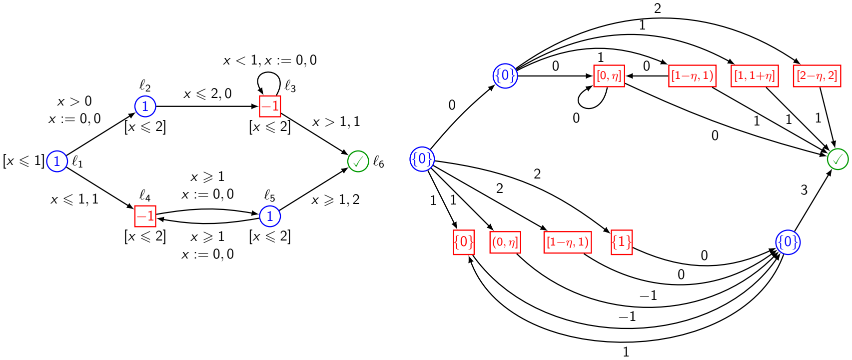

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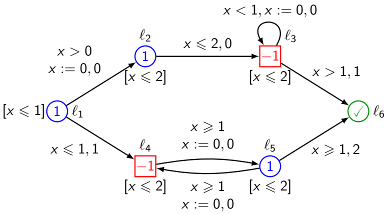

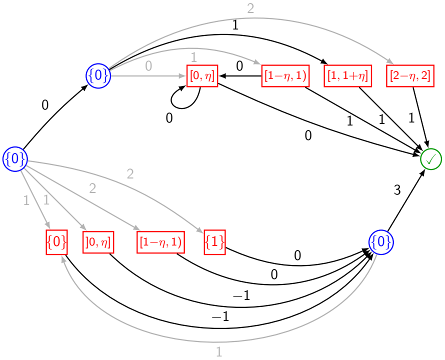

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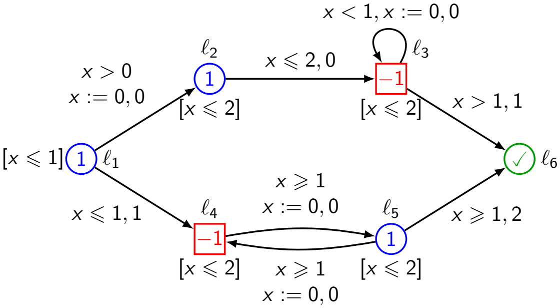
One-clock Bi-Valued PTGs (1BPTGs)

Assumption: rates of locations $\{p^-, p^+\}$ included in $\{0, +d, -d\}$ ($d \in \mathbf{N}$) (no assumption on costs of transitions)



- Techniques of [Bouyer, Cassez, Fleury, and Larsen, 2004, Alur, Bernadsky, and Madhusudan, 2004] not applicable, e.g., because of Zeno costs cycles
- Exponential algorithms of [Bouyer, Larsen, Markey, and Rasmussen, 2006b, Rutkowski, 2011, Hansen, Ibsen-Jensen, and Miltersen, 2013] not working because of presence of negative costs

Results

Theorem:

- Computation of the value $\overline{\mathsf{Val}}(\ell, v)$ of states of a 1BPTG in pseudo-polynomial time
- Synthesis of ε-optimal strategies for player $\bigcirc$ in pseudo-polynomial time

Theorem: Non-negative case

In case of a 1BPTG with only non-negative costs, all complexities drop down to polynomial.

First idea: symmetrize the viewpoint

Value for player $\bigcirc$: $\overline{\mathsf{Val}}(\ell, v) = \inf_{\sigma_\bigcirc \in \mathsf{Strat}_\bigcirc} \sup_{\sigma_\square \in \mathsf{Strat}_\square} \mathsf{Wt}(\mathsf{Play}((\ell, v), \sigma_\bigcirc, \sigma_\square))$

Value for player $\square$: $\underline{\mathsf{Val}}(\ell, v) = \sup_{\sigma_\square \in \mathsf{Strat}_\square} \inf_{\sigma_\bigcirc \in \mathsf{Strat}_\bigcirc} \mathsf{Wt}(\mathsf{Play}((\ell, v), \sigma_\bigcirc, \sigma_\square))$

How to compare them? $\underline{\mathsf{Val}}(\ell, v) \leqslant \overline{\mathsf{Val}}(\ell, v)$

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How to compare them? $\underline{\mathsf{Val}}(\ell, v) \leqslant \overline{\mathsf{Val}}(\ell, v)$

Theorem: (continued)

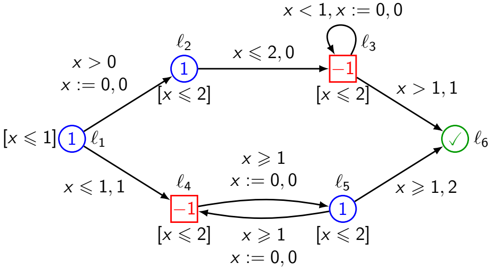
- *Minmax theorem*: 1BPGs are determined, i.e., $\underline{\mathsf{Val}}(\ell, v) = \overline{\mathsf{Val}}(\ell, v)$
- Synthesis of ε-optimal strategies for player $\square$ in pseudo-polynomial time (and polynomial in case of non-negative weights)

Sketch of proof

1. **Reduce the space of strategies in the 1BPTG**
   - restrict to uniform strategies w.r.t. timed behaviors
2. **Build a finite priced game $\mathcal{G}$**
   - based on a refinement of the region abstraction
3. **Study $\mathcal{G}$**
4. **Lift results of $\mathcal{G}$ to the original 1BPTG**

1. Reduce the space of strategies

Intuition: no need for both players to play far from borders of regions



Regions:
$\{0\}, (0,1), \{1\}, (1,2), \{2\}, (2,+\infty)$

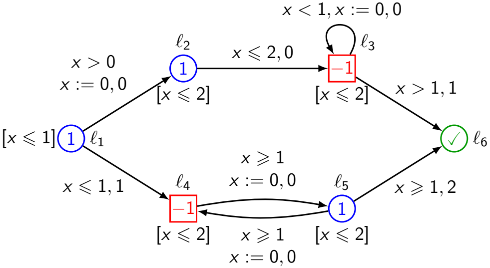
Player ○ wants to leave as soon as possible a state with rate p^+, and wants to stay as long as possible in a state with rate p^-: so, he will always play η-close to a border...

Lemma:

Both players can play arbitrarily close to borders w.l.o.g.: for every η

$$\underline{\mathsf{Val}}^\eta(\ell, v) \leqslant \underline{\mathsf{Val}}(\ell, v) \quad \leqslant \quad \overline{\mathsf{Val}}(\ell, v) \leqslant \overline{\mathsf{Val}}^\eta(\ell, v)$$

2. Finite priced game abstraction



η-regions: $\{0\}, (0, \eta), (1-\eta, 1), \{1\}, (1, 1+\eta), (2-\eta, 2), \{2\}, (2, +\infty)$

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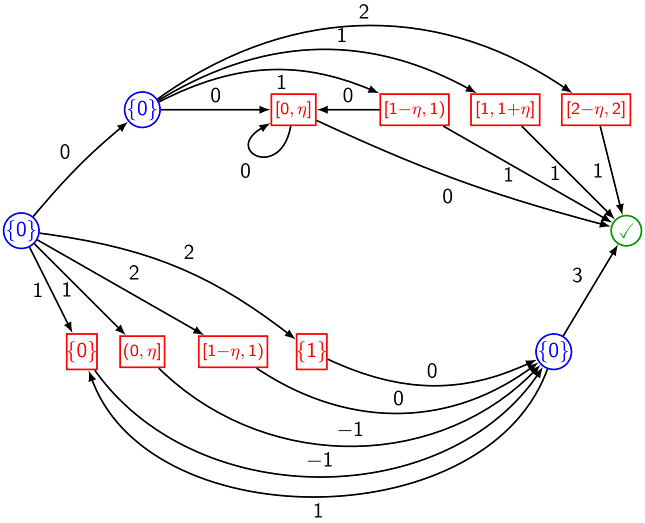
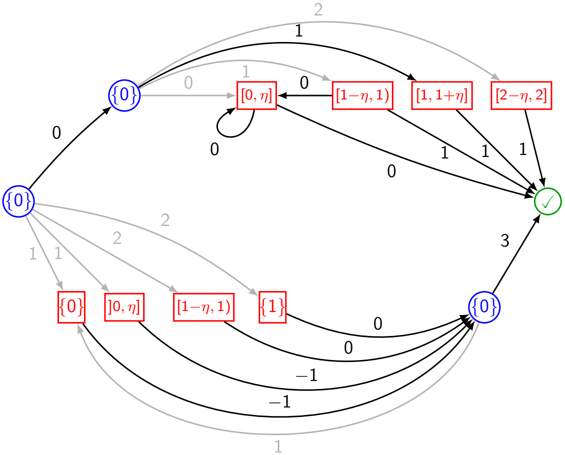


3. Study $\mathcal{G}$: values, optimal strategies of a min-cost reachability game

[Brihaye, Geeraerts, Haddad, and Monmege, 2014]



Optimal value: $\text{Val}_{\mathcal{G}}(\ell_1, \{0\}) = +2$ (for both players)

4. Lift results to the original 1BPTG

Reconstruct strategies in the 1BPTG from optimal strategies of $\mathcal{G}$

Lemma:

For all $\varepsilon > 0$, there exists $\eta > 0$ such that:

$$\mathsf{Val}_{\mathcal{G}}(\ell, \{0\}) - \varepsilon \leqslant \underline{\mathsf{Val}}^{\eta}(\ell, 0) \leqslant \underline{\mathsf{Val}}(\ell, 0) \leqslant \overline{\mathsf{Val}}(\ell, 0) \leqslant \overline{\mathsf{Val}}^{\eta}(\ell, 0) \leqslant \mathsf{Val}_{\mathcal{G}}(\ell, \{0\}) + \varepsilon$$

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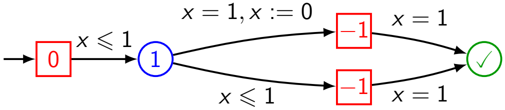
- So $\underline{\mathsf{Val}}(\ell, 0) = \overline{\mathsf{Val}}(\ell, 0)$, i.e., determination
- ε-optimal strategies for both players
  - Finite memory for player ○ (finite memory in finite priced games)
  - Infinite memory for player □ (even though memoryless in finite priced games), because it needs to ensure convergence of its differences between the 1BPTG and $\mathcal{G}$
- Overall complexity: pseudo-polynomial (polynomial if non-negative costs) in the size of $\mathcal{G}$, which is polynomial in the 1BPTG (because 1 clock)

1BPTG: maximal fragment for corner-point abstraction

Players may need to play far from corners...

- With 3 weights in $\{-1, 0, +1\}$: value $1/2$...



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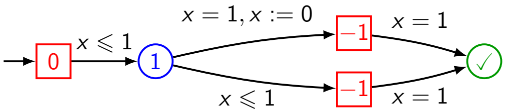
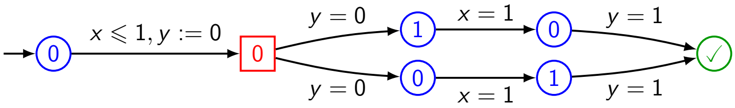


- With 2 weights in $\{-1, 0, +1\}$ but 2 clocks: value $1/2$...



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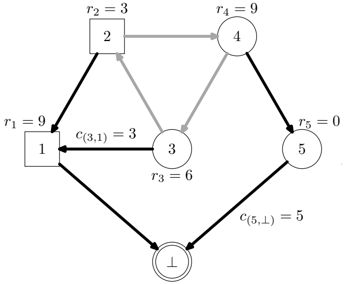

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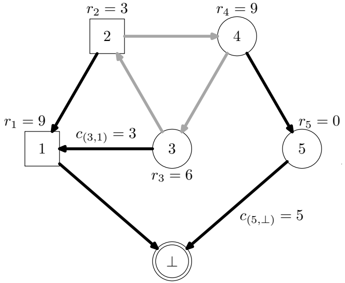

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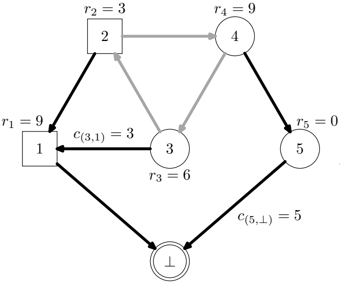

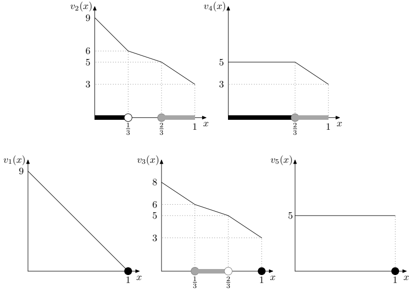

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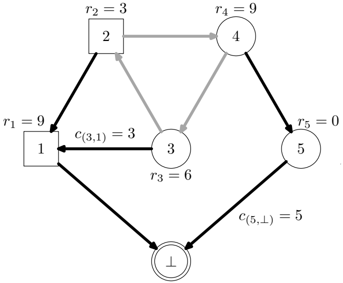

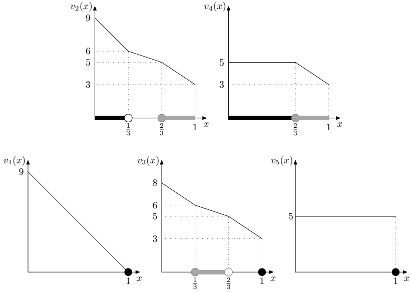

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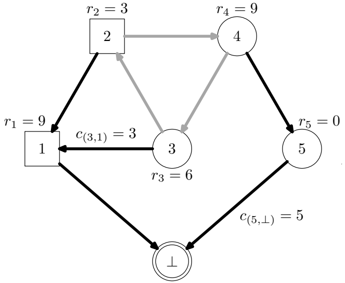

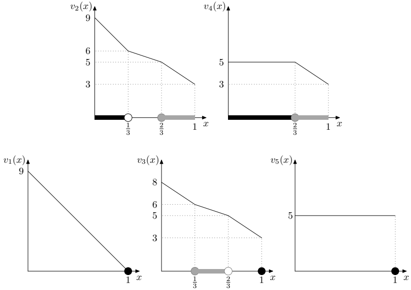

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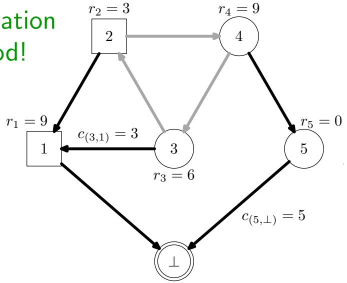

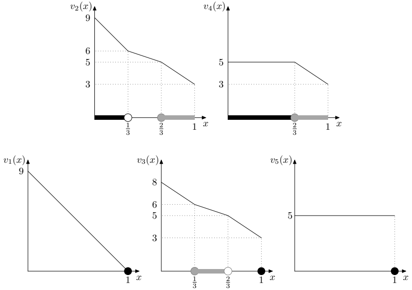

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Proof using the extension of Martin's determinacy result for games with arbitrary branching [Martin, 1990]

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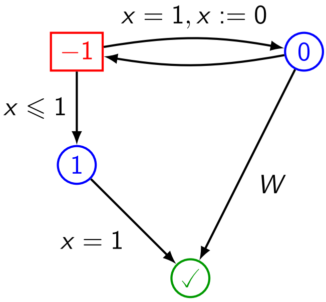

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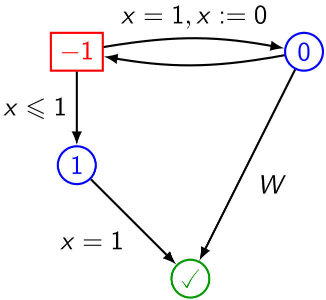

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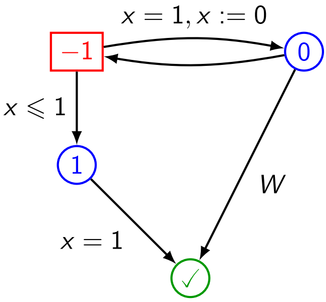

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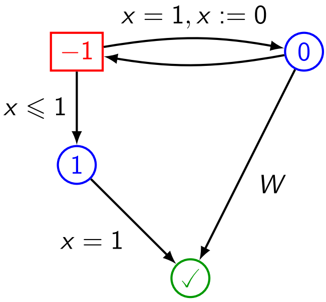

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... yet, to obtain ε, ○ needs to loop at most $W \cdot \lceil 1/\varepsilon \rceil - 1$ times before reaching ✓!

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- Refined undecidability results due to the presence of negative costs
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