To Reach or not to Reach? Efficient Algorithms for Total-Payoff Games

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and **Benjamin Monmege (ULB – Aix-Marseille University)**

Madrid meet 2015 — CONCUR

Game theory for synthesis

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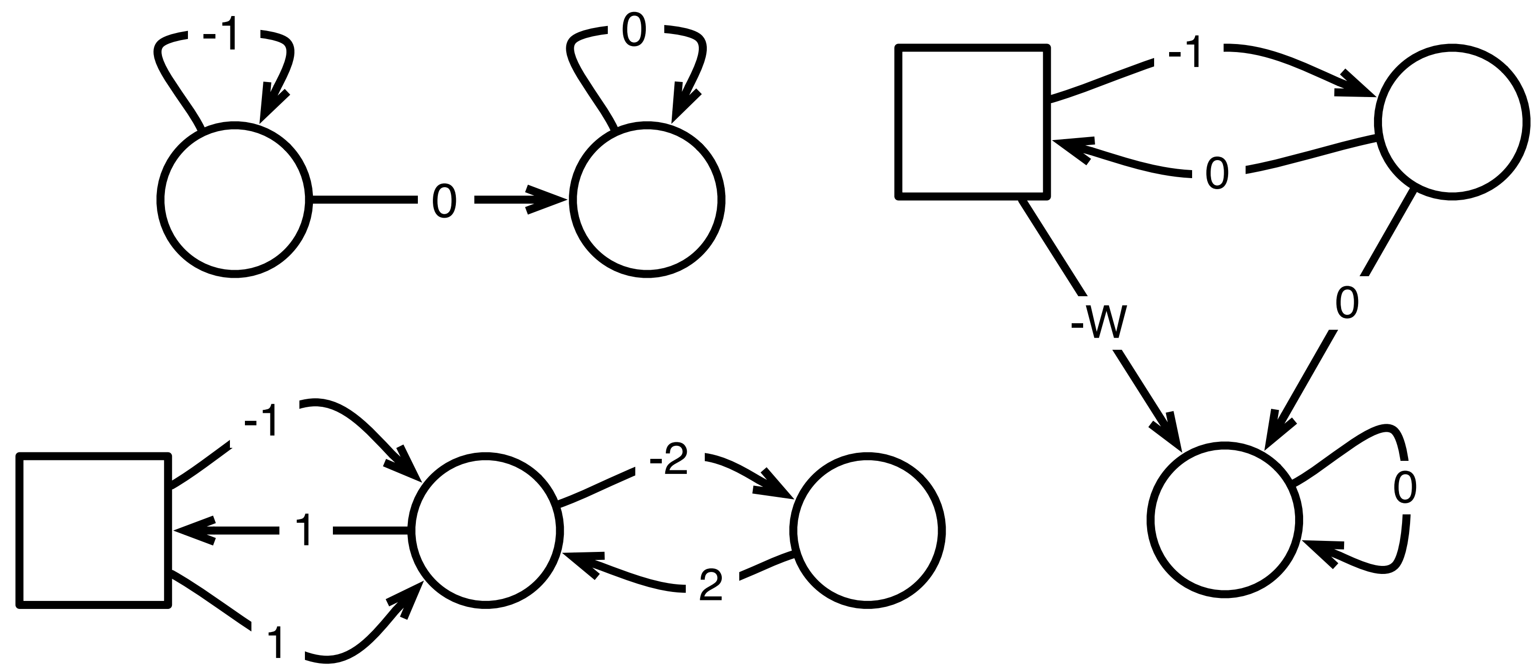
- Interested with energy consumption, reliability, lifetime…
  Quantitative synthesis with games on weighted graphs

Games on weighted graphs

$$(V, E, w)$$

$$V = V_{\min} \uplus V_{\max}$$

$$w : E \to \mathbf{Z}$$



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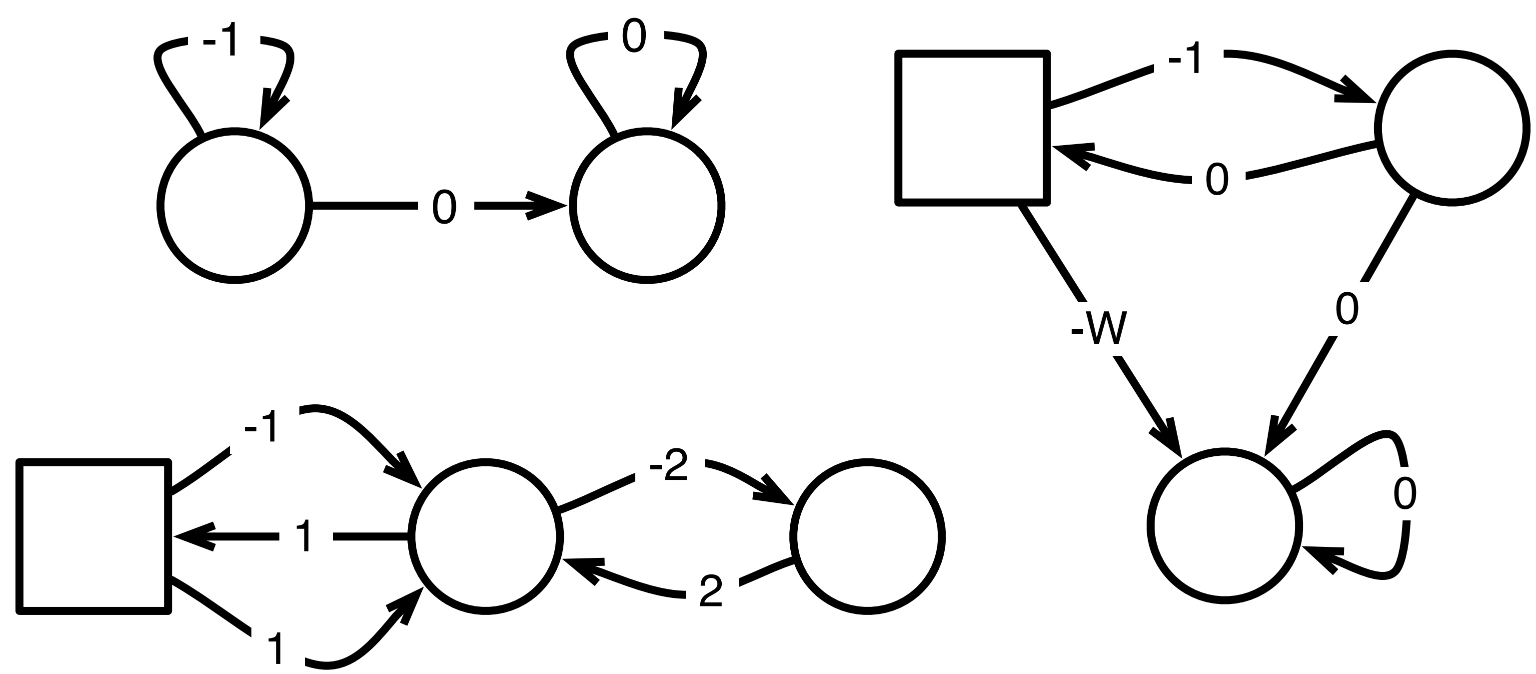
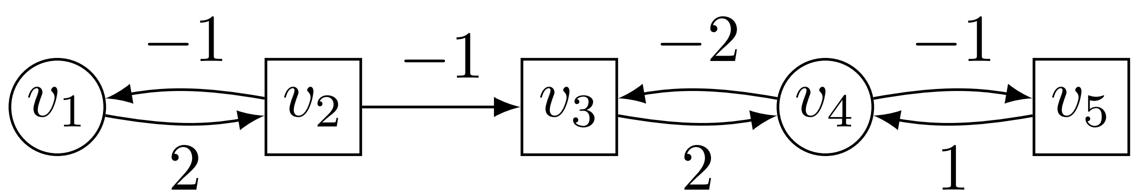


- Quantitative objective of the controller: maximising his payoff, accumulated along the computation of the system
    - Mean-payoff: *good in average*.

      Abundantly studied, NP∩co-NP, pseudo-polynomial time algorithm by Zwick & Paterson…
    - Total-payoff: *good in total*. Refinement of mean-payoff
    - Discounted-payoff…

Total-payoff games



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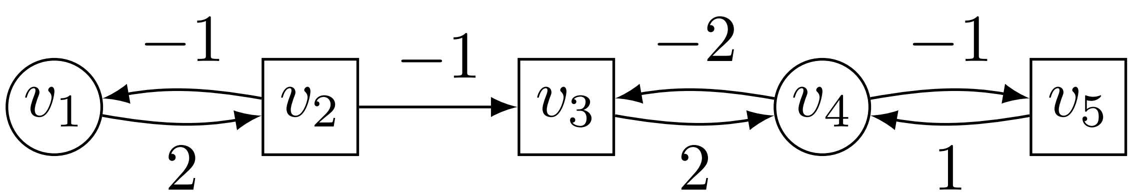

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- Play: $\pi = v_0 v_1 v_2 \cdots$

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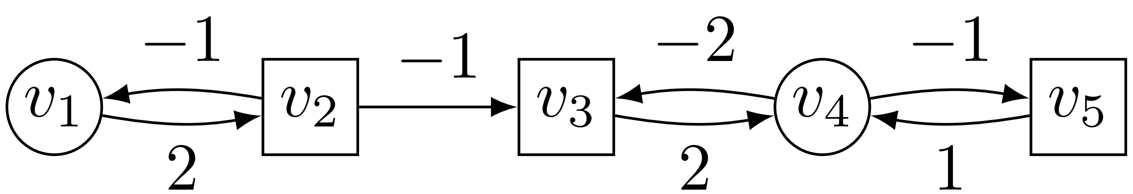

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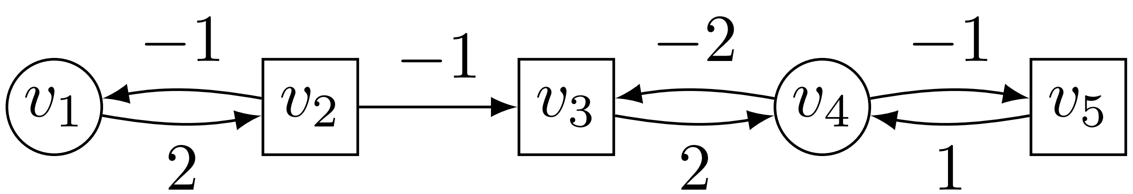

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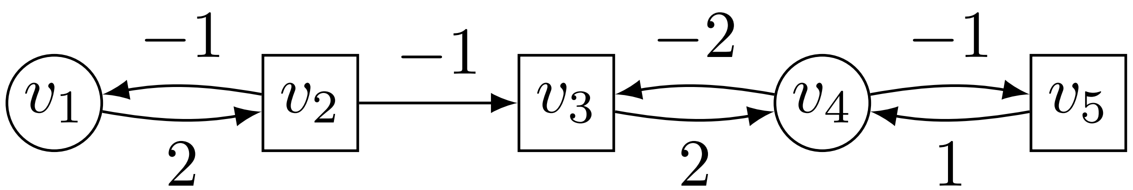

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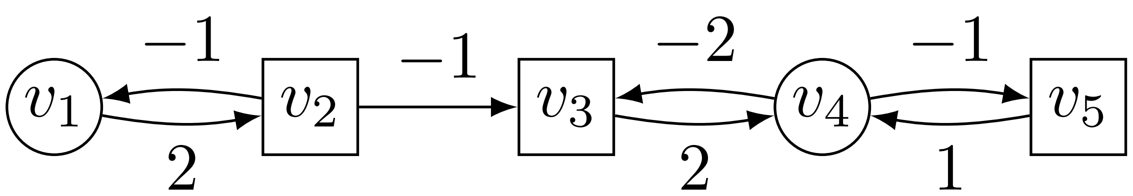

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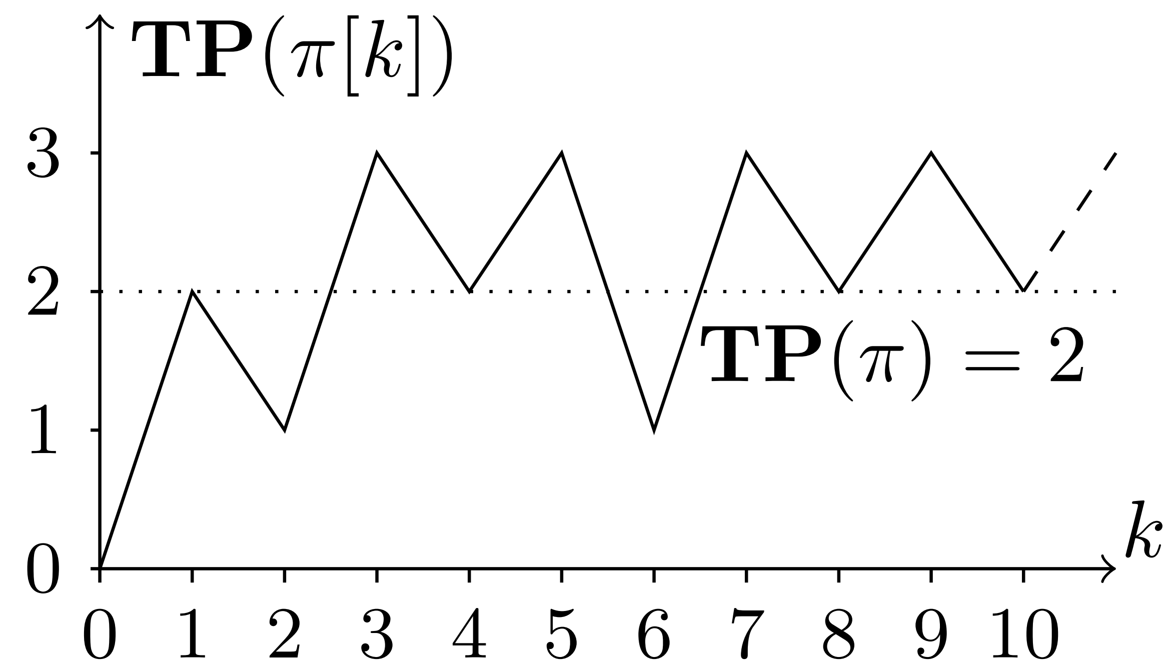

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Known results:

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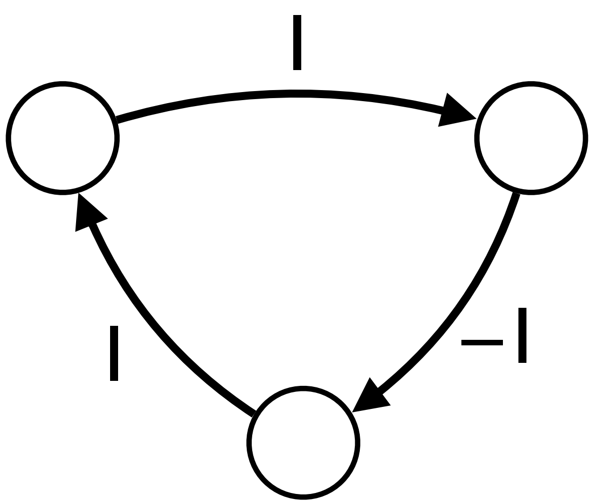
Our contribution:

- First pseudo-polynomial time algorithm for total-payoff games + heuristics
- Requires the study of a variant with reachability

Mean-payoff vs Total-payoff



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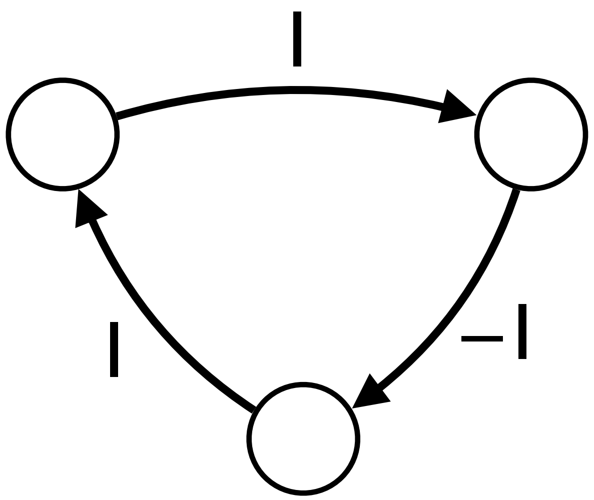

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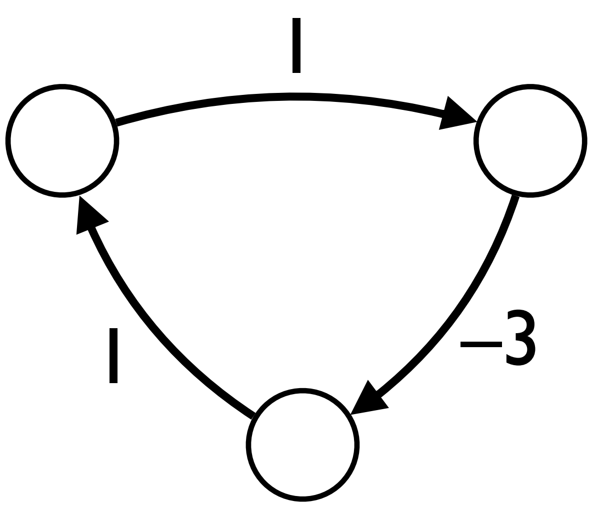

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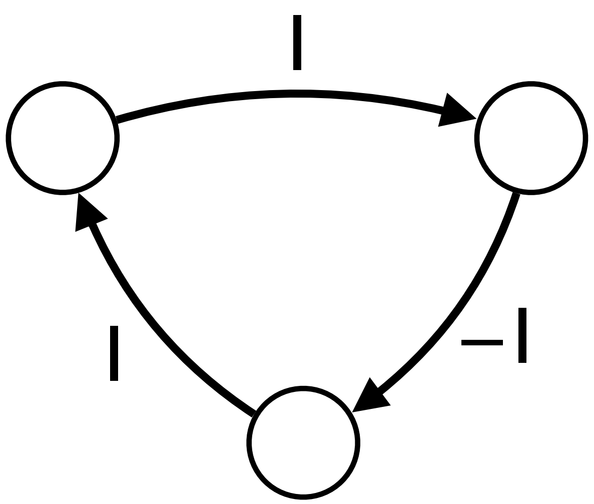

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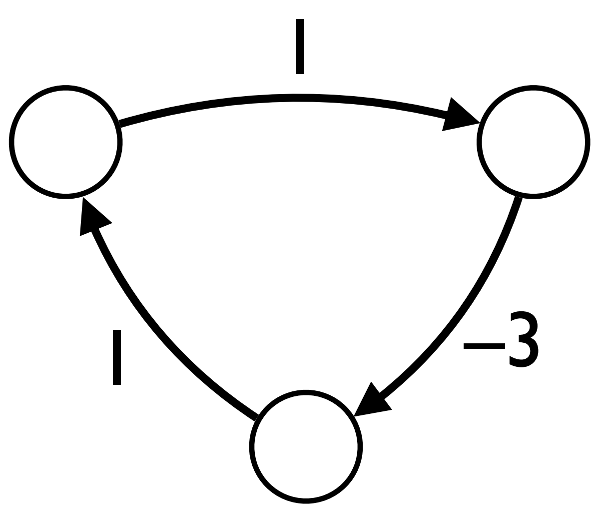

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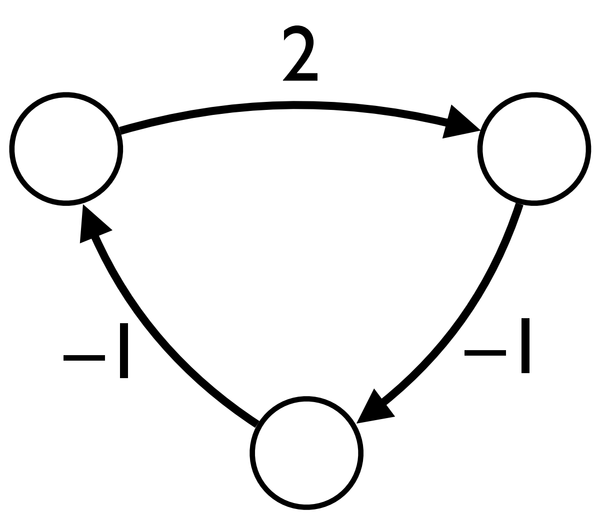

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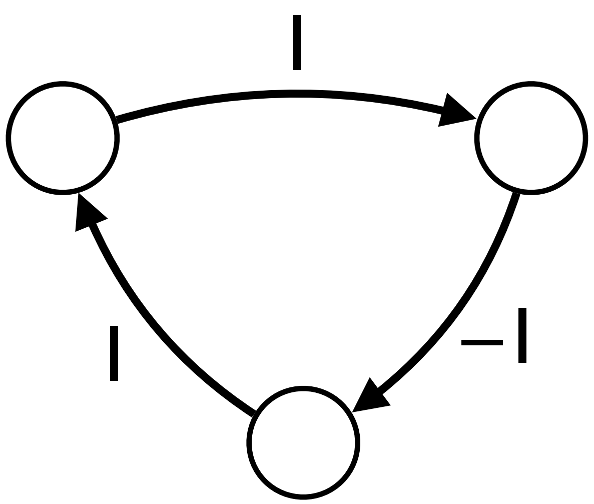

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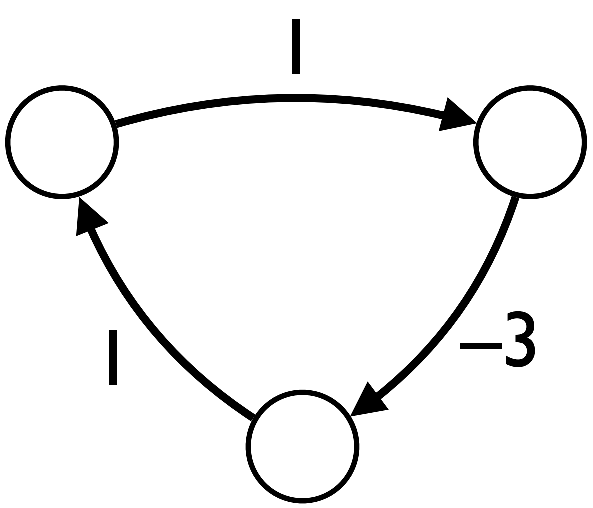

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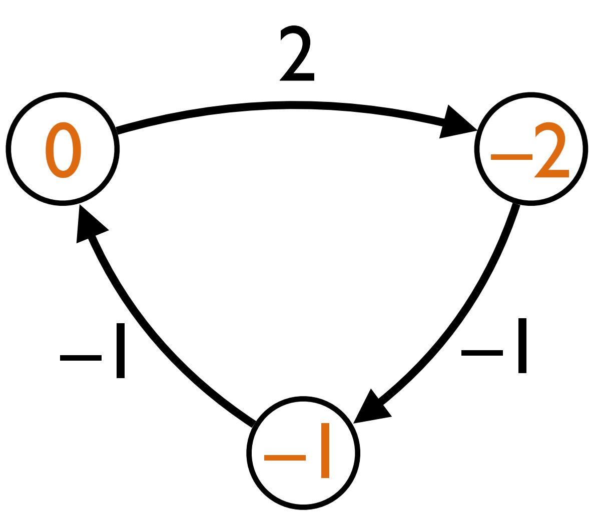

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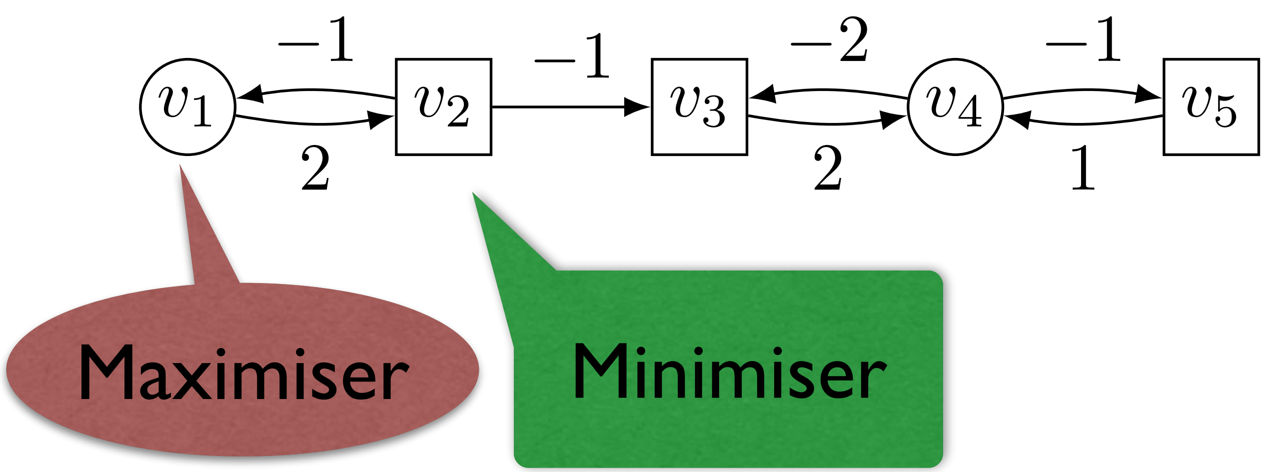
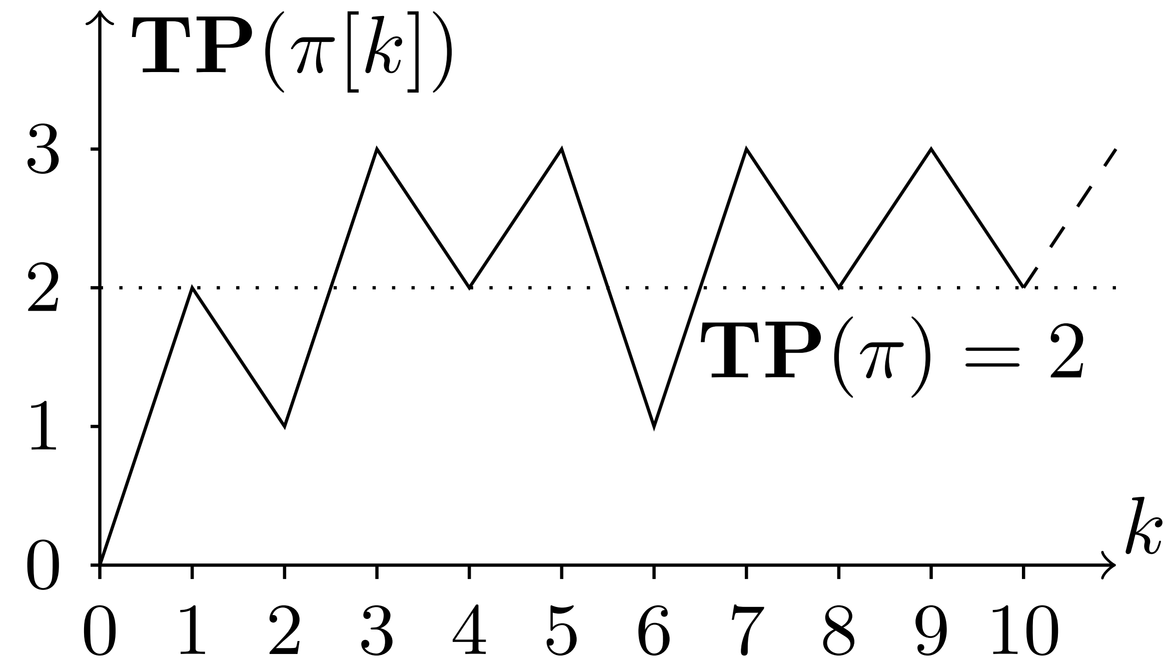
Our result in a nutshell





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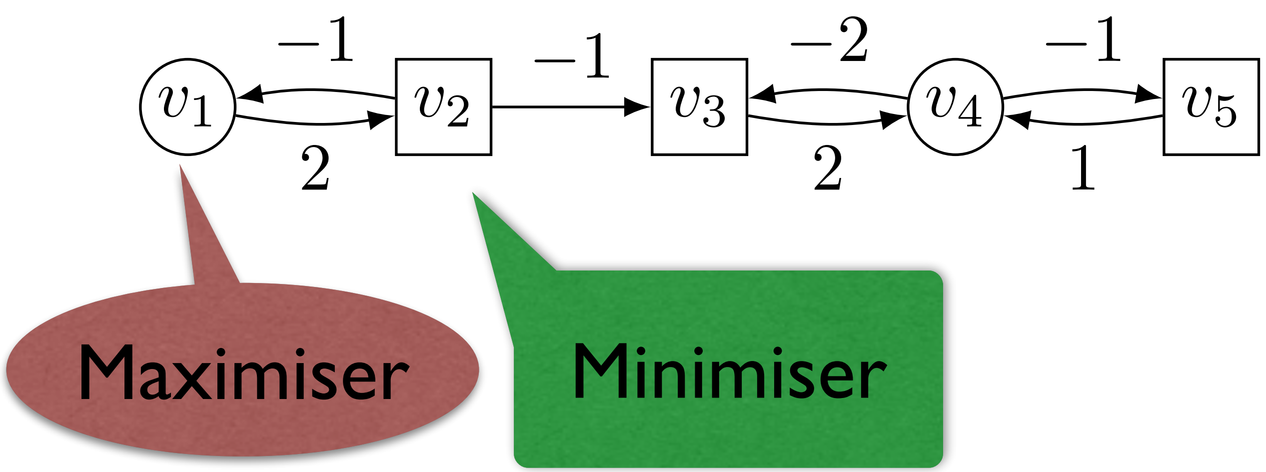

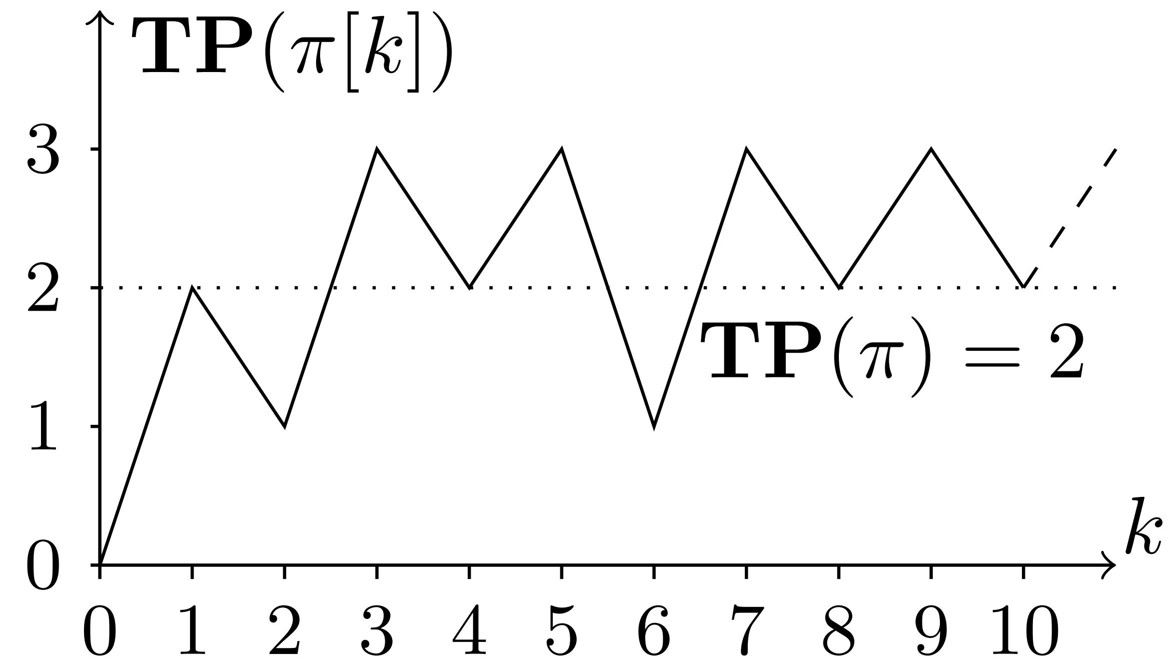

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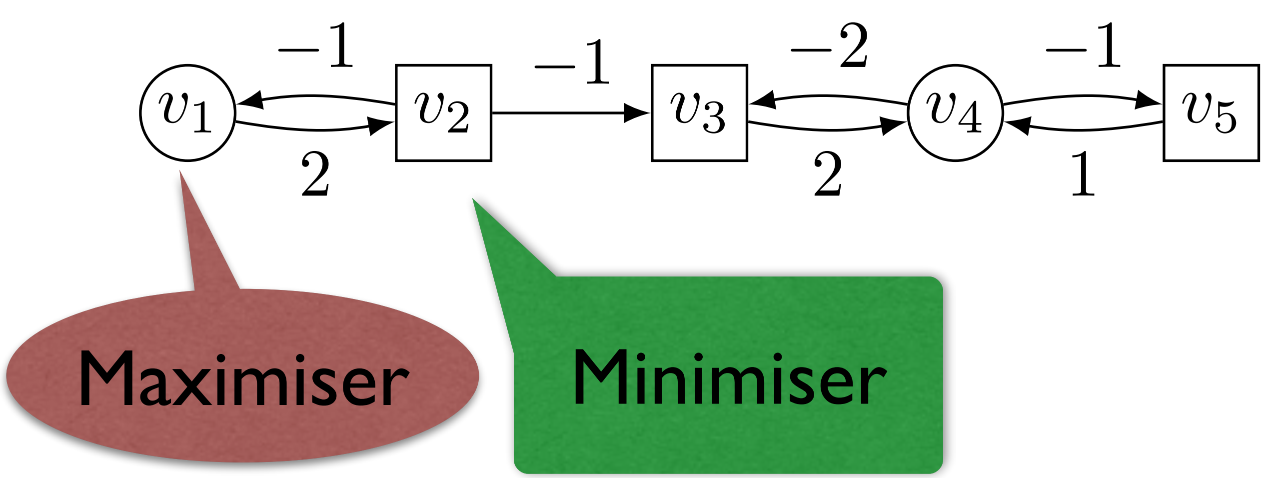

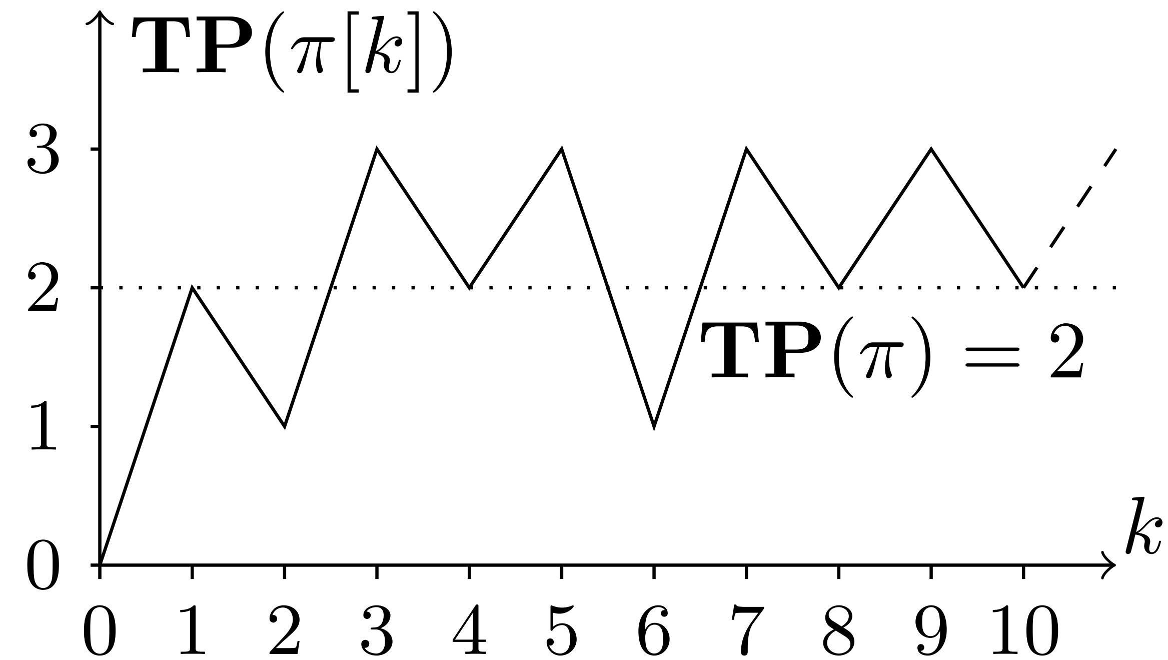

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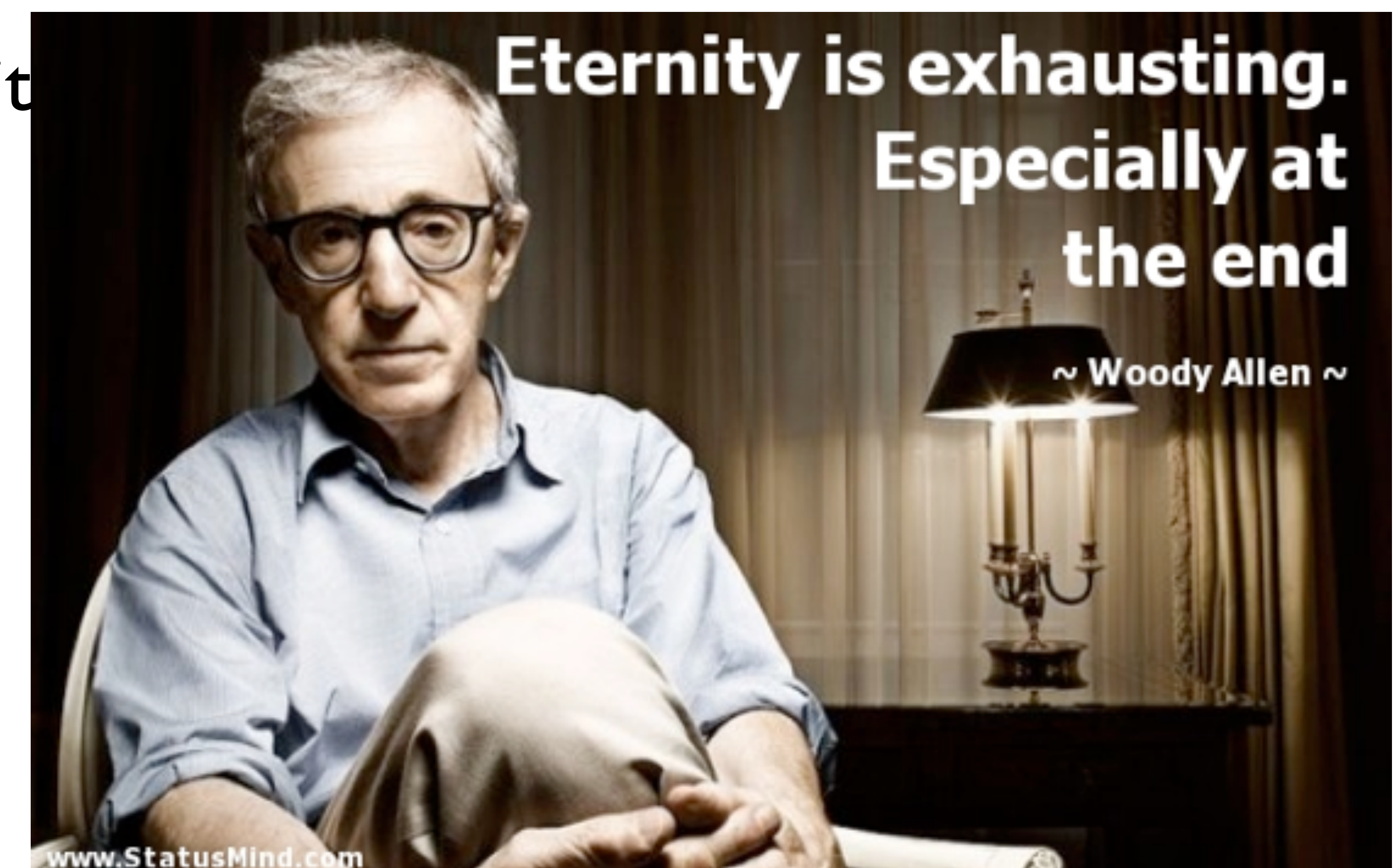

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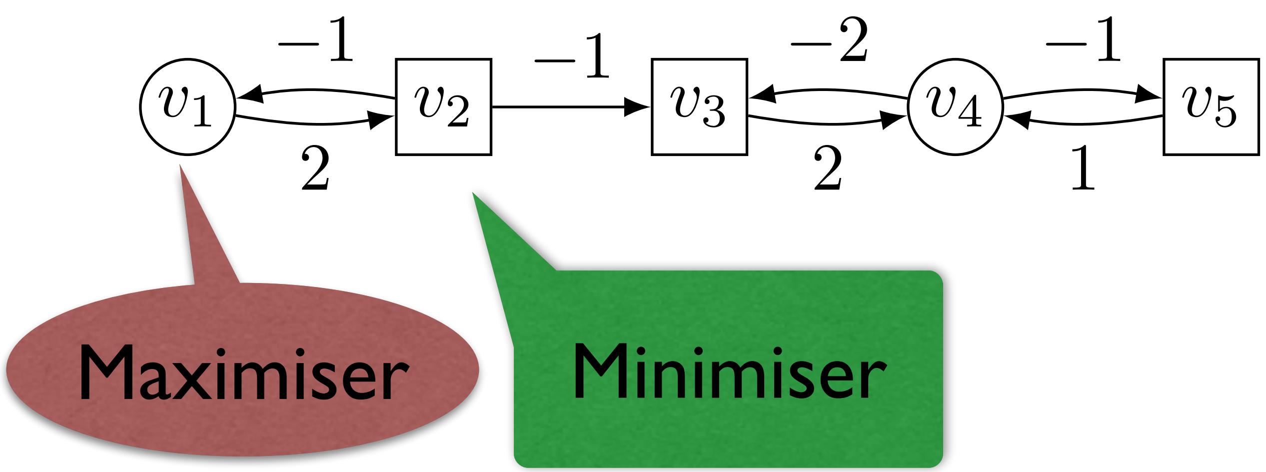

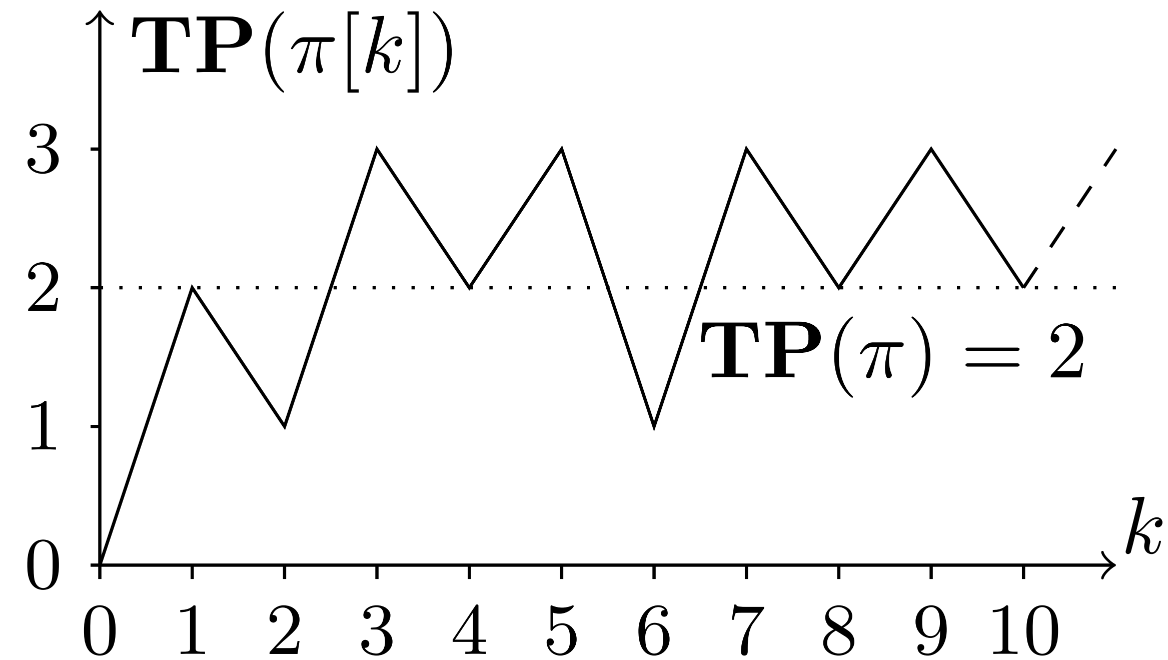

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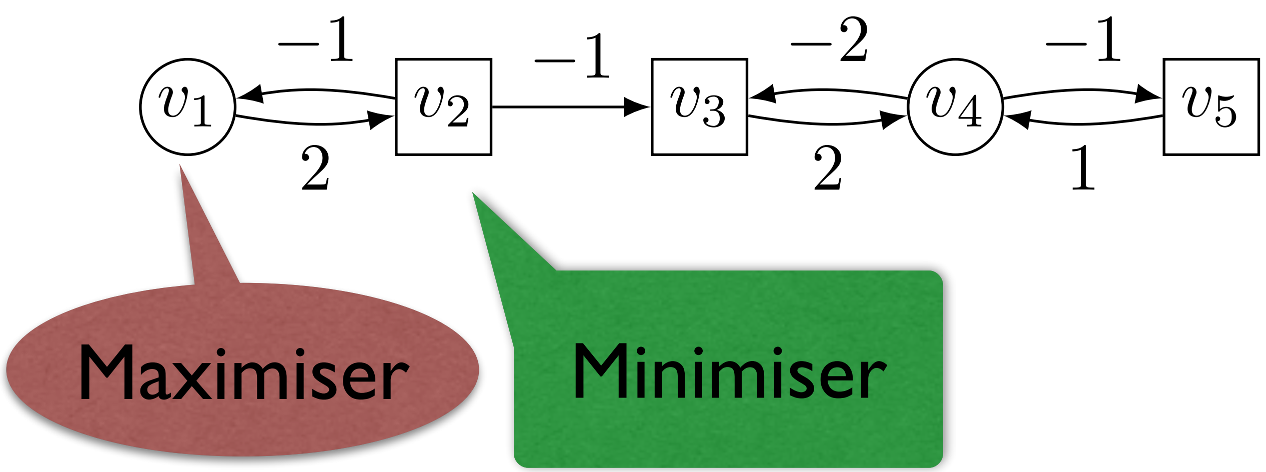

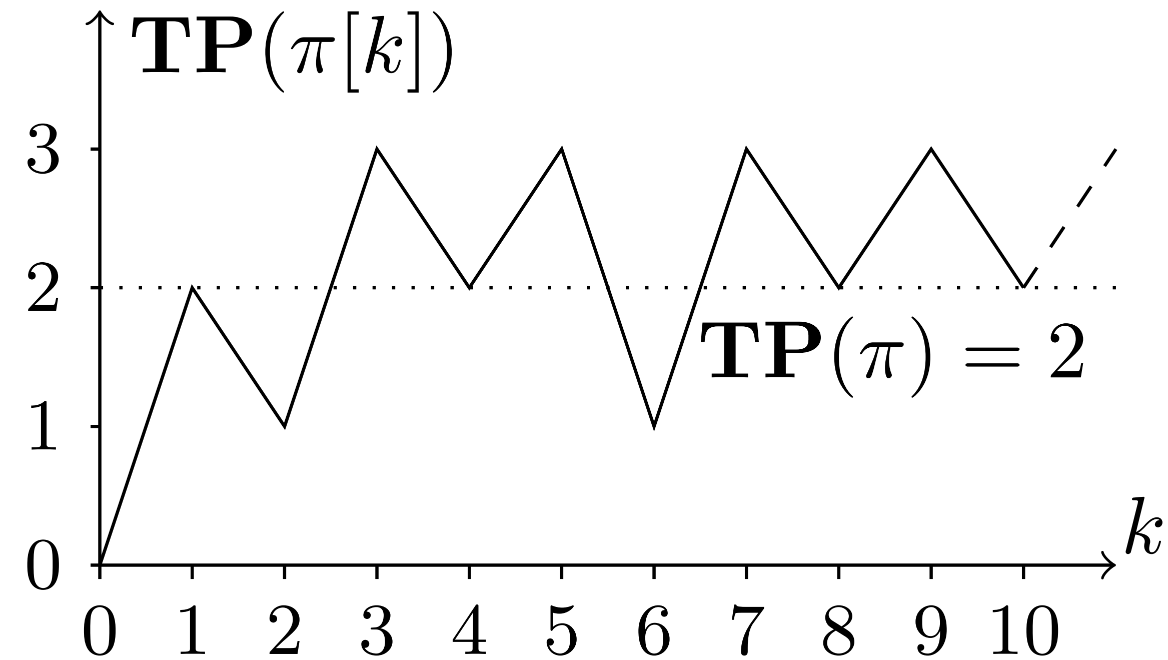

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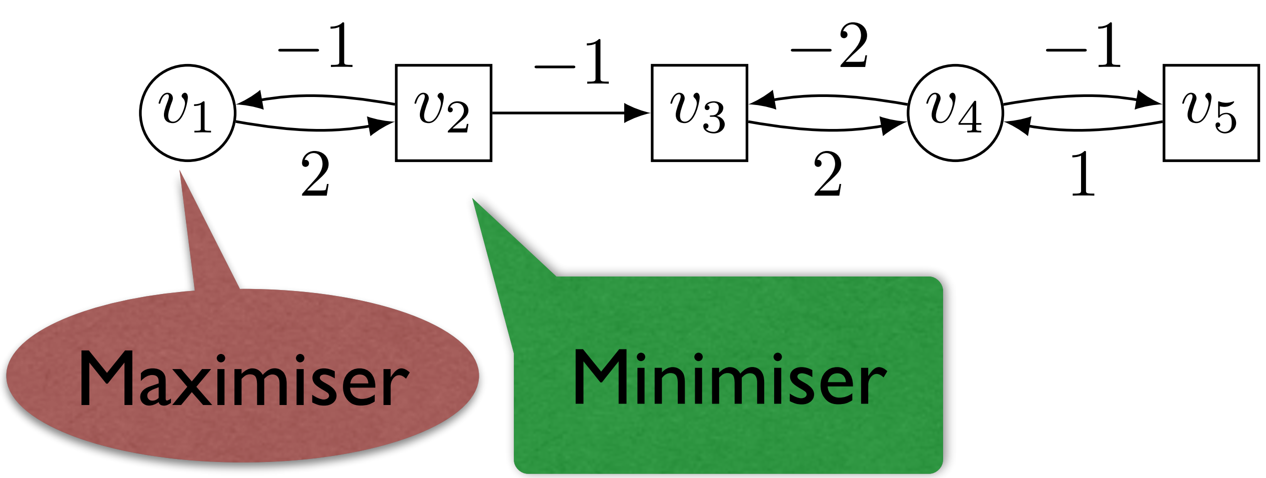

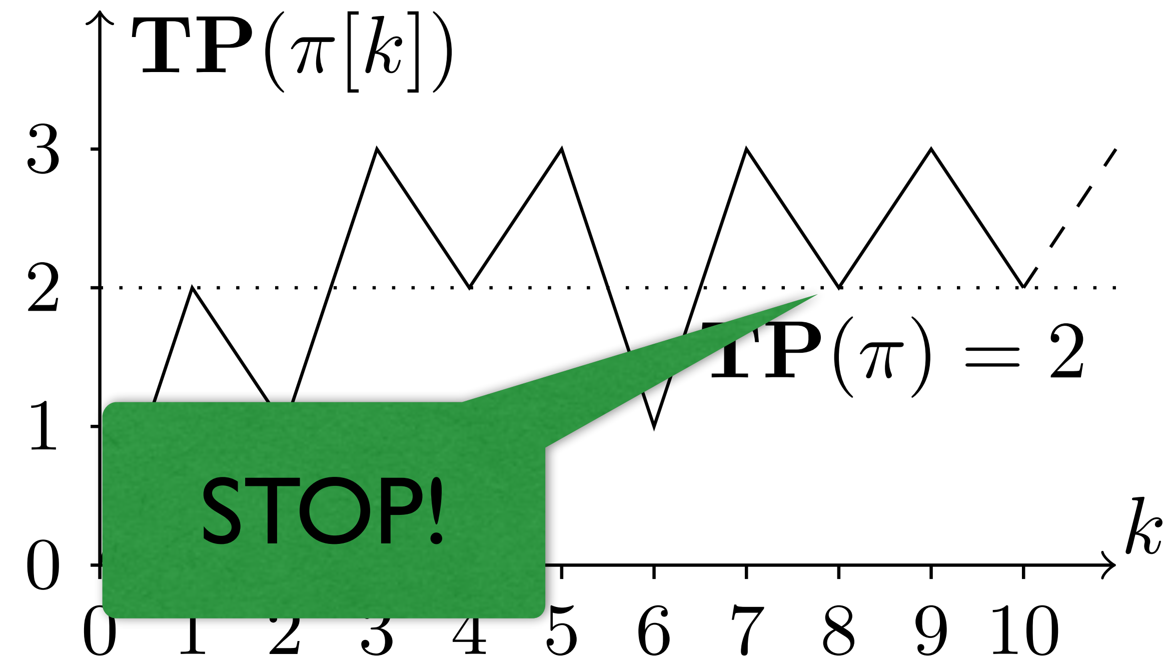

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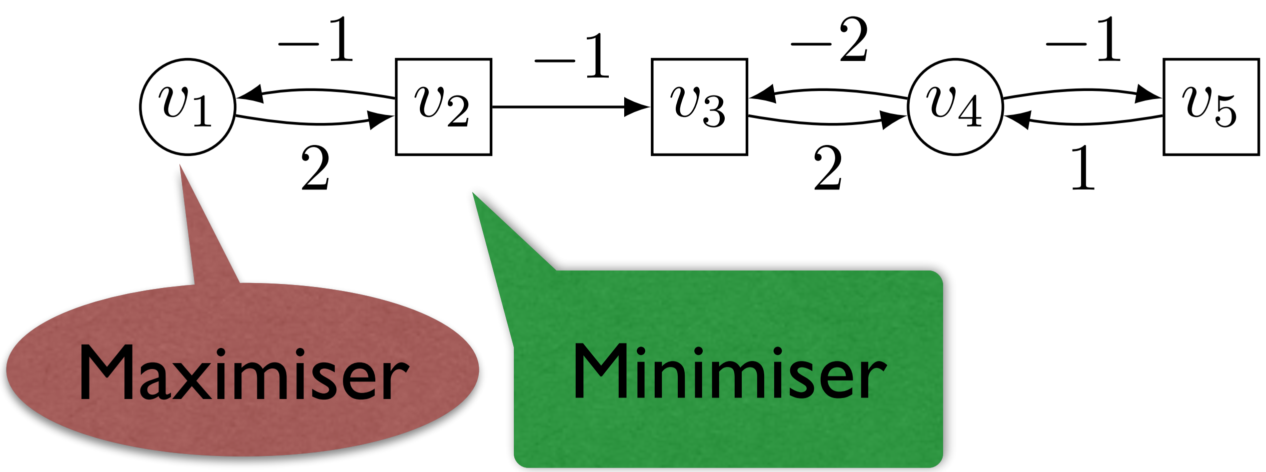

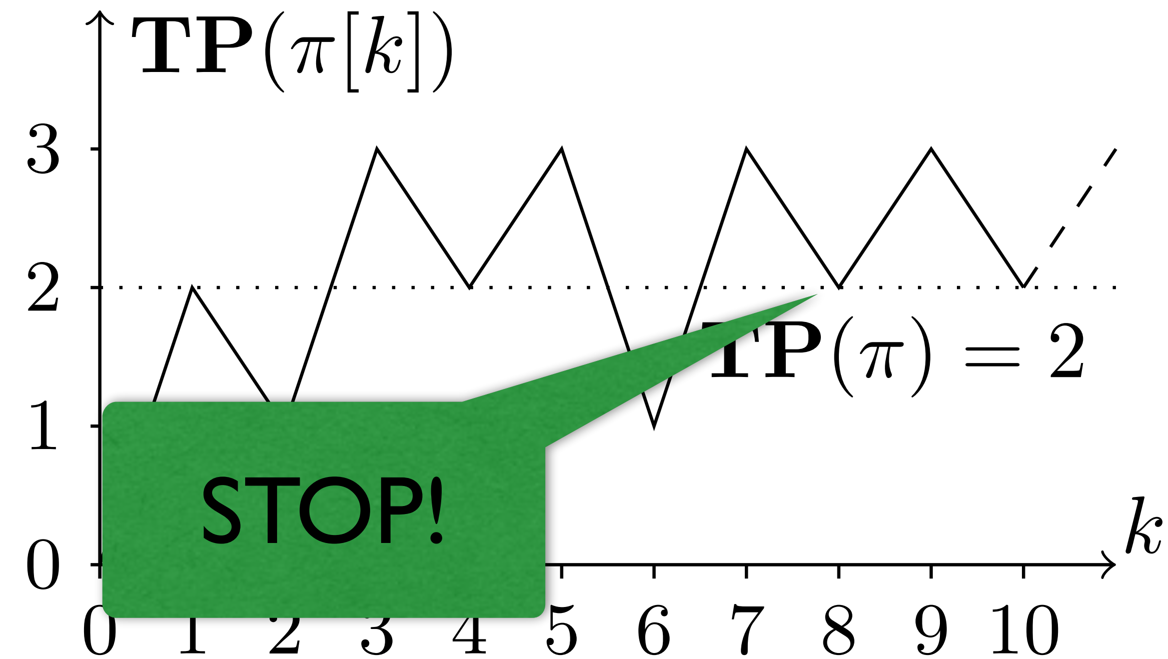

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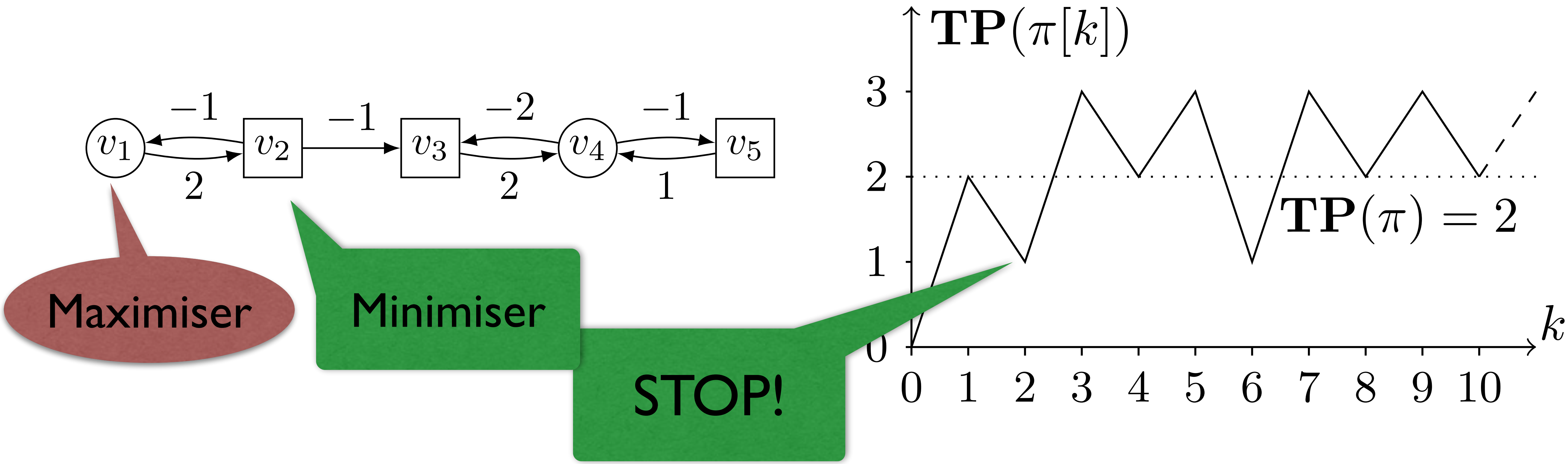

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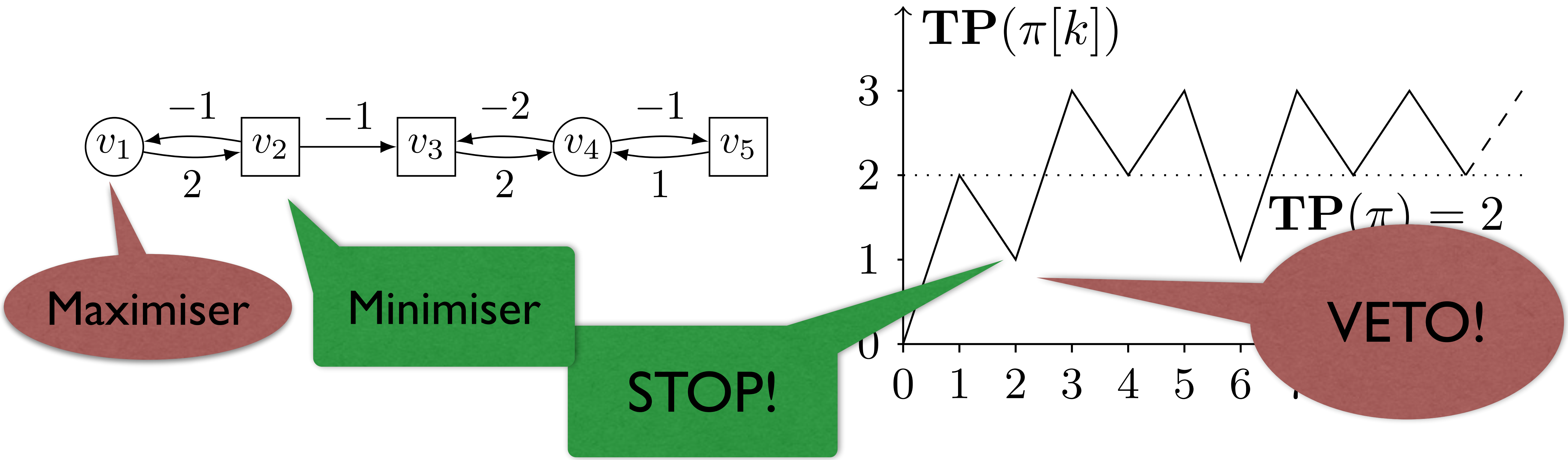

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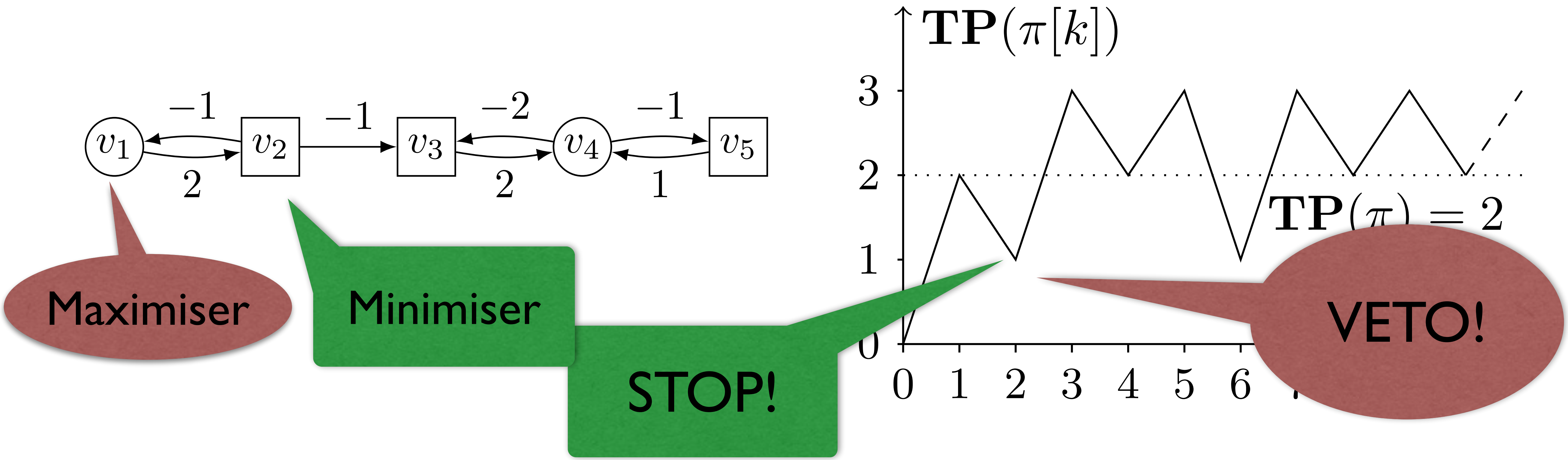

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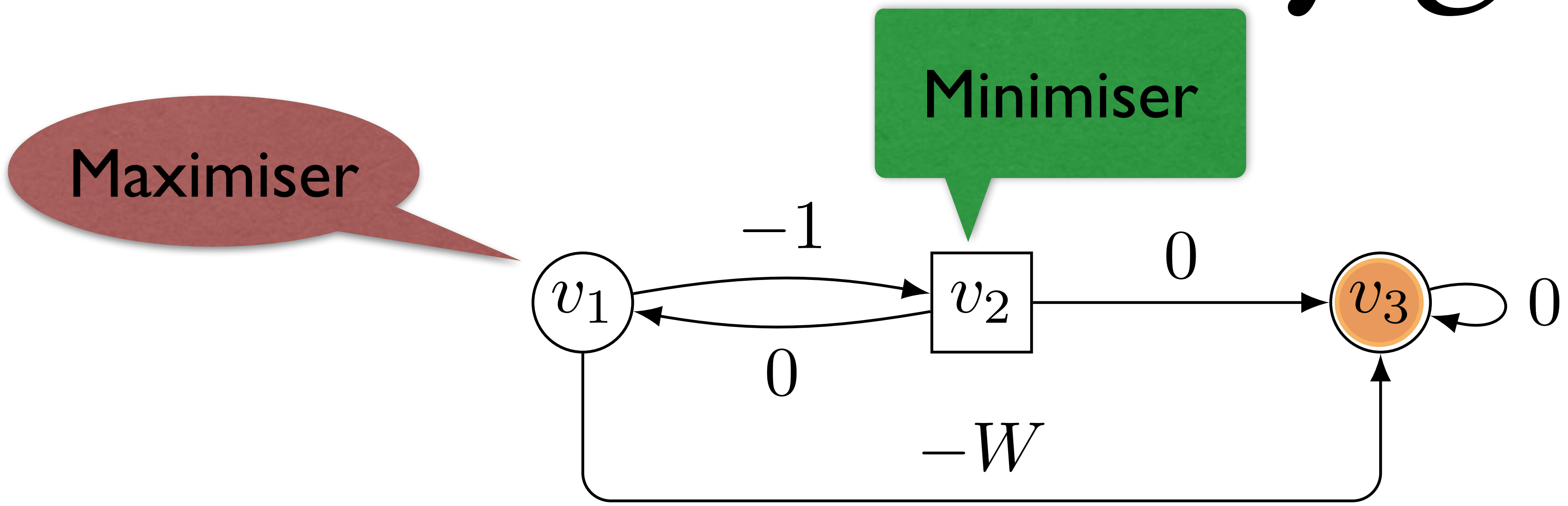
- Is there always a good value of *K* so that both games are equivalent?

Min-cost reachability games



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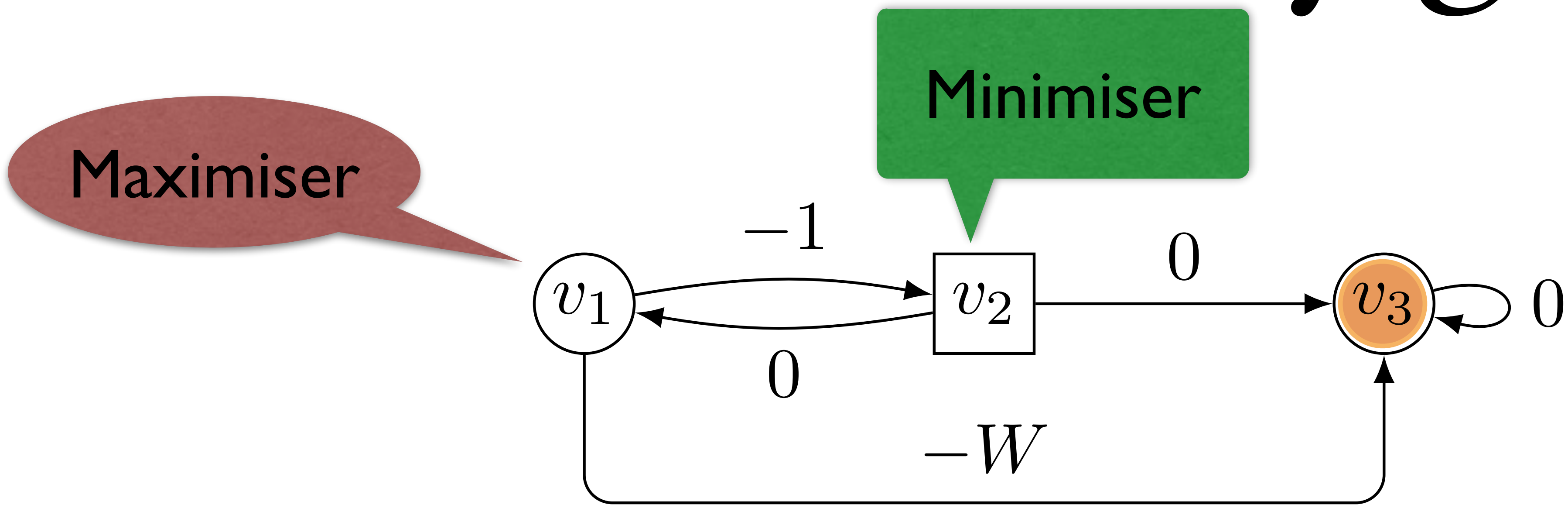

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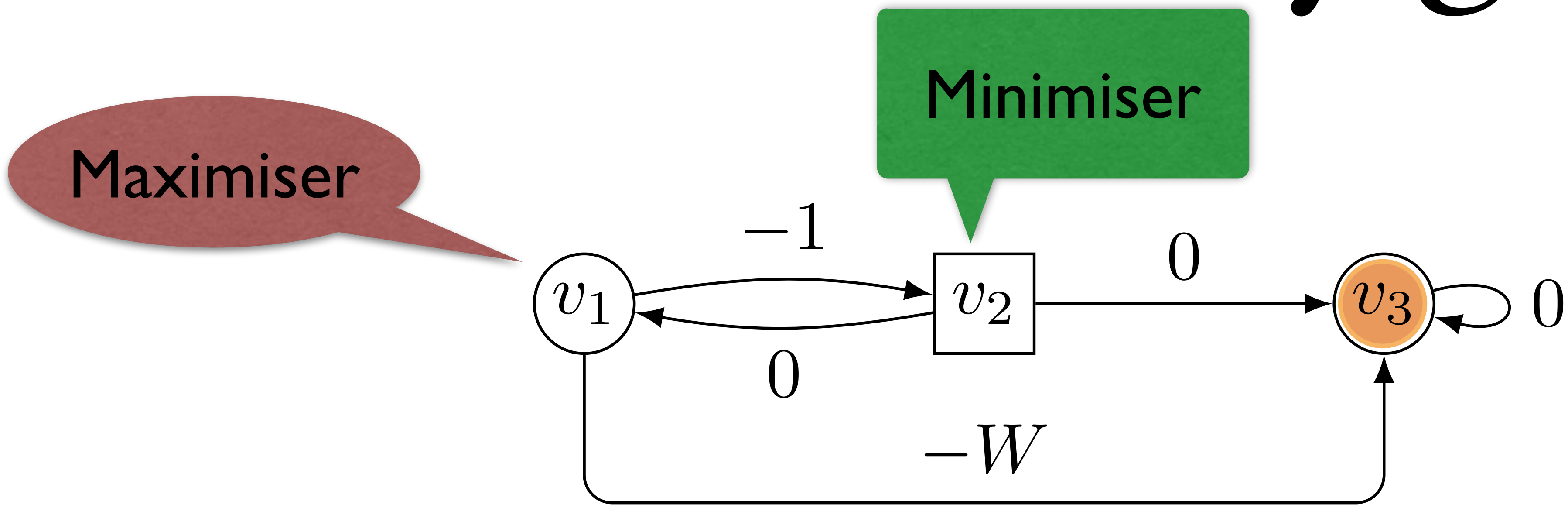

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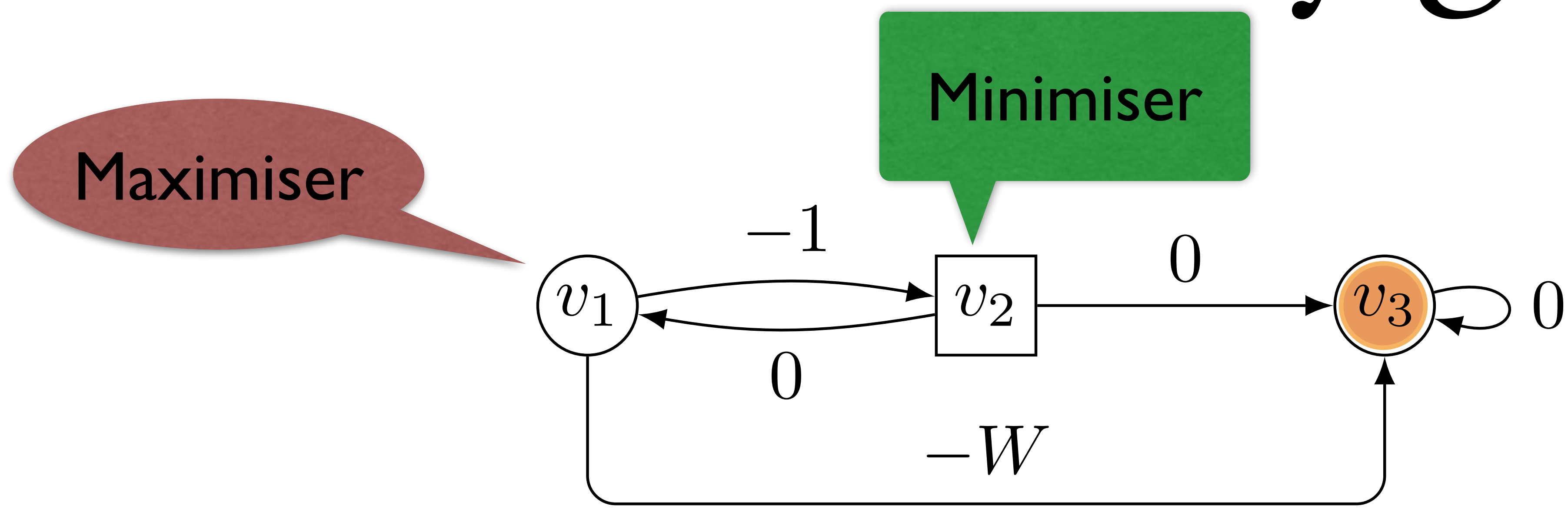

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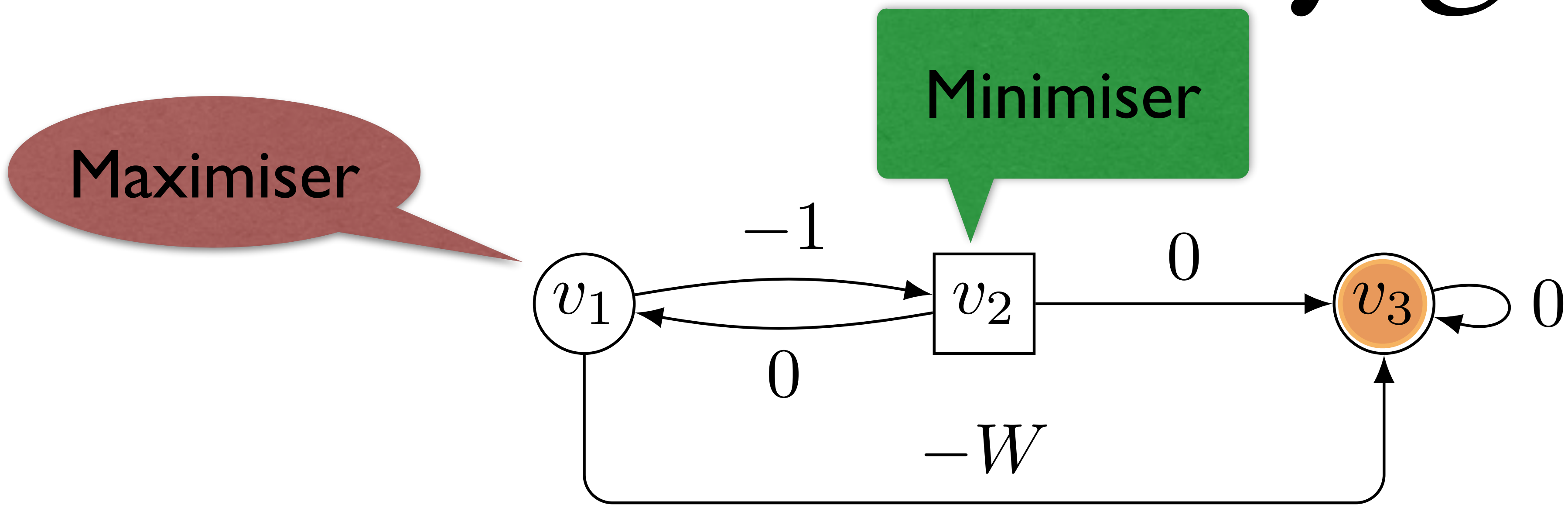

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- Example: value of v_1 and v_2 is $-W$… and Minimiser needs memory to ensure it!

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- Case one player: shortest path in a weighted graph…
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- General case? Not known…
  ➡ Our contribution: pseudo-polynomial time and as hard as solving mean-payoff games

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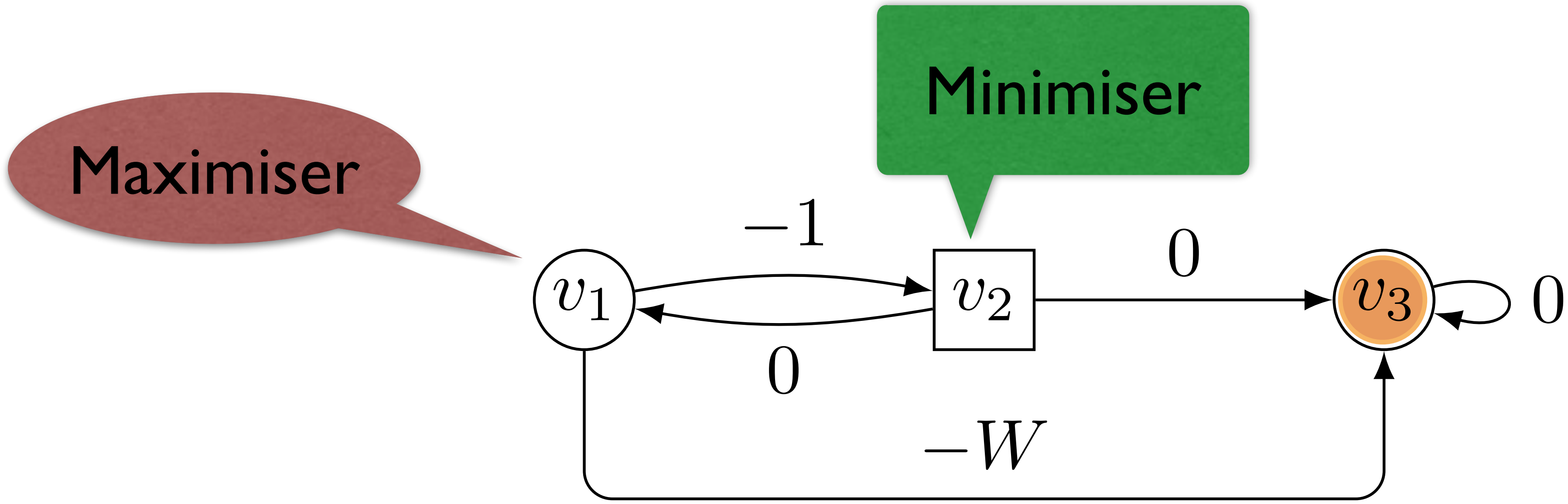
  - ➡ Maximiser: memoryless optimal strategy
  - ➡ Minimiser: finite memory suffices, and may be required

Value iteration on an example



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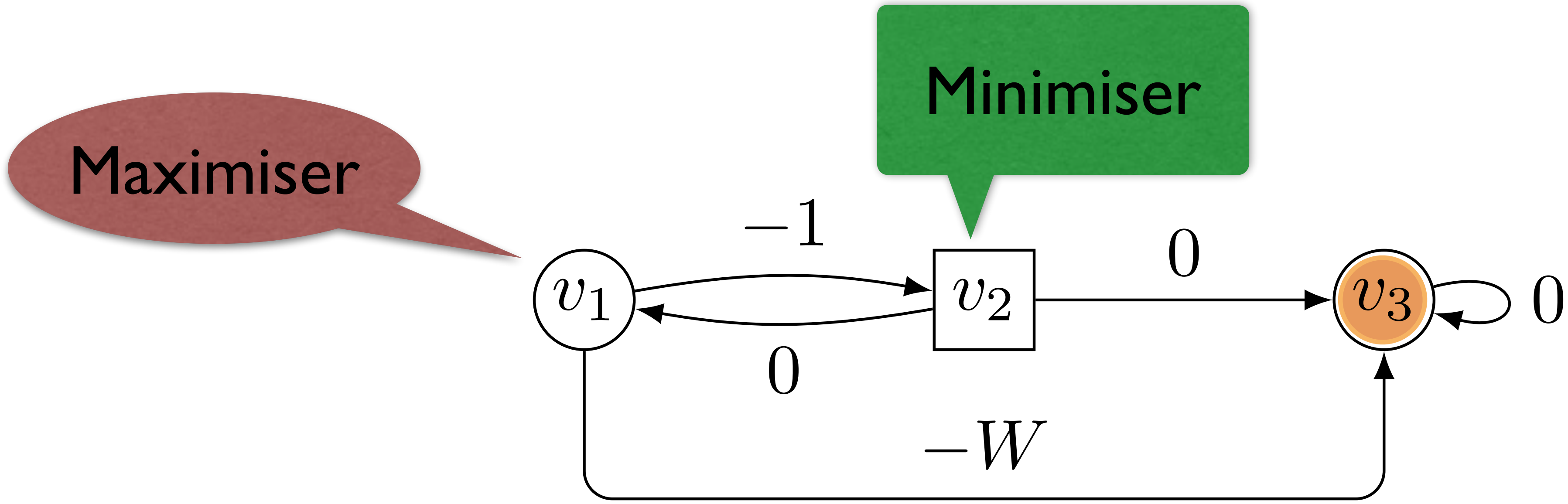

$+\infty$	$+\infty$	0

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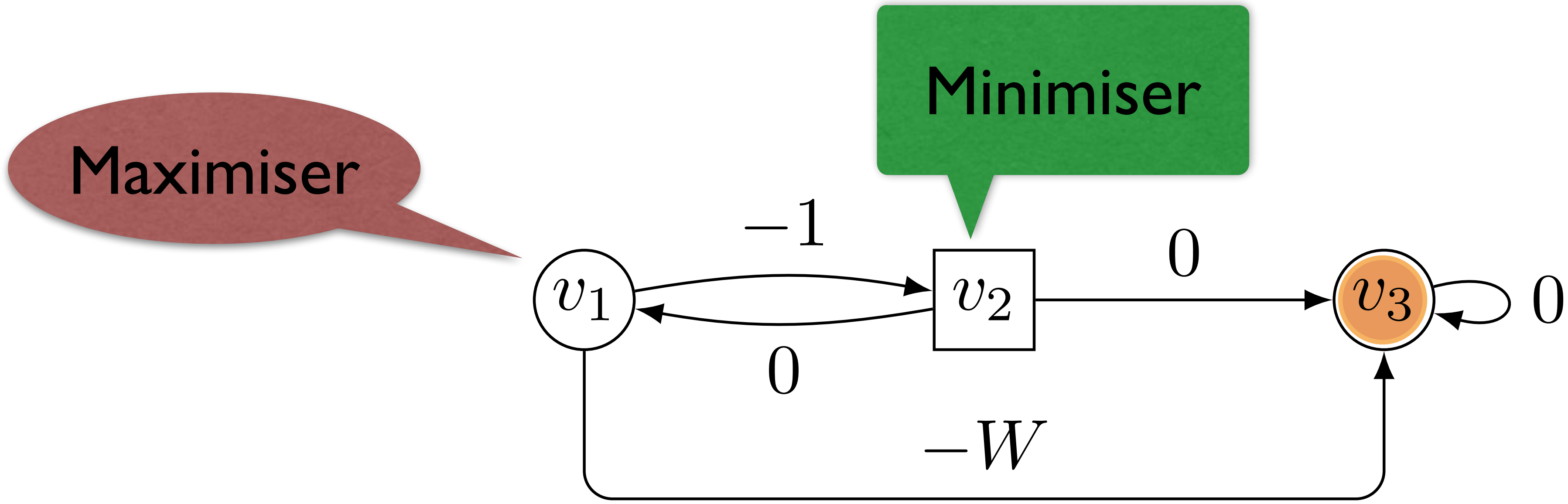
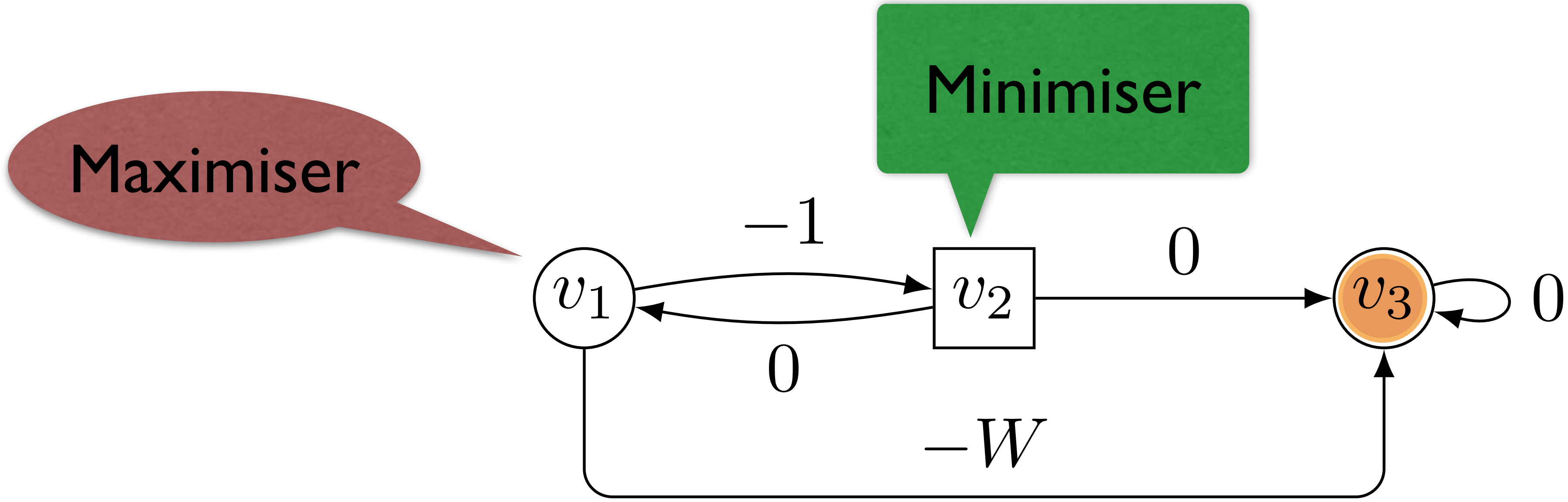


what may both players achieve in 1 step

$+\infty$	$+\infty$	0
$+\infty$	0	0

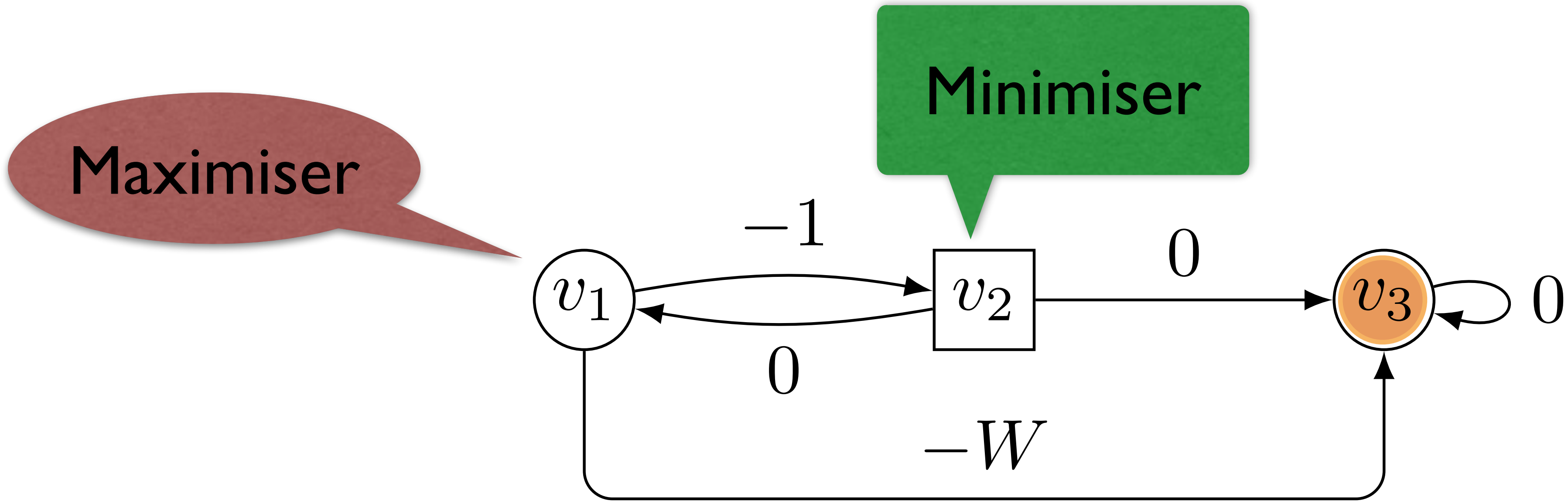
Value iteration on an example



what may both players achieve in 1 step

$+\infty$	$+\infty$	0
$+\infty$	0	0
-1	0	0

Value iteration on an example



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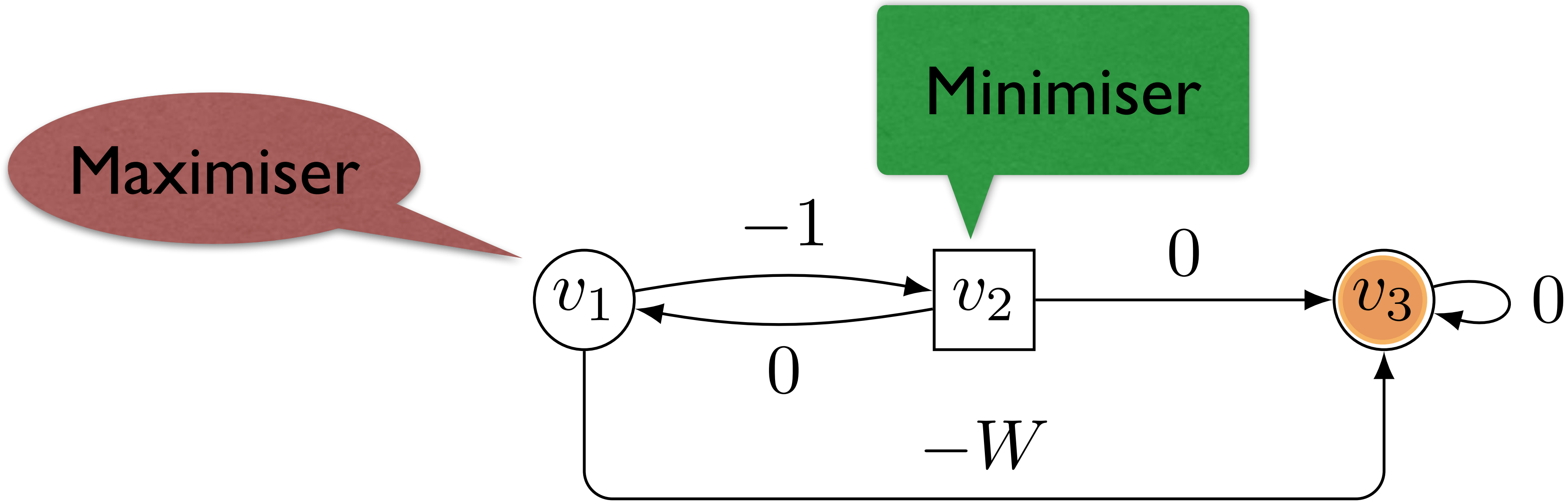
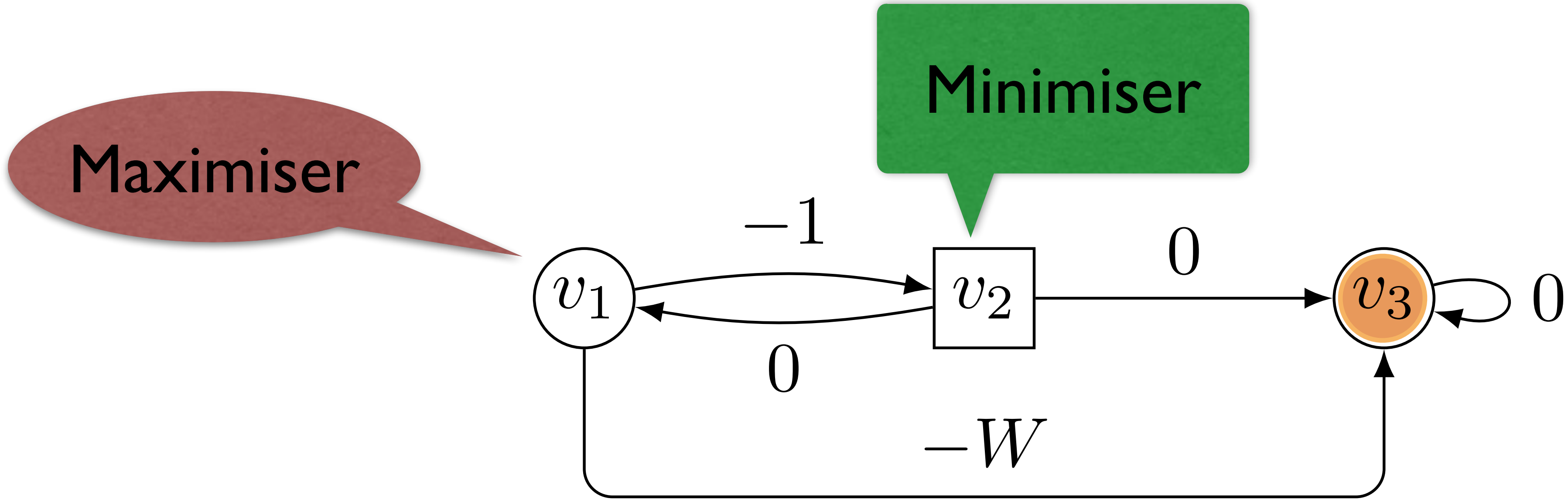


what may both players achieve in 1 step

$+\infty$	$+\infty$	0
$+\infty$	0	0
-1	0	0
-1	-1	0
-2	-1	0

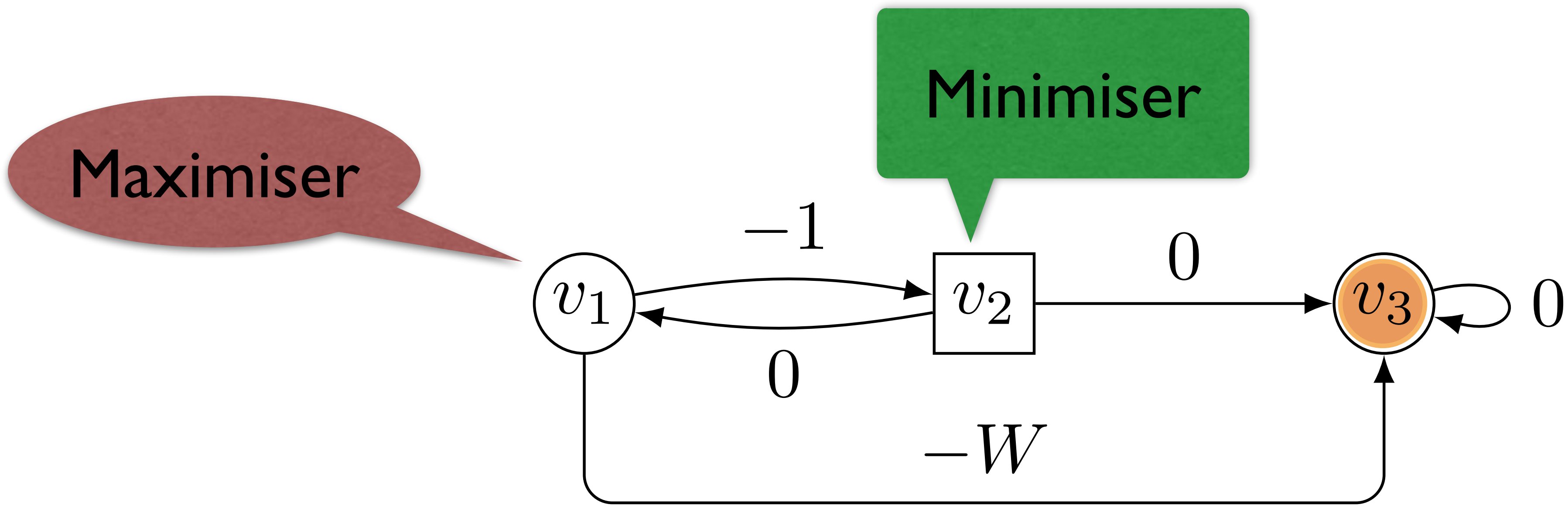
Value iteration on an example



what may both players achieve in 1 step

$+\infty$	$+\infty$	0
$+\infty$	0	0
–1	0	0
–1	–1	0
–2	–1	0
…	…	…
$-W$	$-W$	0

Value iteration on an example

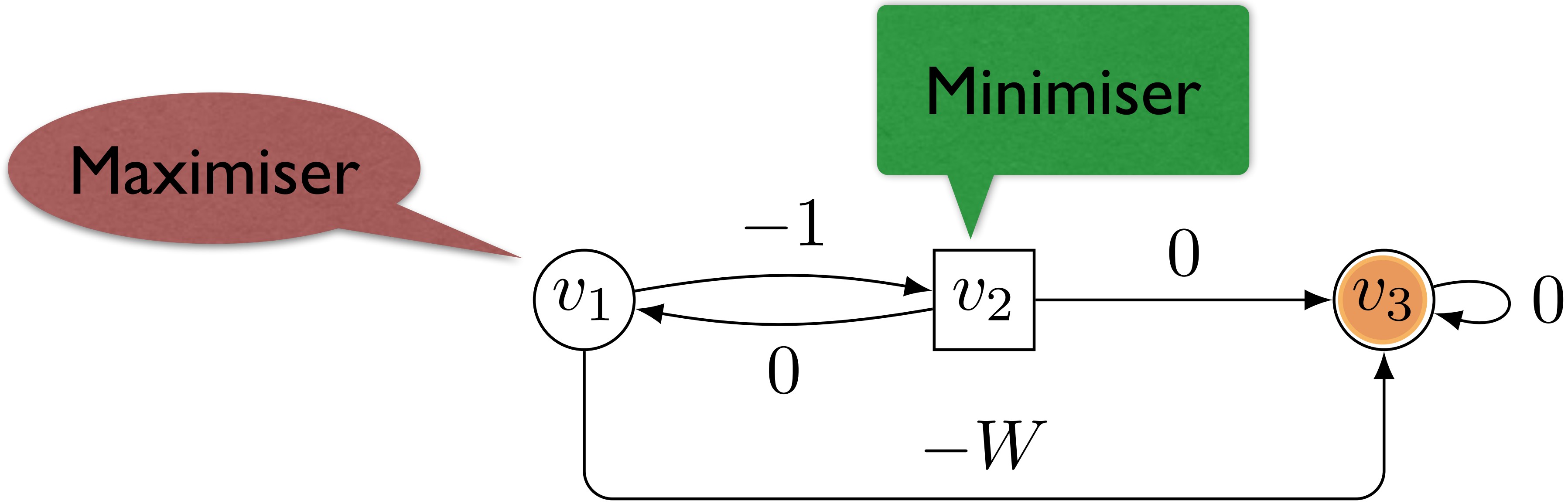


what may both players achieve in 1 step

$+\infty$	$+\infty$	0
$+\infty$	0	0
–1	0	0
–1	–1	0
–2	–1	0
...	...	...
–W	–W	0
–W	–W	0

stabilisation is proved always to happen in pseudo-polynomial time and the result is the value

Value iteration on an example



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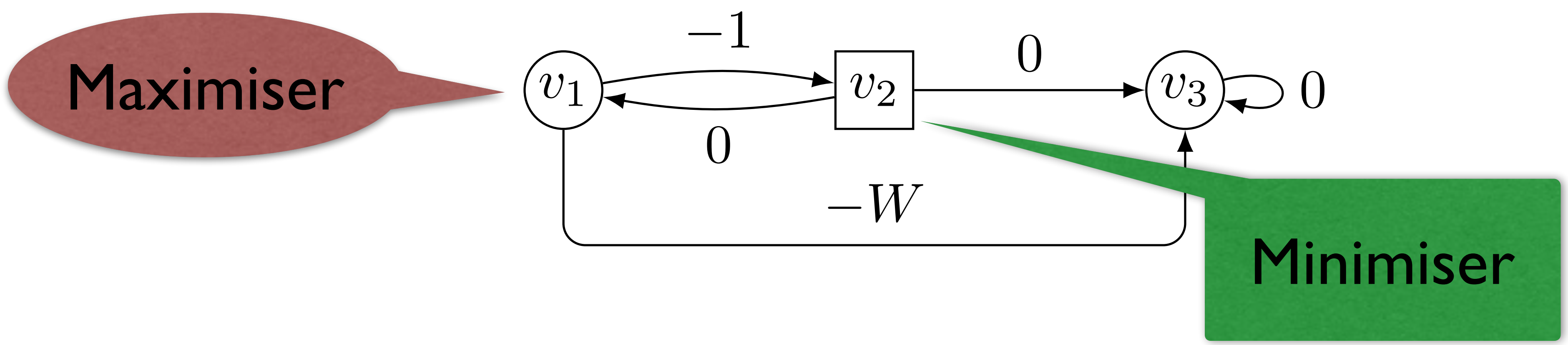
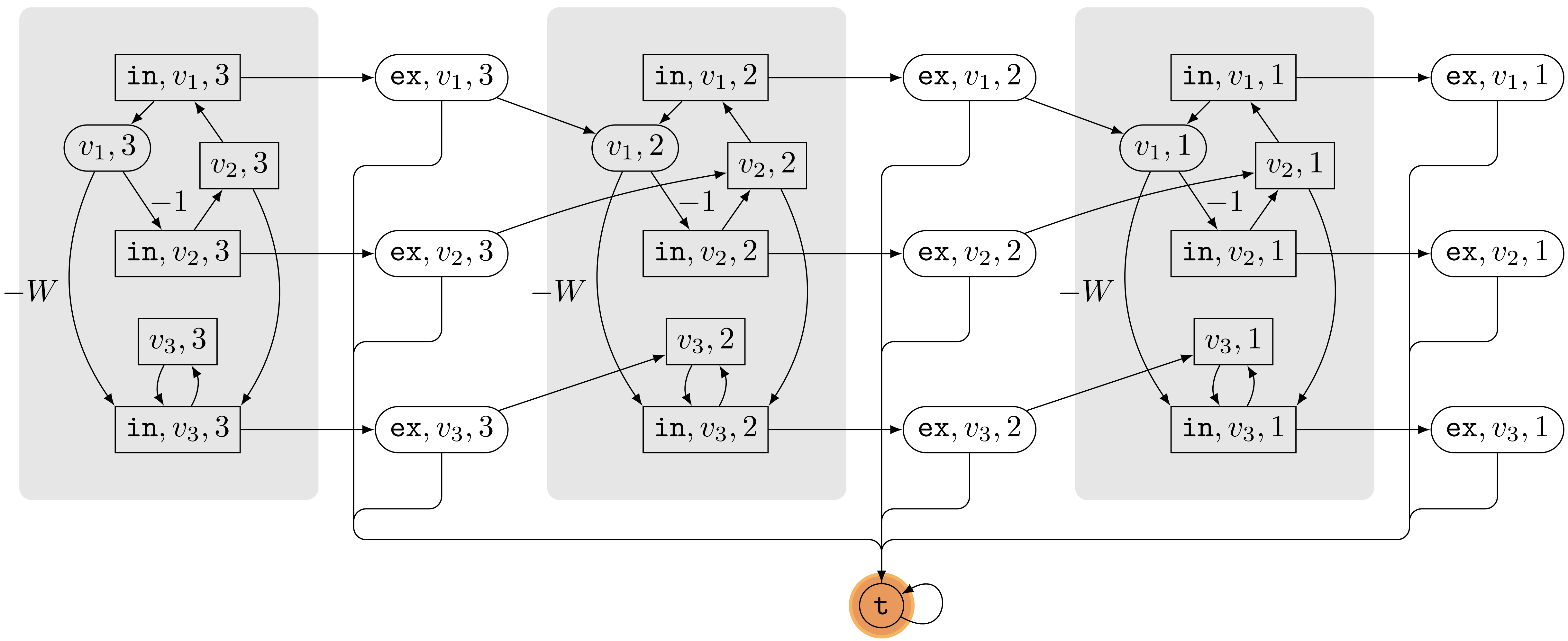
$+\infty$	$+\infty$	0
$+\infty$	0	0
-1	0	0
-1	-1	0
-2	-1	0
...	...	...
$-W$	$-W$	0
$-W$	$-W$	0

Strategy of Minimiser

From total-payoff to MCR games



- For all *K*, unfold *K* times the arena, allowing Minimiser to ask to go to target t



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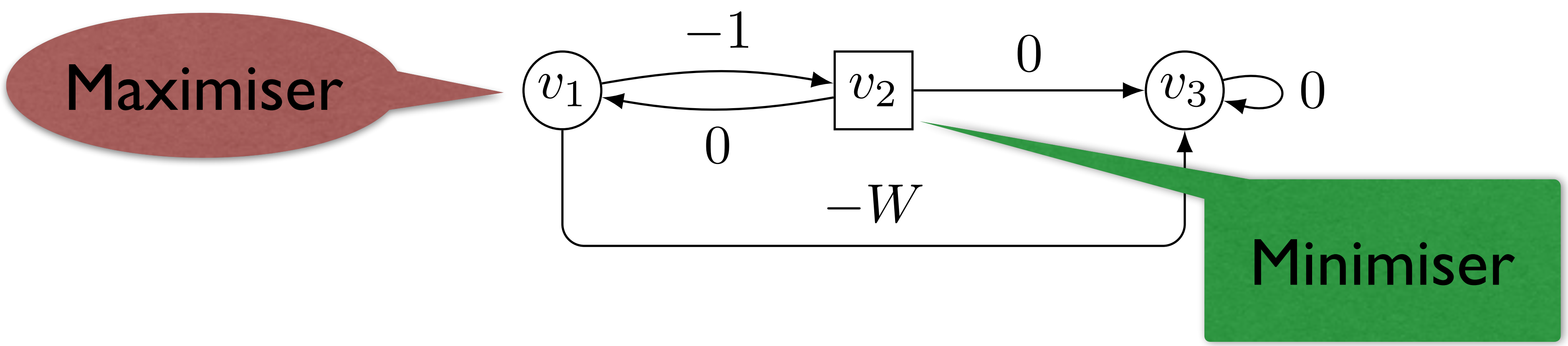

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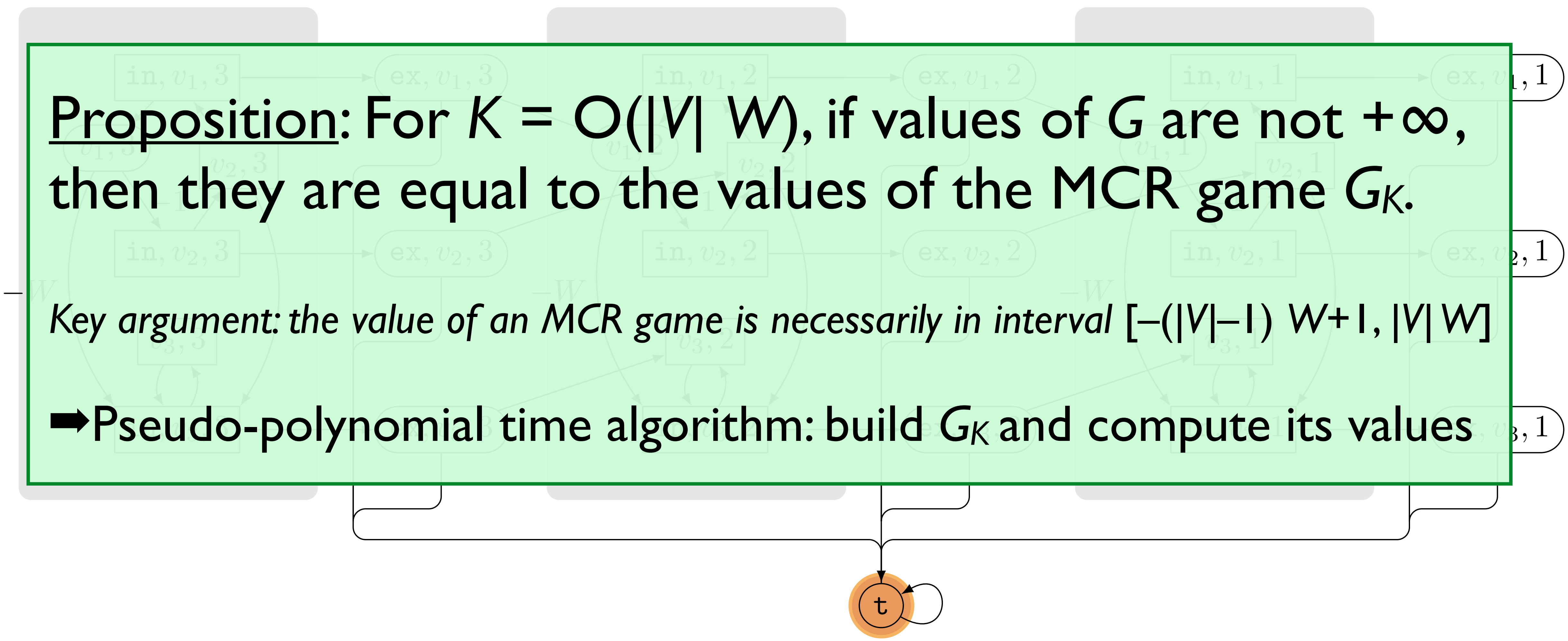
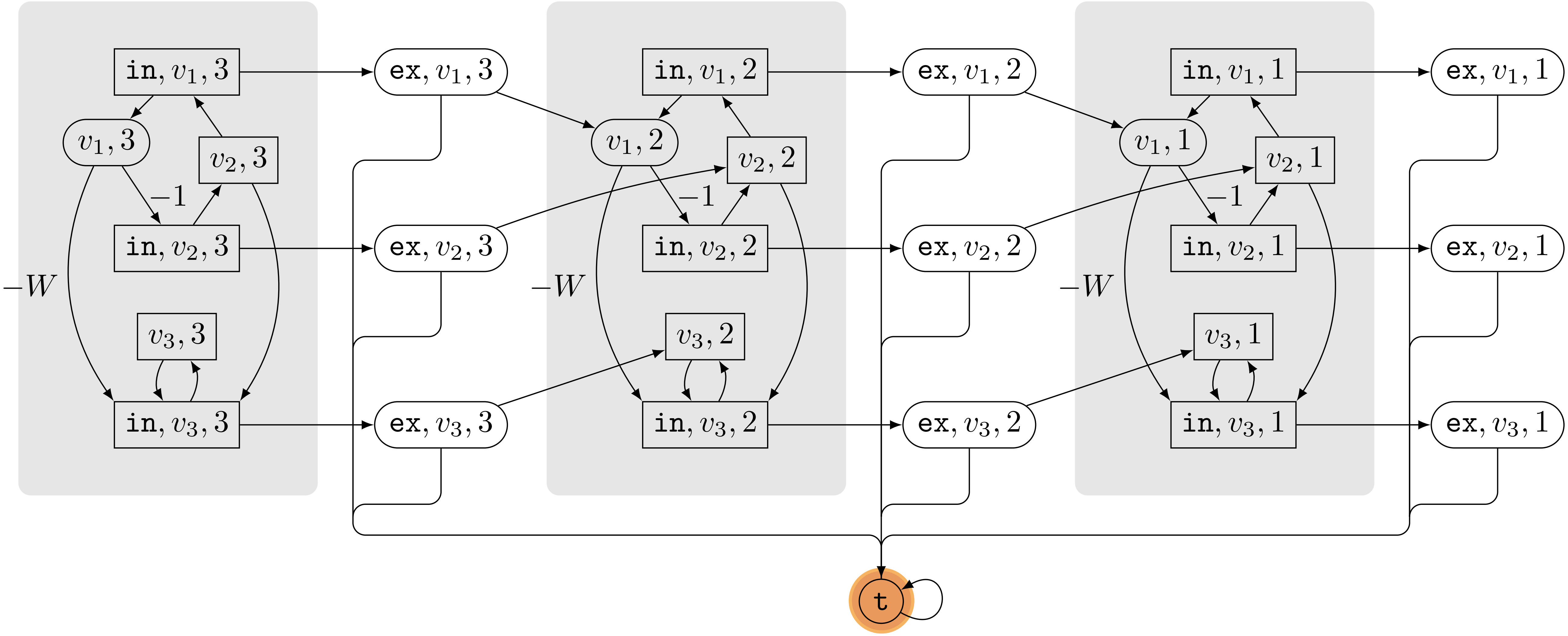


<u>Proposition</u>: For *K* = O(|*V*| *W*), if values of *G* are not +∞, then they are equal to the values of the MCR game G_K.

Key argument: the value of an MCR game is necessarily in interval [–(|*V*|–1) *W*+1, |*V*| *W*]

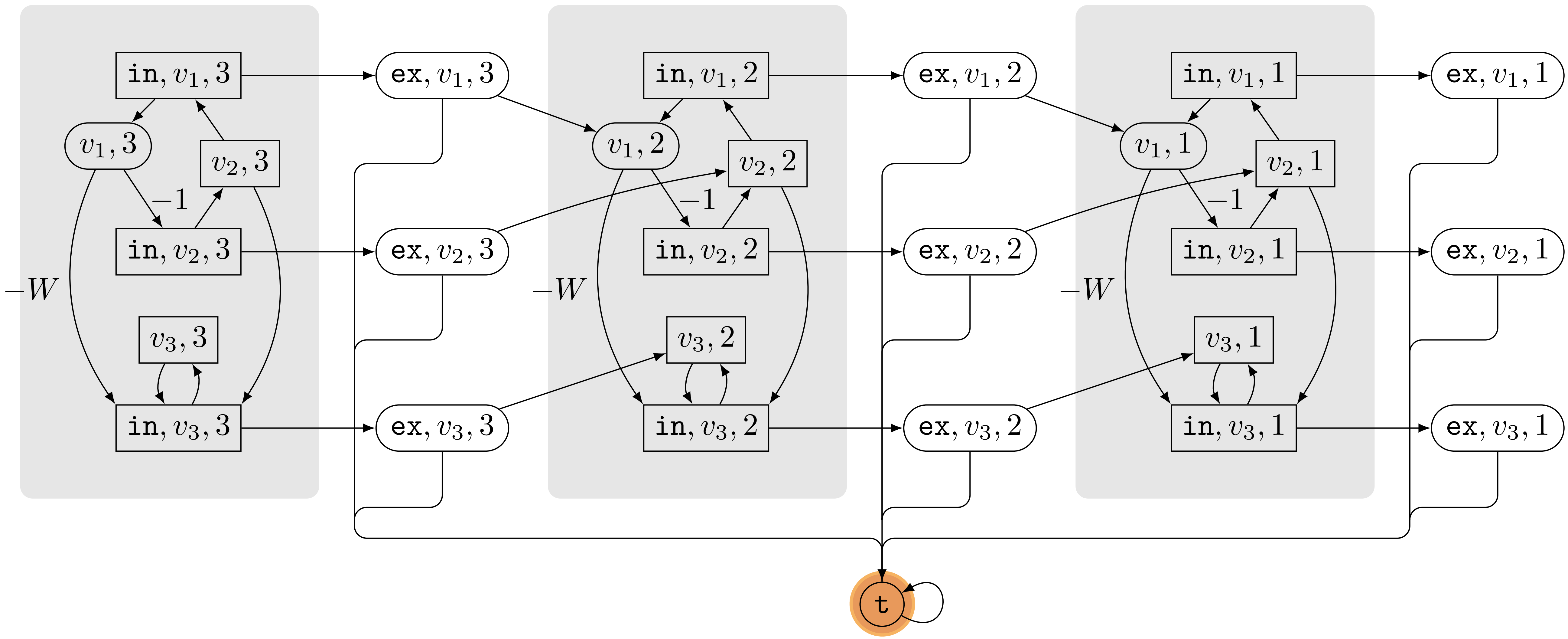
➡Pseudo-polynomial time algorithm: build G_K and compute its values

How not to build G_K?...



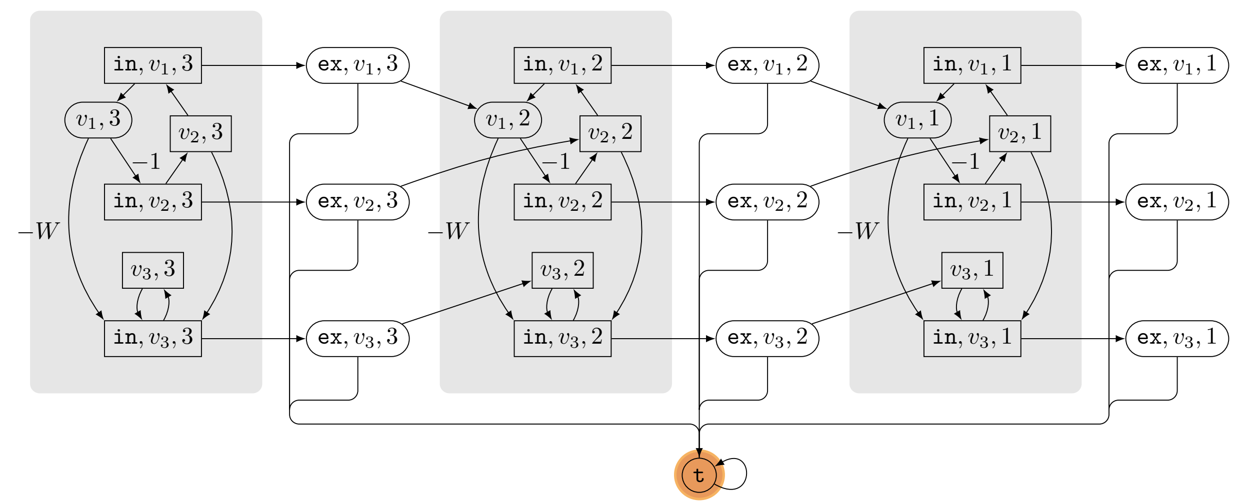
How not to build G_K?...

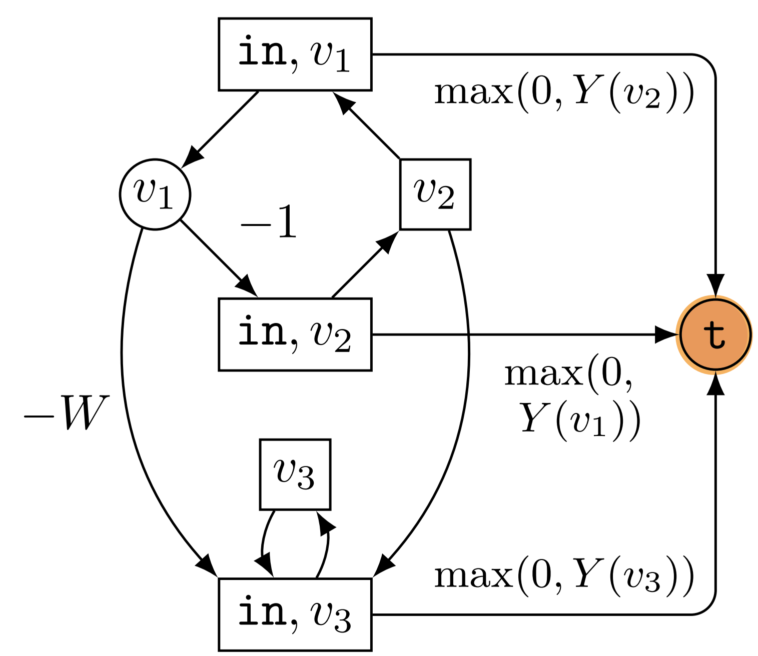
- In the value iteration for MCR games, we may compute the value from the last copy of the game to the first one (outer loop)



How not to build G_K?…

- In the value iteration for MCR games, we may compute the value from the last copy of the game to the first one (outer loop)
- Each time, it is the same arena: only the *exit* values evolve… Compute the values of a linear size MCR game (inner loop)





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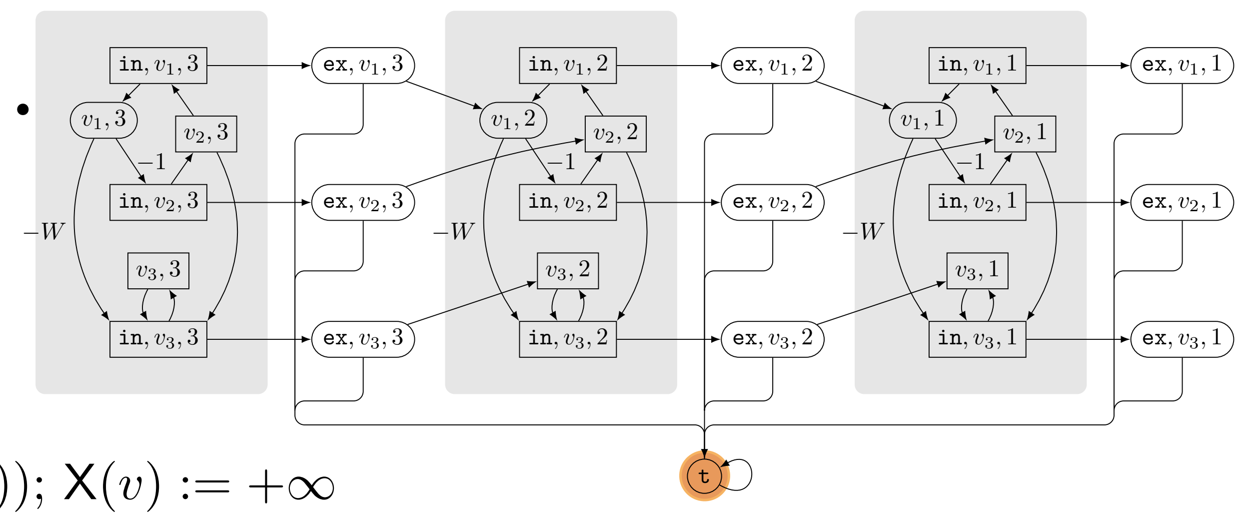
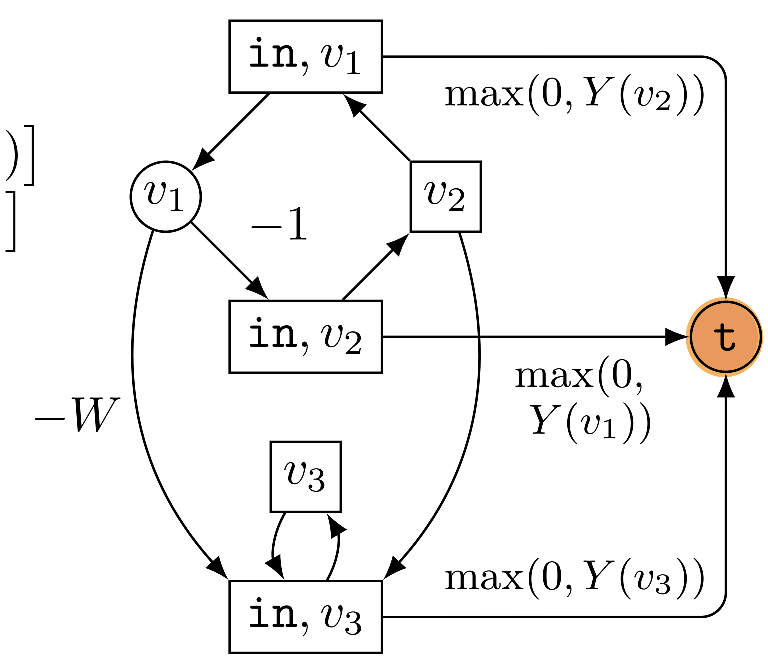
- In the value iteration for MCR games, we may compute the value from the last copy of the game to the first one (outer loop)
- Each time, it is the same arena: only the *exit* values evolve… Compute the values of a linear size MCR game (inner loop)
- Stop early inner and outer loops…



foreach $v \in V$ **do** $\mathsf{Y}(v) := -\infty$
repeat
  foreach $v \in V$ **do** $\mathsf{Y}_{pre}(v) := \mathsf{Y}(v)$; $\mathsf{Y}(v) := \max(0, \mathsf{Y}(v))$; $\mathsf{X}(v) := +\infty$
  repeat
    $\mathsf{X}_{pre} := \mathsf{X}$
    foreach $v \in V_{\mathsf{Max}}$ **do** $\mathsf{X}(v) := \max_{v' \in E(v)} \left[\omega(v, v') + \min(\mathsf{X}_{pre}(v'), \mathsf{Y}(v'))\right]$
    foreach $v \in V_{\mathsf{Min}}$ **do** $\mathsf{X}(v) := \min_{v' \in E(v)} \left[\omega(v, v') + \min(\mathsf{X}_{pre}(v'), \mathsf{Y}(v'))\right]$
    foreach $v \in V$ such that $\mathsf{X}(v) < -(|V| - 1)W$ **do** $\mathsf{X}(v) := -\infty$
  until $\mathsf{X} = \mathsf{X}_{pre}$
  $\mathsf{Y} := \mathsf{X}$
  foreach $v \in V$ such that $\mathsf{Y}(v) > (|V| - 1)W$ **do** $\mathsf{Y}(v) := +\infty$
until $\mathsf{Y} = \mathsf{Y}_{pre}$
return Y



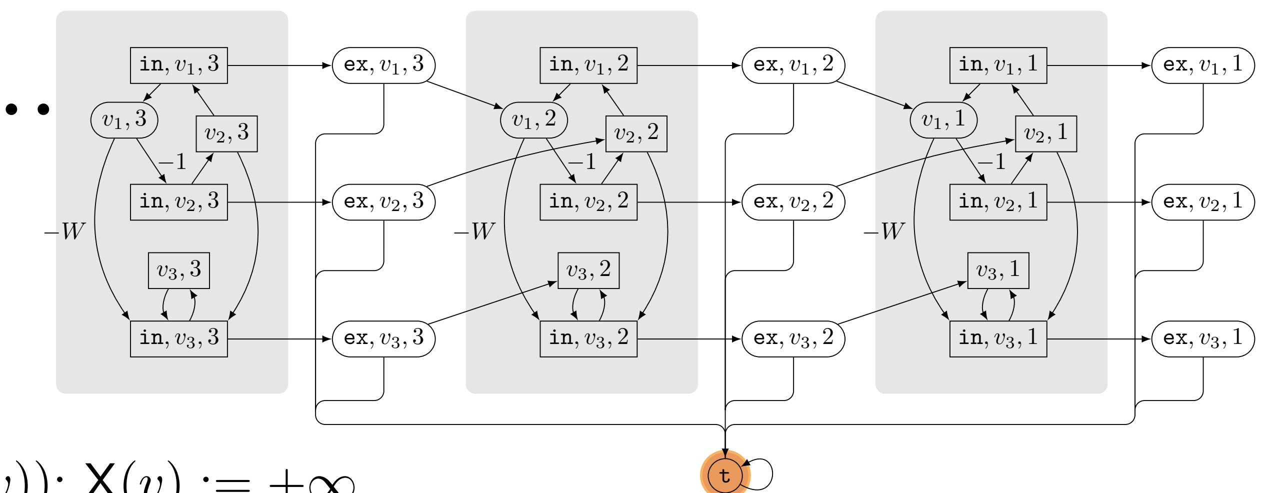
How not to build G_K?...

- In the value iteration for MCR games, we may compute the value from the last copy of the game to the first one (outer loop)
- Each time, it is the same arena: only the *exit* values evolve… Compute the values of a linear size MCR game (inner loop)

Requires very few memory (no need to construct G_K)

➡Pseudo-polynomial time: $O(|V|^4\ |E|\ W^2)$

- Stop early inner and outer loops…



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    foreach $v \in V_{\mathsf{Max}}$ **do** $\mathsf{X}(v) := \max_{v' \in E(v)} \left[\omega(v, v') + \min(\mathsf{X}_{pre}(v'), \mathsf{Y}(v'))\right]$
    foreach $v \in V_{\mathsf{Min}}$ **do** $\mathsf{X}(v) := \min_{v' \in E(v)} \left[\omega(v, v') + \min(\mathsf{X}_{pre}(v'), \mathsf{Y}(v'))\right]$
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  until $\mathsf{X} = \mathsf{X}_{pre}$
  $\mathsf{Y} := \mathsf{X}$
  foreach $v \in V$ such that $\mathsf{Y}(v) > (|V| - 1)W$ **do** $\mathsf{Y}(v) := +\infty$
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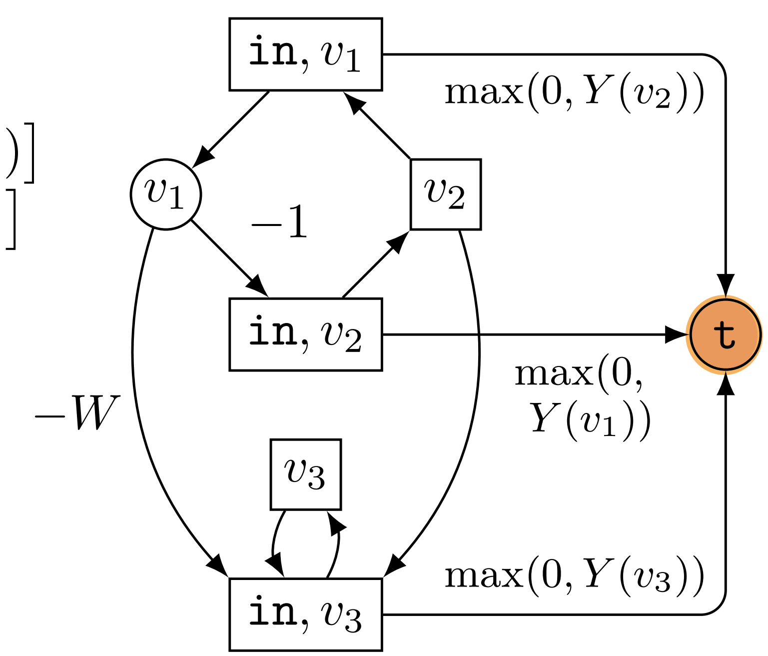
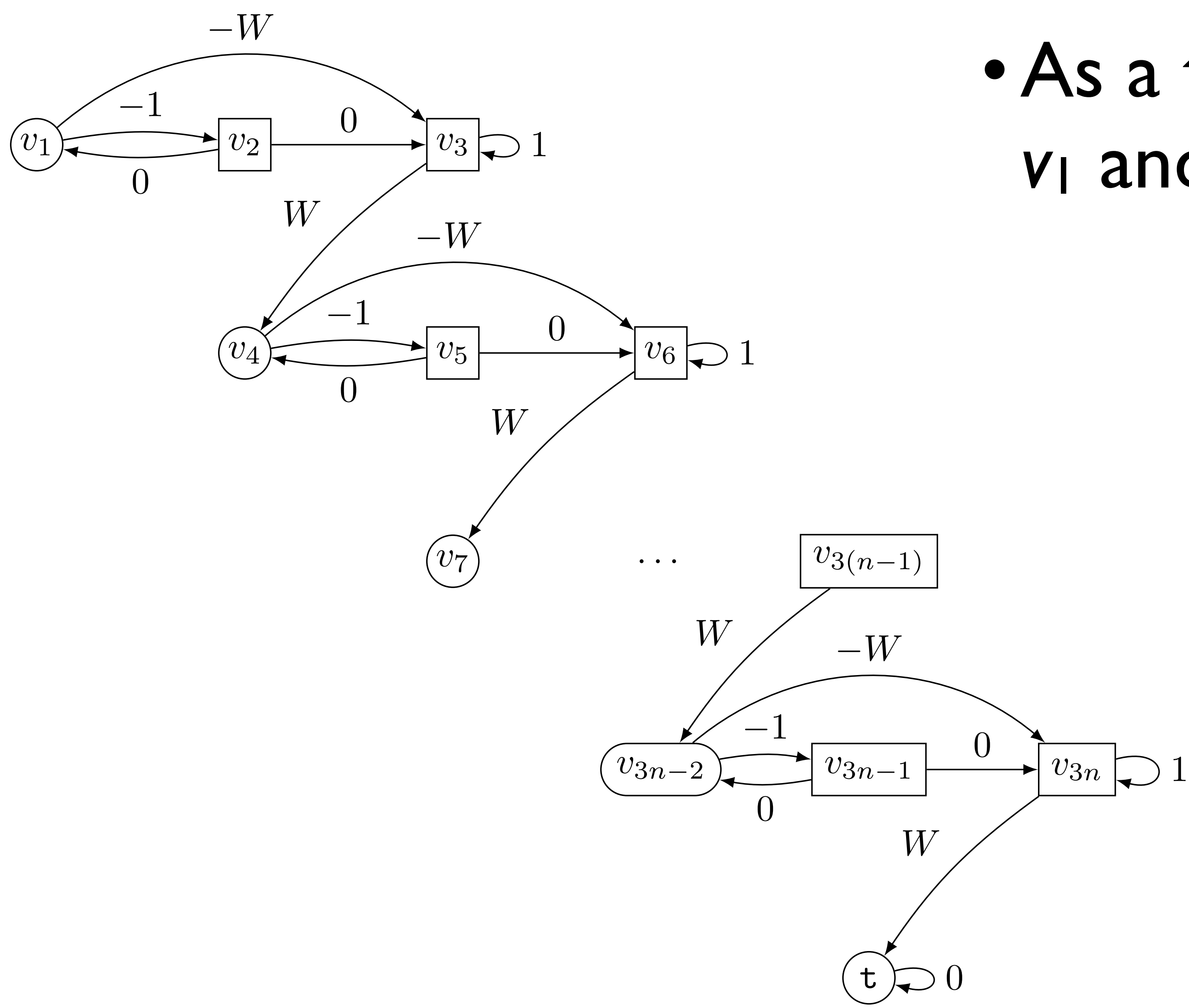
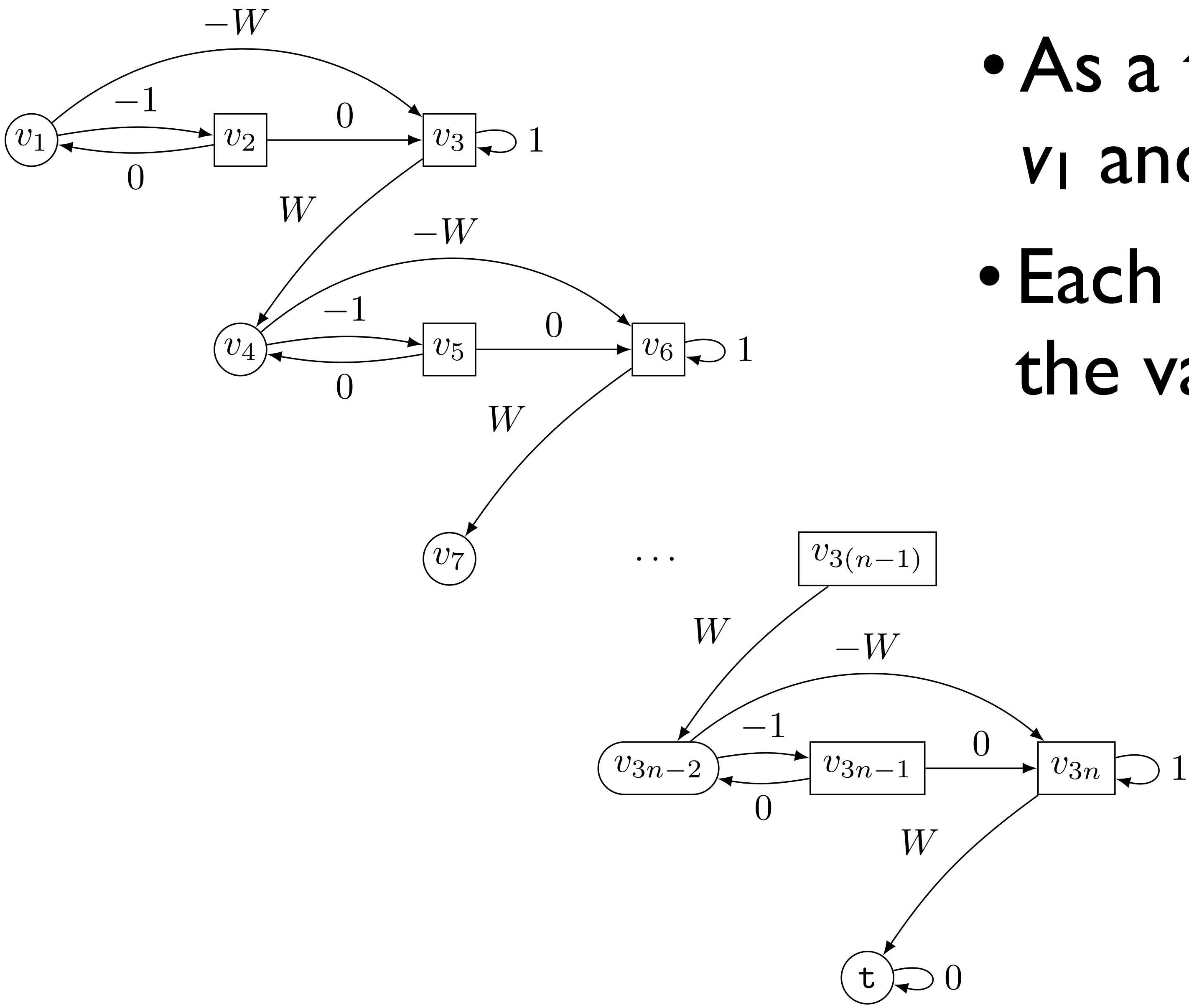


On an example



- As a total-payoff game, values of v_1 and v_2 are 0, value of v_3 is W…

On an example



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- Each inner loop computes all the values (needs $O(nW)$ steps)

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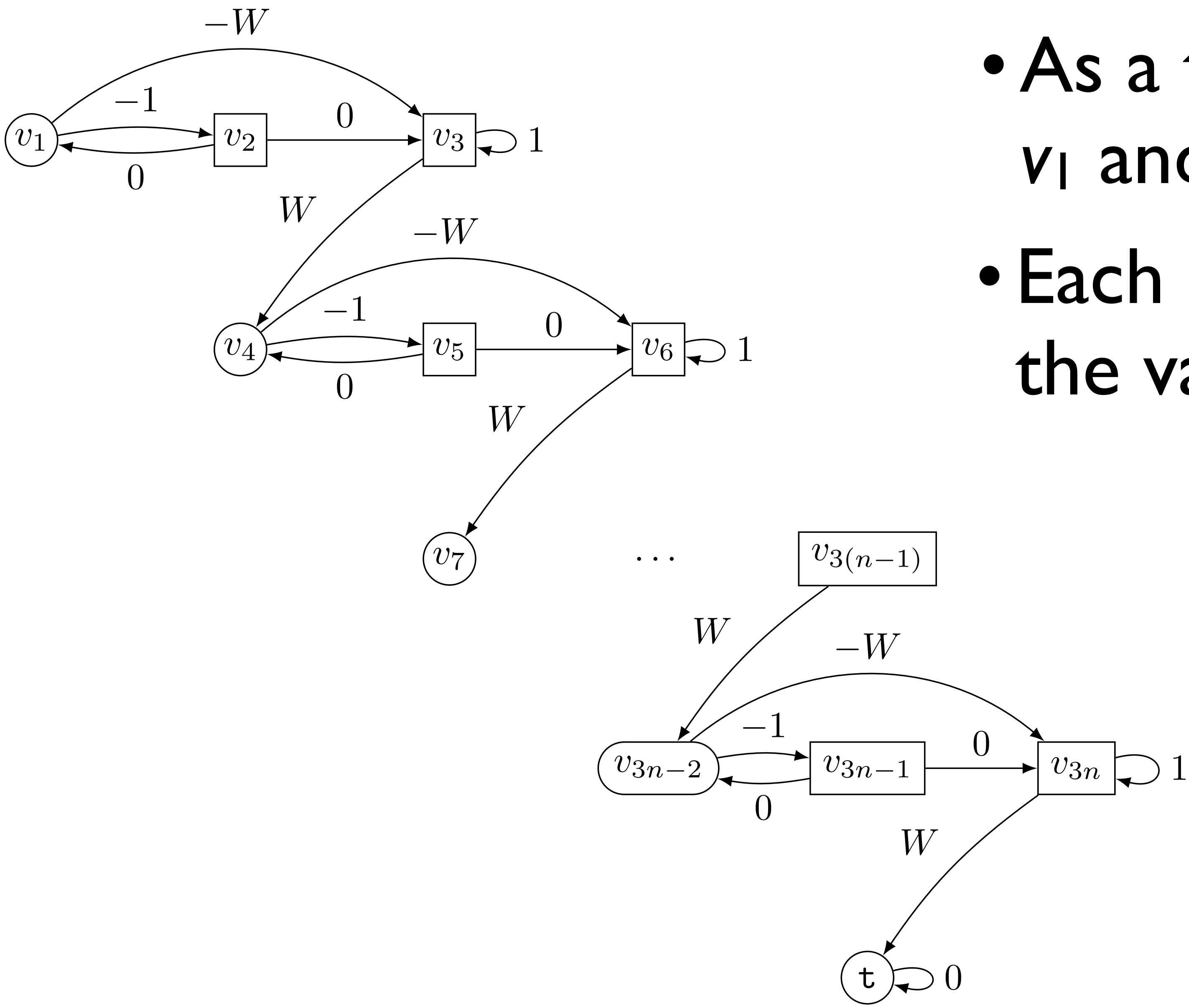

- As a total-payoff game, values of v_1 and v_2 are 0, value of v_3 is W…
- Each inner loop computes all the values (needs $O(n\,W)$ steps)
  - After 1 outer loop, only the values of the last component are correct…

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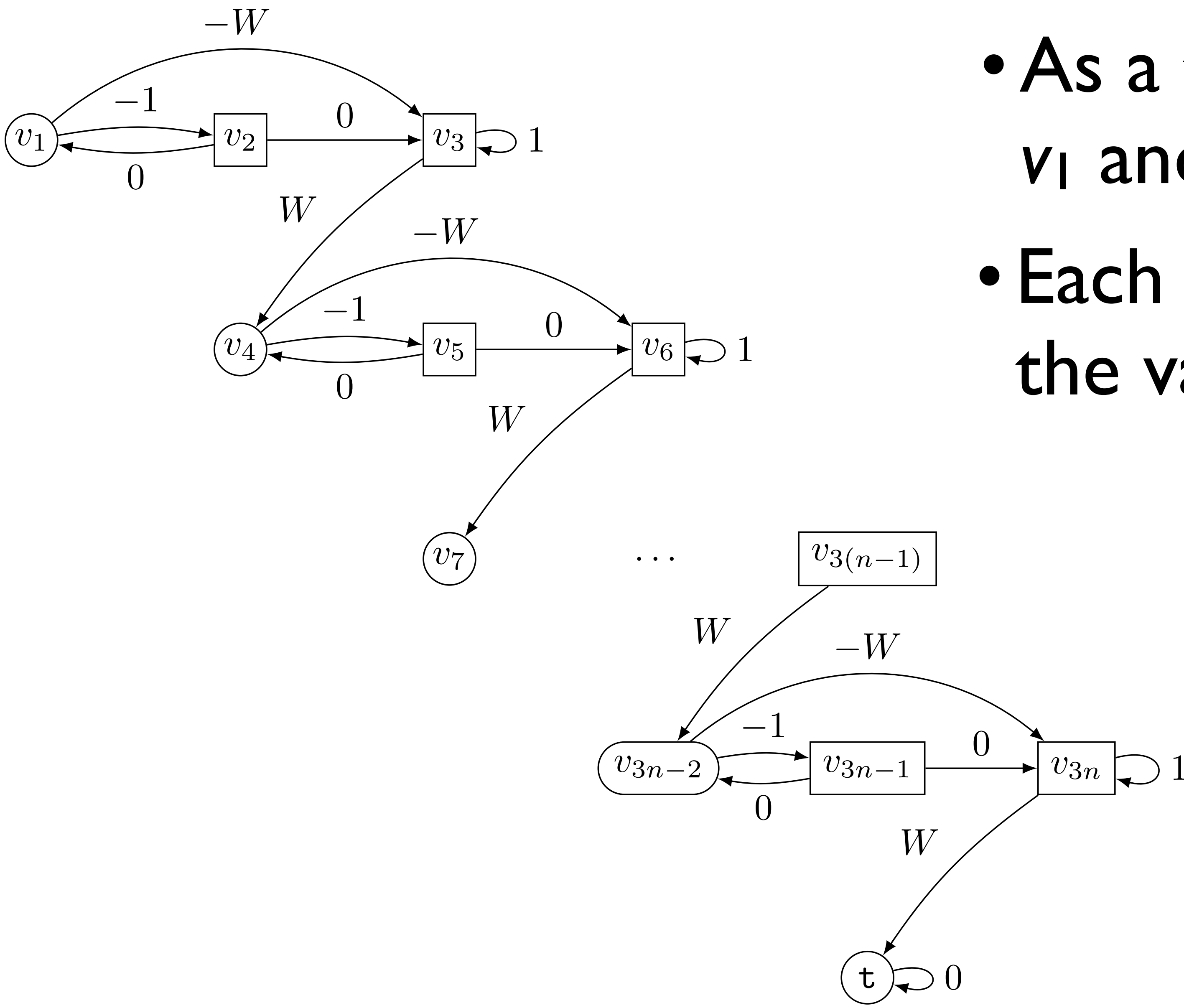

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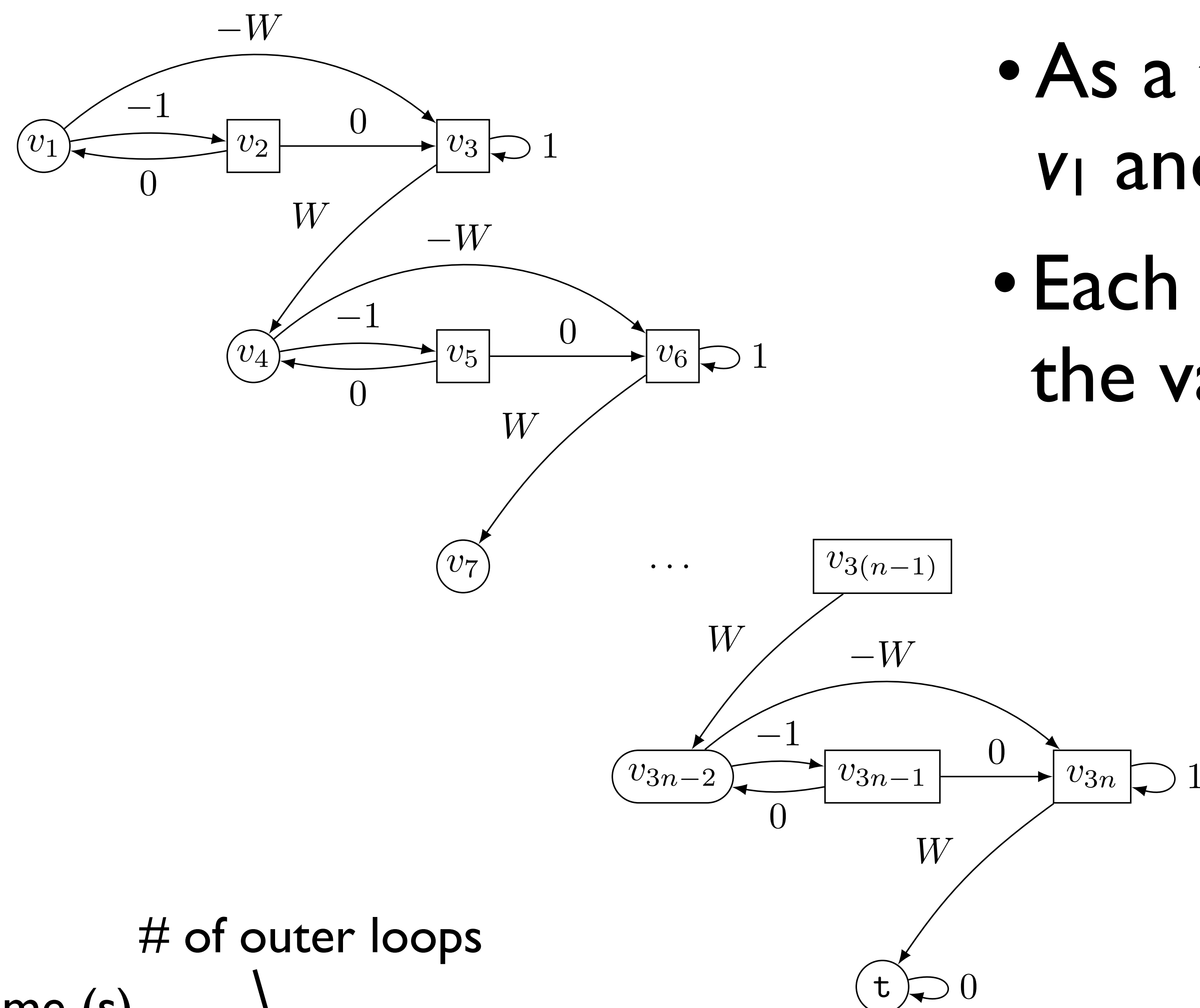
  - Requires n outer loops to converge

On an example



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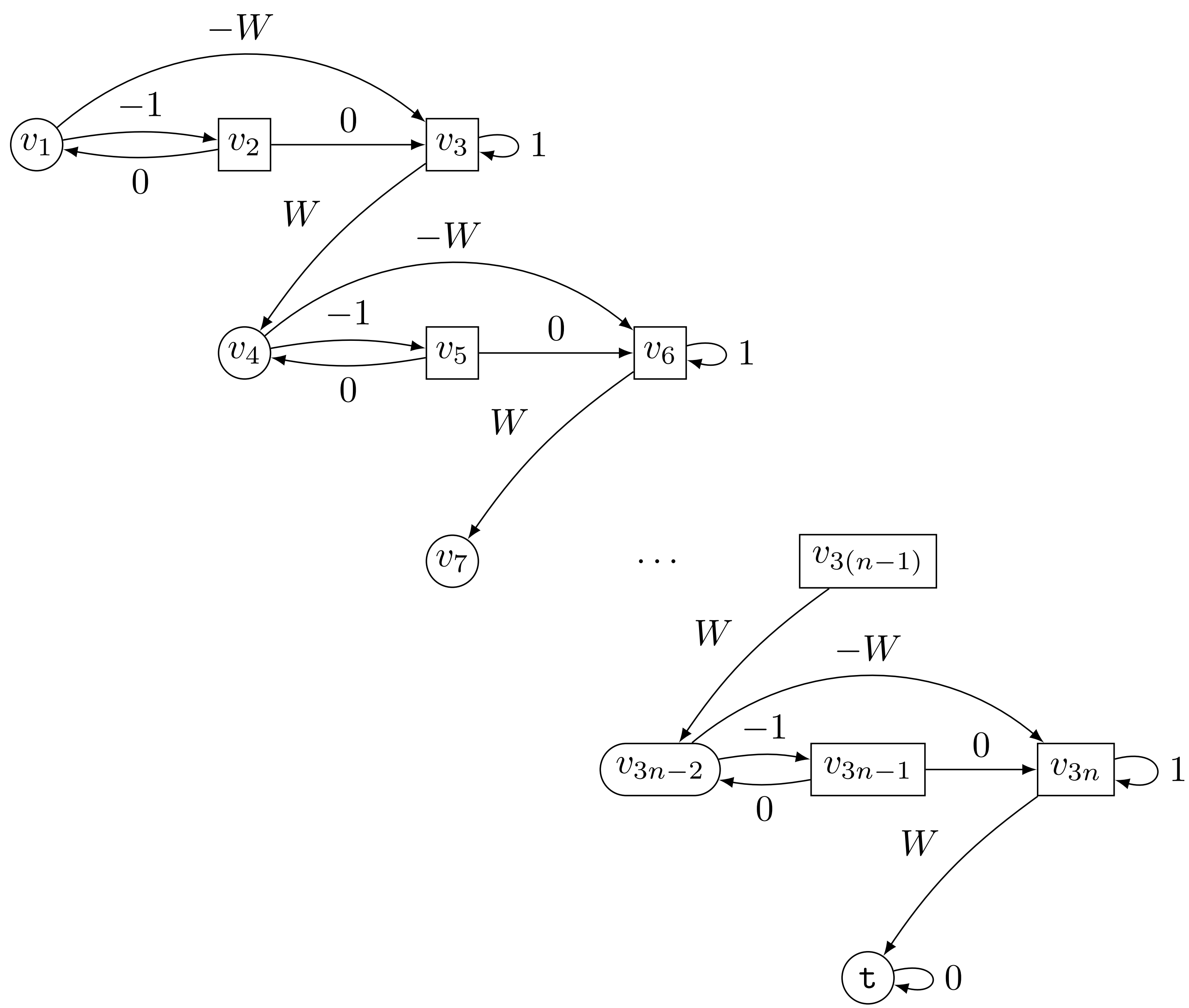
# of outer loops

time (s)

total # of inner loops

$W \backslash n$	100	200	300	400	500
50	0.52 / 151 / 12603	1.90 / 251 / 22703	3.84 / 351 / 32803	6.05 / 451 / 42903	9.83 / 551 / 53003
100	1.00 / 201 / 30103	3.48 / 301 / 50203	8.64 / 401 / 70303	13.53 / 501 / 90403	22.64 / 601 / 110503
150	1.89 / 251 / 52603	6.02 / 351 / 82703	12.88 / 451 / 112803	22.13 / 551 / 142903	34.16 / 651 / 173003
200	2.96 / 301 / 80103	9.62 / 401 / 120203	18.33 / 501 / 160303	30.42 / 601 / 200403	45.64 / 701 / 240503
250	3.92 / 351 / 112603	13.28 / 451 / 162703	25.18 / 551 / 212803	46.23 / 651 / 262903	71.51 / 751 / 313003

Heuristics: compute as little as possible!



- In the outer loop, compute SCC by SCC

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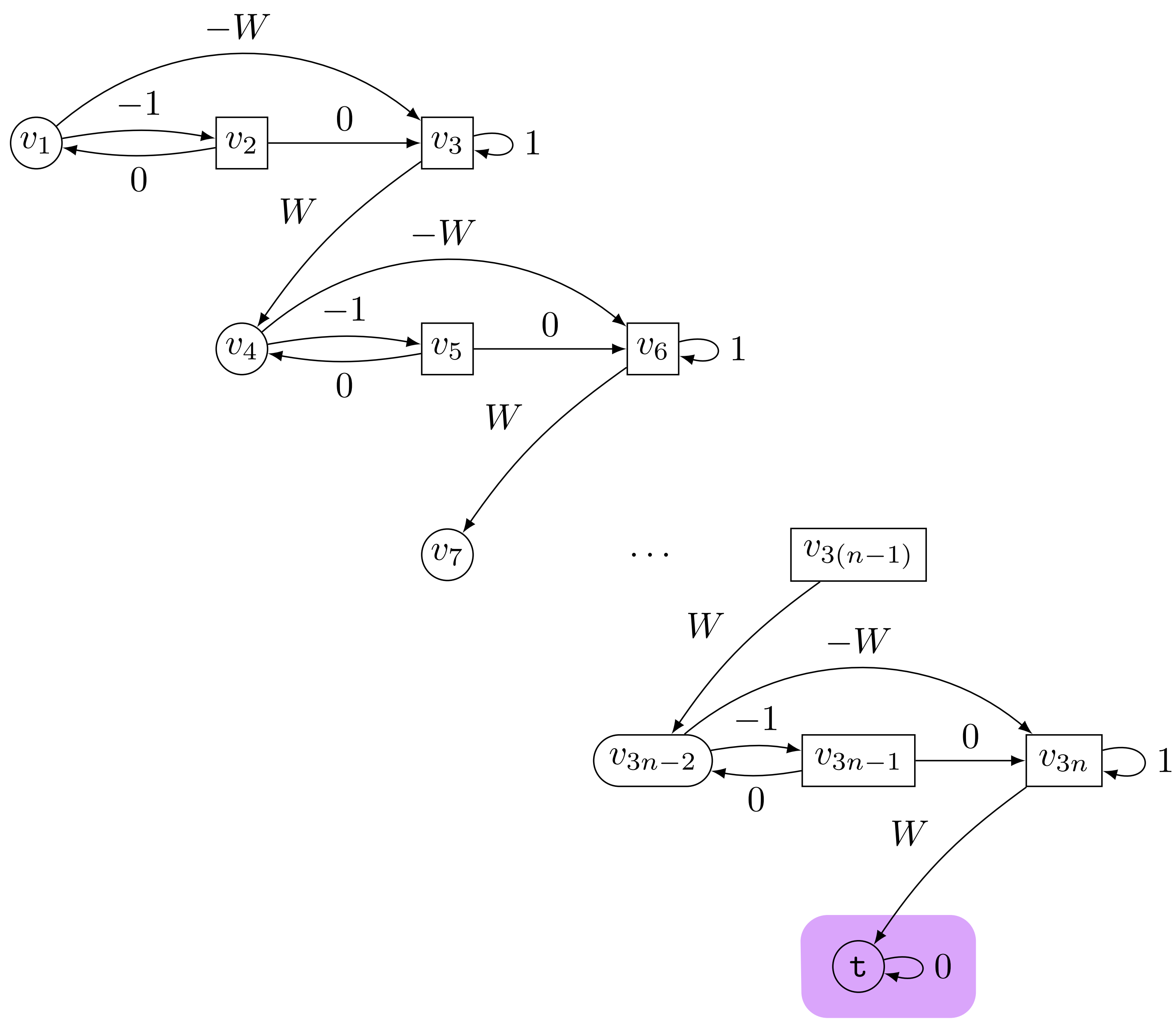

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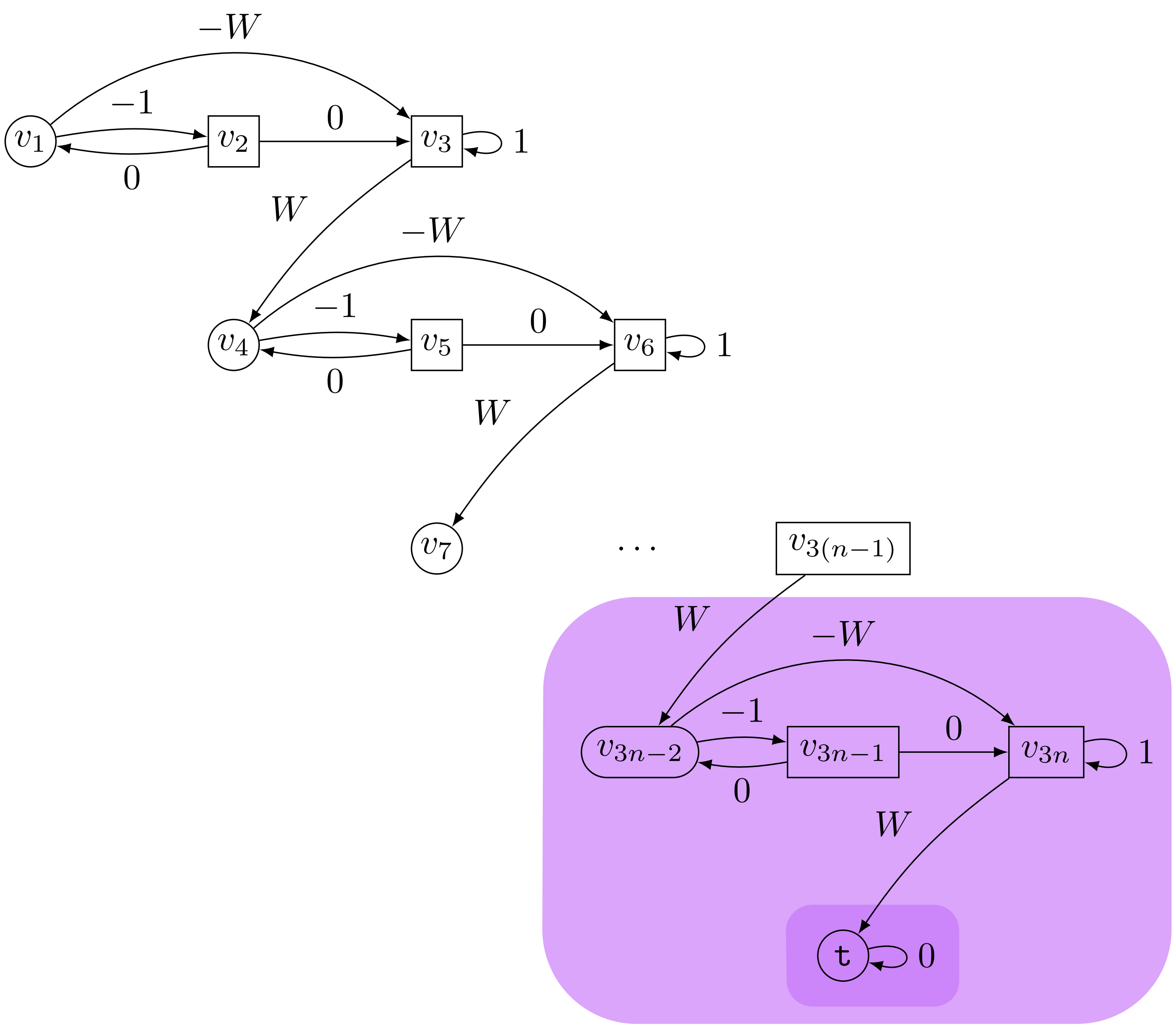

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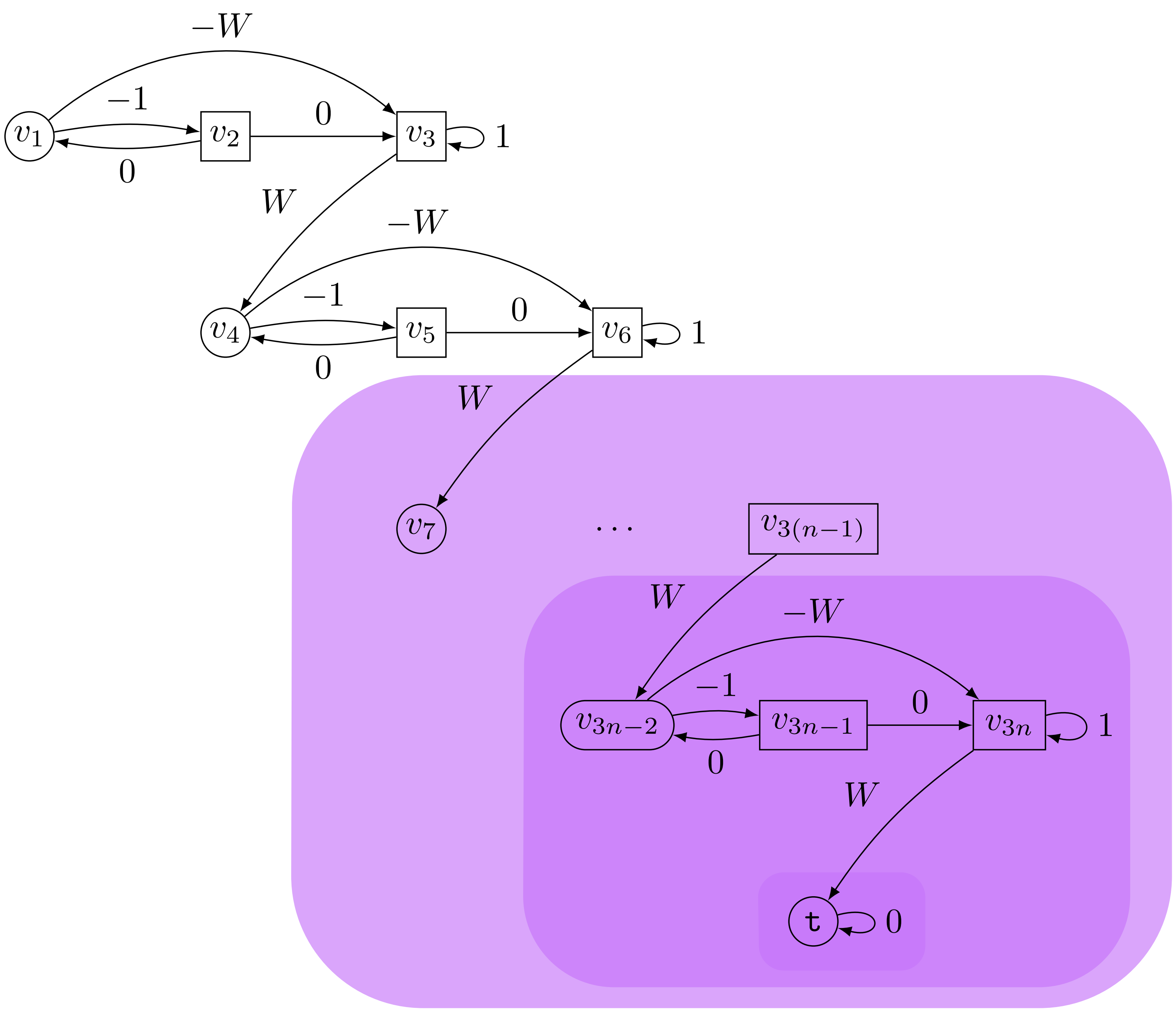

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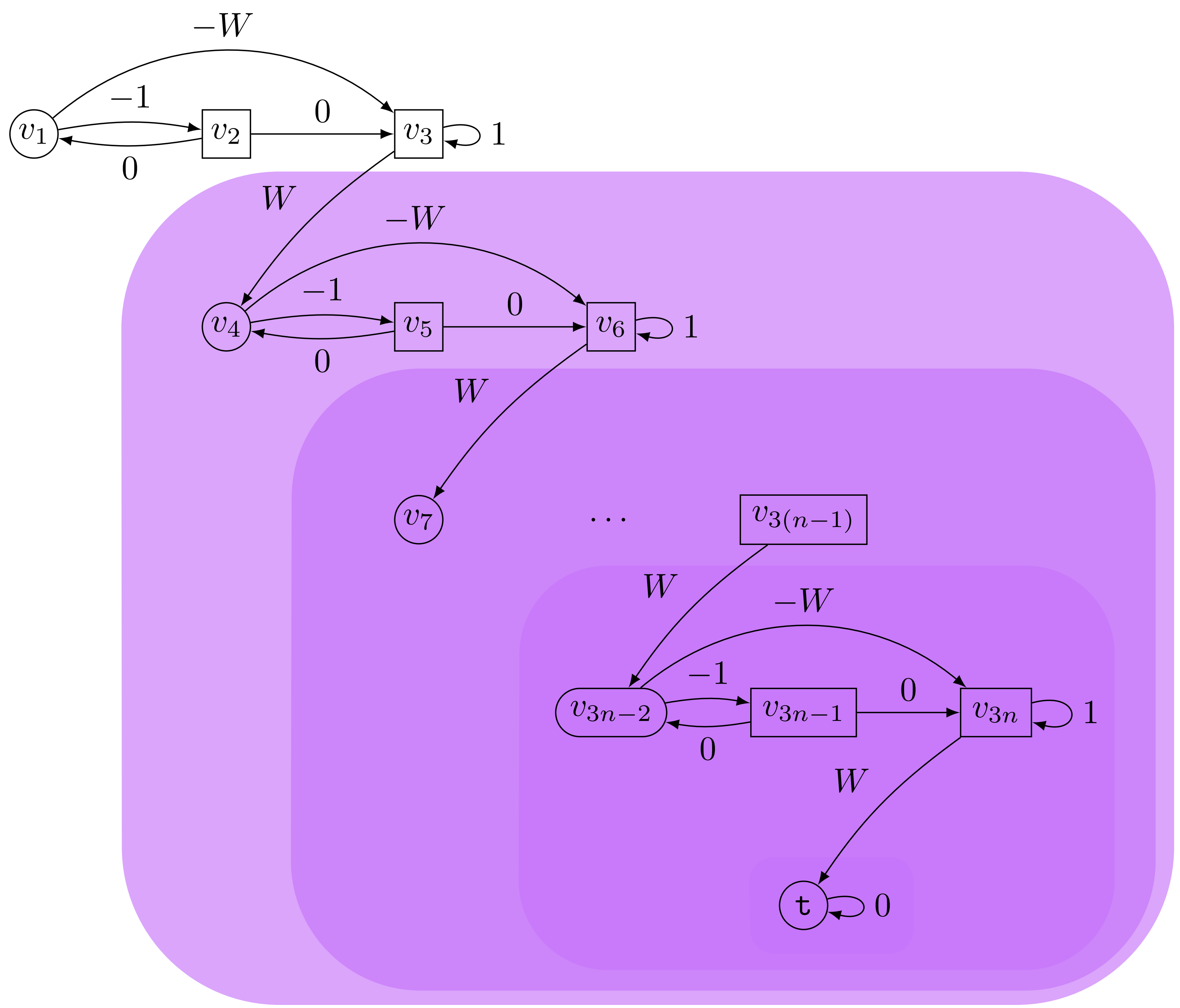

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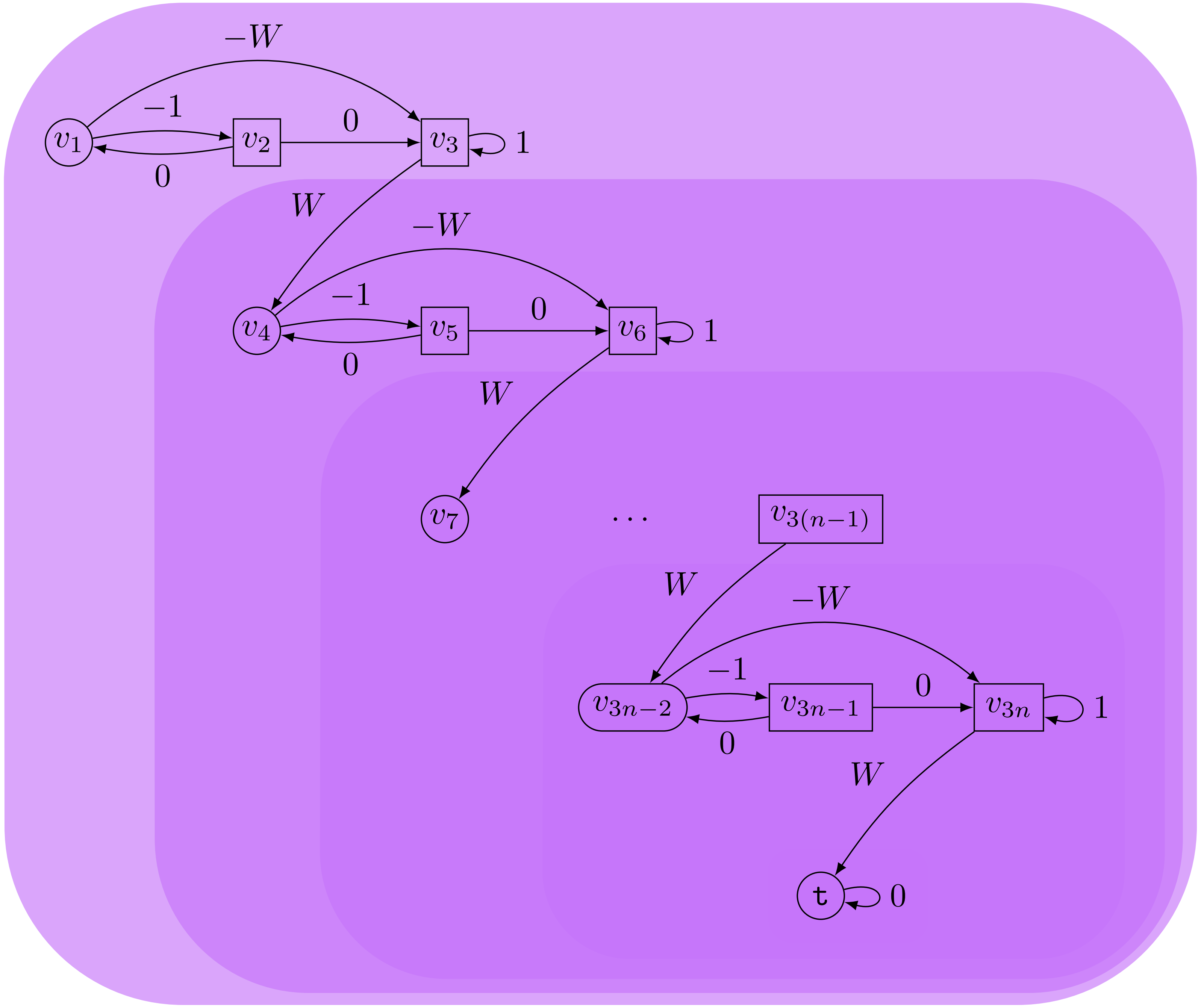

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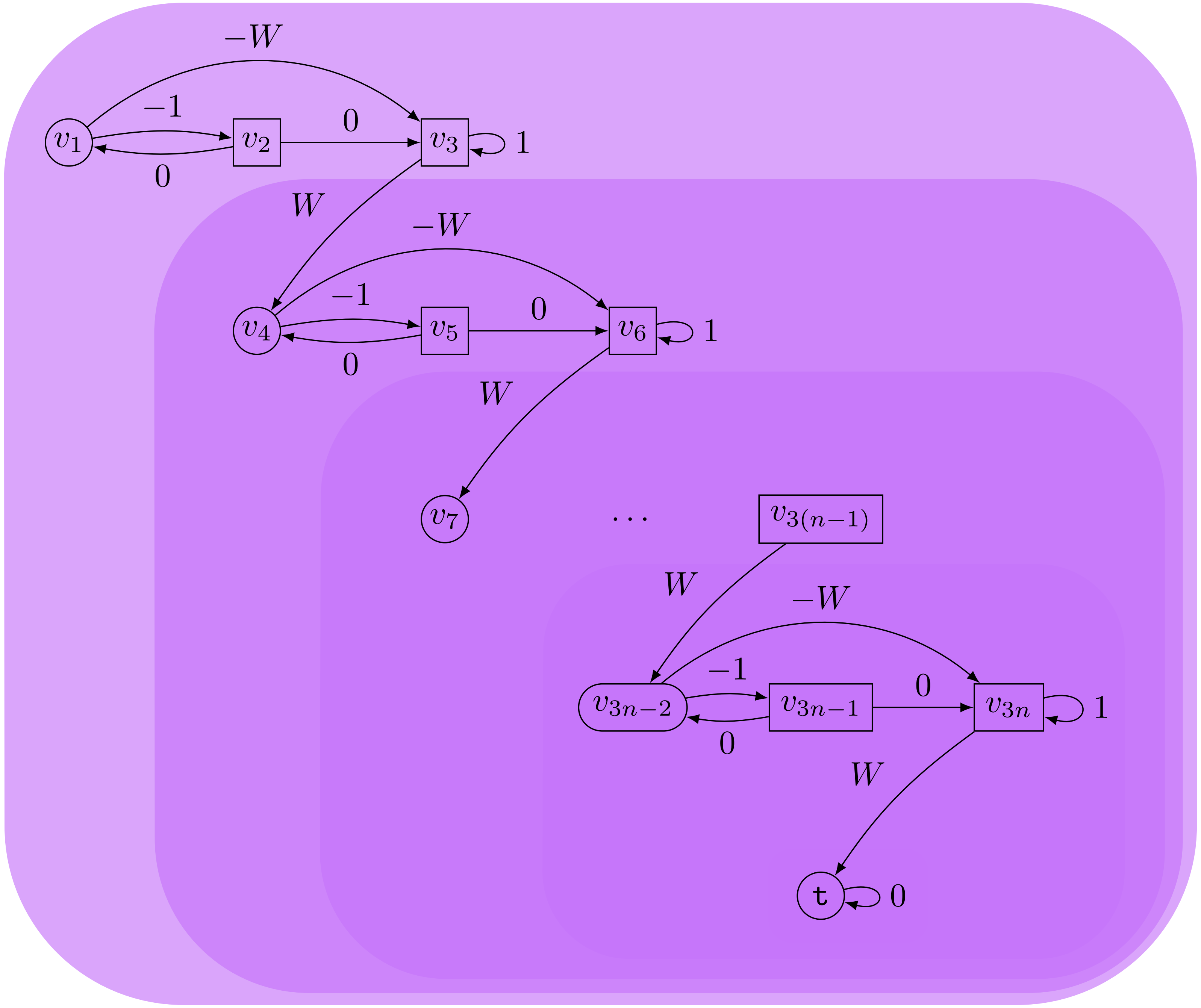
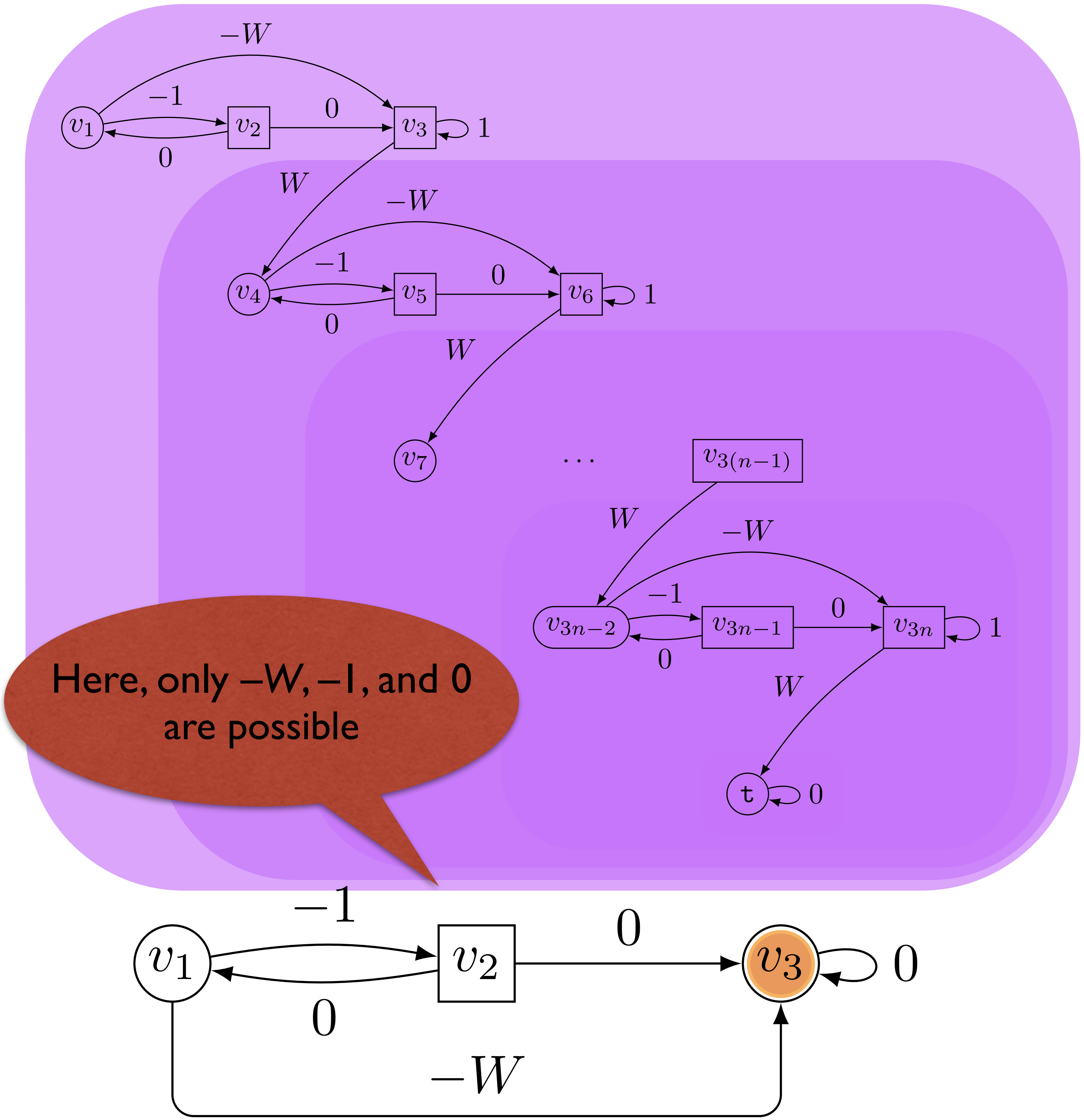


- In the outer loop, compute SCC by SCC
- For each inner loop, we solve an MCR game: optimal memoryless strategies, so value is weight of a simple path…

$+\infty$	$+\infty$	0
$+\infty$	0	0
-1	0	0
-1	-1	0
-2	-1	0
-2	-2	0
-3	-2	0
-3	-3	0
…	…	…
$-W$	$-W$	0
$-W$	$-W$	0

Heuristics: compute as little as possible!



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–2	–2	0
–3	–2	0
–3	–3	0
…	…	…
–W	–W	0
–W	–W	0

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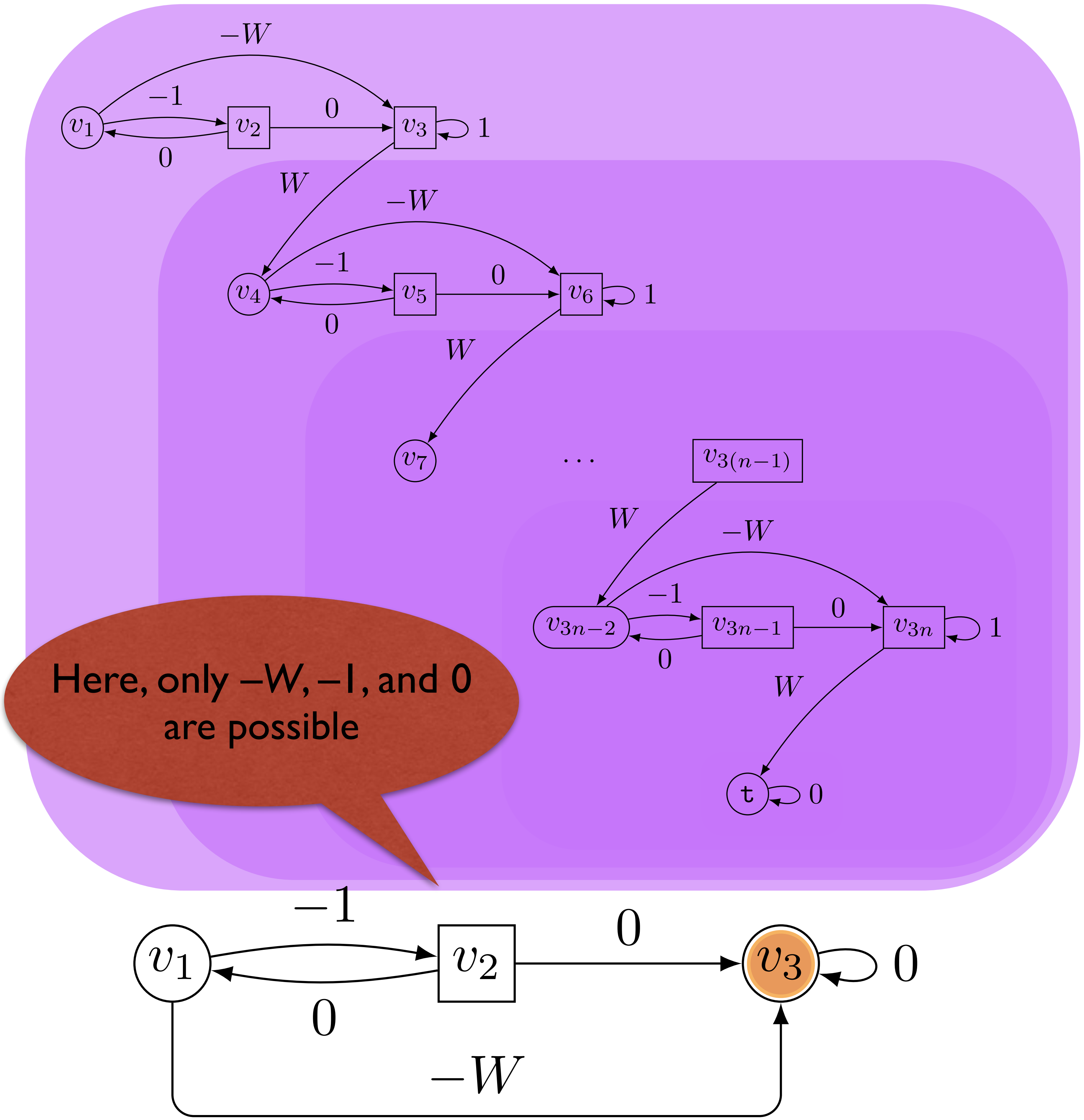

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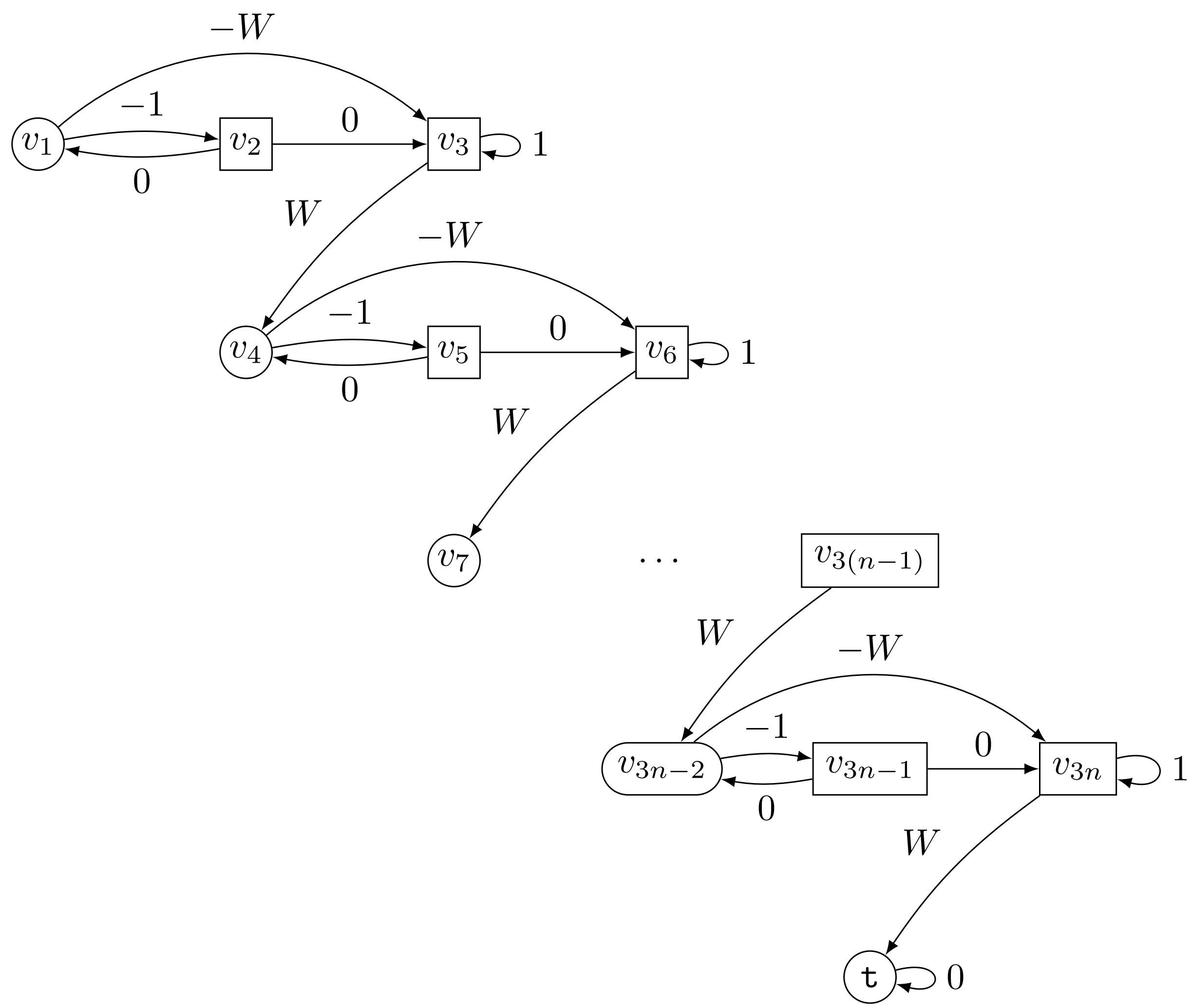
–1	0	0
–1	–1	0
–2	–1	0
–2	–2	0
–3	–2	0
–3	–3	0
…	…	…
–W	–W	0
–W	–W	0

skip!

Some total-payoff games in polynomial time



- Combination of both heuristics

- If all SCC uses at most *L* distincts weights (that can be arbitrarily large in absolute values), algorithm with heuristics runs in polynomial time.

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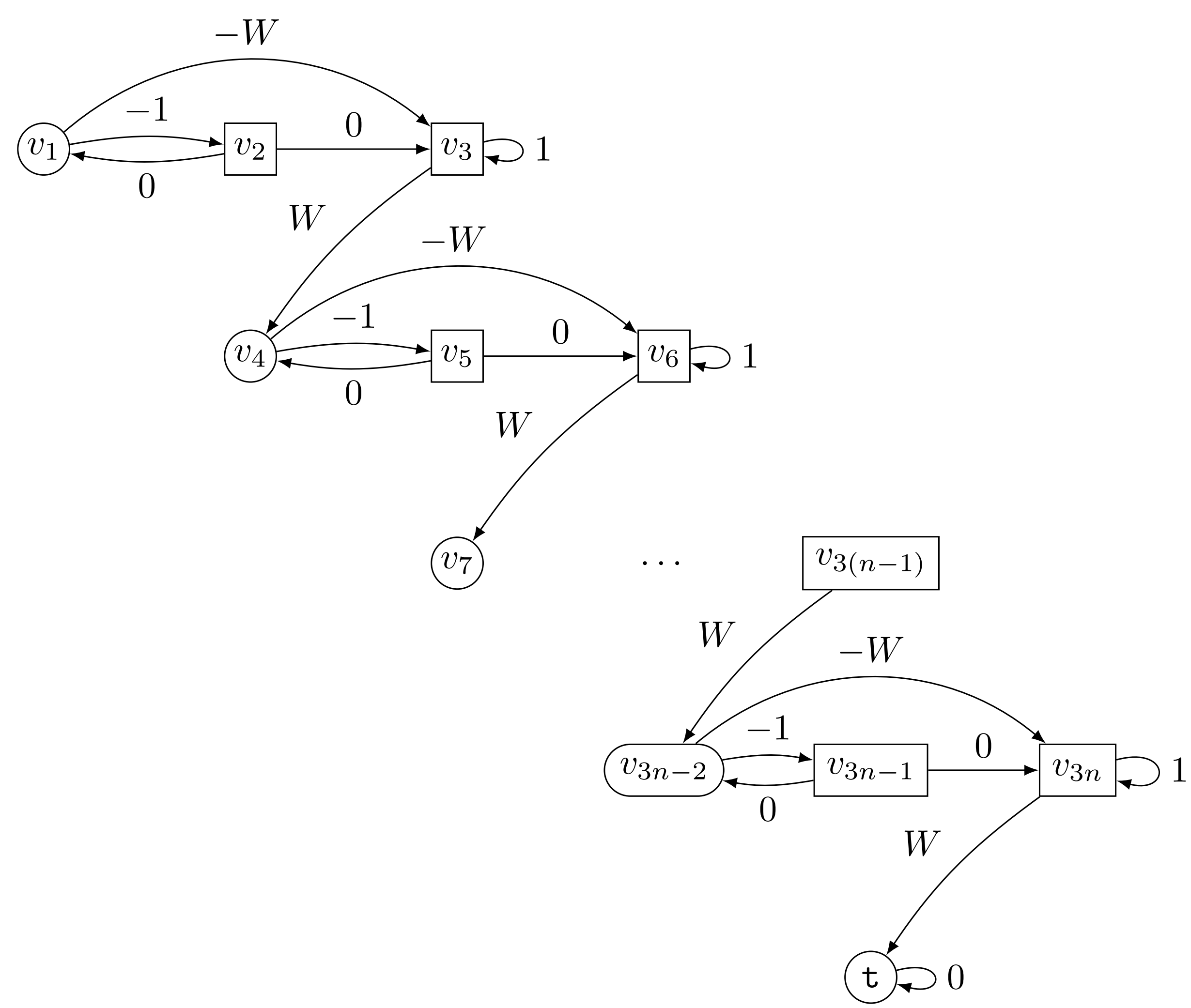

- Combination of both heuristics
- If all SCC uses at most *L* distincts weights (that can be arbitrarily large in absolute values), algorithm with heuristics runs in polynomial time.
- Implementation as an add-on to PRISM games available at *http://www.ulb.ac.be/di/verif/monmege/tool/TP- MCR/*

		without heuristics			with heuristics		
W	n	t	k_e	k_i	t	k_e	k_i
50	100	0.52s	151	12,603	0.01s	402	1,404
50	500	9.83s	551	53,003	0.42s	2,002	7,004
200	100	2.96s	301	80,103	0.02s	402	1,404
200	500	45.64s	701	240,503	0.47s	2,002	7,004
500	1,000	536s	1,501	1,251,003	2.37s	4,002	14,004

Conclusion and future works

- First pseudo-polynomial time algorithm to solve total-payoff games, by nested fixed point computation with value iteration
- By means of a reachability variant (MCR games), interesting on their own
- Large subclasses with polynomial time complexity
- Tool: add-on of PRISM games

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Thank you for your attention!