Efficient Reactive Synthesis of MITL Properties

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Benjamin Monmege
Aix-Marseille Université, LIF, CNRS, France

Thomas Brihaye, Morgane Estiévenart (UMONS)
Gilles Geeraerts, Hsi-Ming Ho (ULB), Nathalie Sznajder (UPMC, LIP6)

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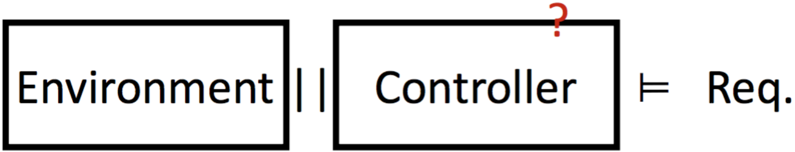
Controller synthesis problem

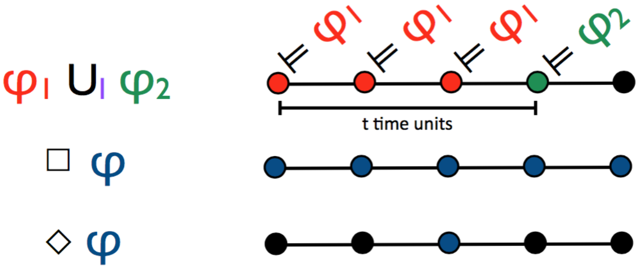


Metric Temporal Logic (MTL)

$$\varphi ::= \top \mid a \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \,\mathsf{U}_I\, \varphi$$

with $a \in \Sigma$, I interval of $\mathbb{R}^+$ with bounds in $\mathbb{N} \cup \{+\infty\}$

Model of a formula: (in)finite timed word $\sigma = (a_1, t_1)(a_2, t_2)\cdots$ with $a_i \in \Sigma$, (t_i) non-decreasing sequence of time stamps



Synthesis with plant: example of lift

$\Sigma = \Sigma_c \uplus \Sigma_e$

- controller's actions: closing of the doors, moving of the lift...
- environment's actions: pushing of the buttons, uncertainty on responses of the lift...

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(t, req) $(t', grant)$

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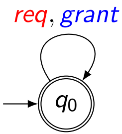
Reactive synthesis problem (RS): *find strategy of controller such that every play verifies the specification*

A toy example

Universal plant $\mathcal{P}$:



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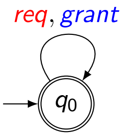

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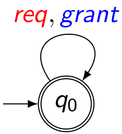

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- left hand side of the specification = fairness condition to give the time to the controller to answer...
- controller requires unbounded memory: unboundedly many events to remember as "*to be granted*" + infinite precision "$= 1$"

Implementable Reactive Synthesis (IRS)

Controller $=$ time-deterministic symbolic transition system $\mathcal{T}$

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- finite set of clocks
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if $\sigma \cdot (c, T)$ is possible,
and $\mathcal{T}$ may fire (t, grant) currently,
  then $\sigma \cdot (c, T) \cdot (\mathit{grant}, T + t)$ is possible if readable in the plant,
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Implementable reactive synthesis problem (IRS): *find a set of clocks X, a precision, and a td STS $\mathcal{T}$ of controller such that every* **possible** *play* **accepted by the plant** *verifies the specification*

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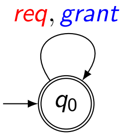

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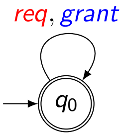

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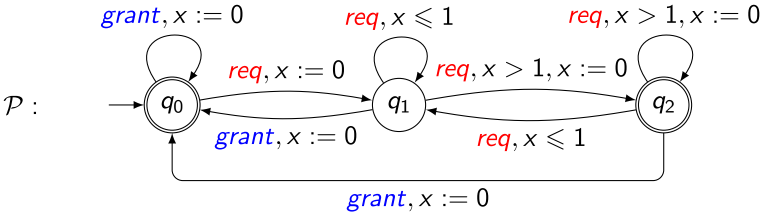
NOT CONTROLLABLE for IRS: requires infinite set of clocks, or infinite precision...

Another example



- every timed word fireable;
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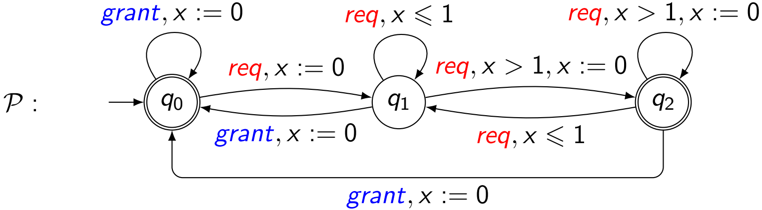

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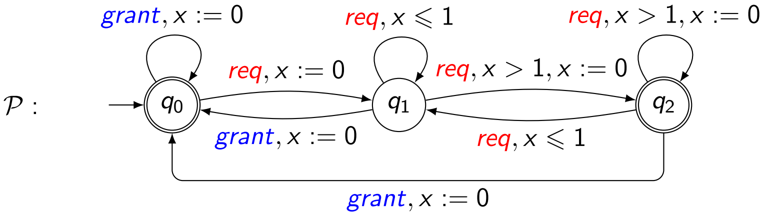

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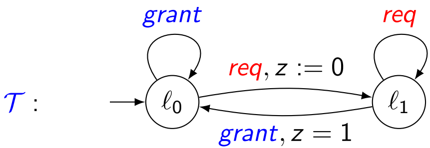
CONTROLLABLE for IRS: controller only keeps track of the first *req* in the sequence, and proposes to grant it 1 time unit later with a *grant*



Unfortunately...

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Theorem: [D'Souza and Madhusudan, 2002]

IRS is undecidable for specifications given as **timed regular languages**, or **complement of timed regular languages** (over infinite words, and also finite words).

- Reduction of the universality of non-deterministic timed automata

Recovering decidability...

Bounding *a priori* the resources: set of clocks X and precision (m, K) of the controller
Comparisons with maximal guards in $G^{\max}_{m,K}(X)$

$$g ::= \top \mid g \wedge g \mid x < \alpha/m \mid x \leqslant \alpha/m \mid x = \alpha/m \mid x \geqslant \alpha/m \mid x > \alpha/m$$

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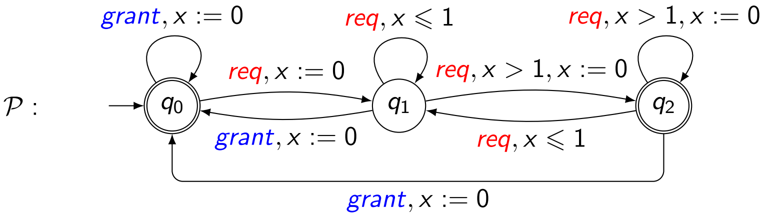
Bounded-resources reactive synthesis problem (BRessRS): *find a td STS $\mathcal{T}$ of controller with* **a given set of clocks X and precision (m, K)** *such that every possible play accepted by the plant verifies the specification*

Example



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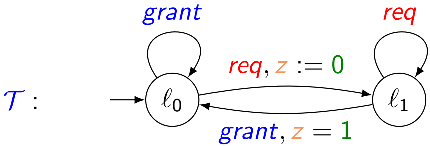
CONTROLLABLE for BRessRS: a single clock $X = \{z\}$, and precision $(m = 1, K = 1)$



Previous results

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BRessRS is decidable for specifications given as **complement of timed regular languages** (over infinite words, and also finite words), with a 2-EXPTIME complexity.

- Build the region automaton, determinise and complement it, and solve a timed game on the synchronous product with the plant and all possible behaviours of the controller

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Restrict the specification language: MITL

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Theorem: [Doyen, Geeraerts, Raskin, and Reichert, 2009]

RS is undecidable for specifications in MITL (over infinite words), even without plants.

- Reduction of a lossy 3-counter machine

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Our result

Practical algorithm for BRessRS of MITL over finite words, with 3-EXPTIME theoretical complexity.

- Via [D'Souza and Madhusudan, 2002], BRessRS of MITL is 3-EXPTIME
  - build non-deterministic timed automaton equivalent to the negation of the MITL formula...
  - requires the determinisation of the full region automaton!

Alternating automata combine:

From MTL to One-Clock Alternating Timed Automata [Ouaknine and Worrell, 2007]

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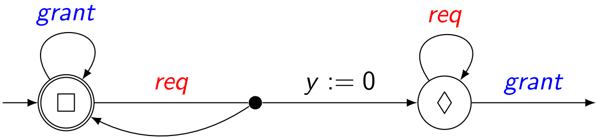
- conjunctive transitions = parallelism = the suffix must be accepted from **all** successor states

From MTL to OCATA

$$\varphi = \Box(\textit{req} \Rightarrow \Diamond_{[1,2]}\, \textit{grant})$$



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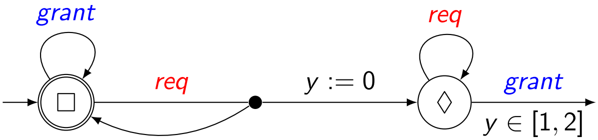

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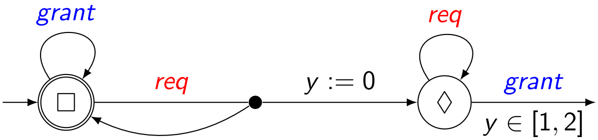

Execution on the timed word $(\mathit{req}, 0.5)(\mathit{req}, 0.6)(\mathit{req}, 1.2)(\mathit{grant}, 2.3)$:

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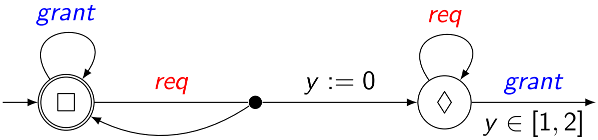

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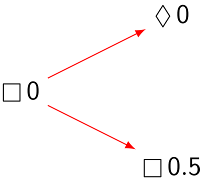

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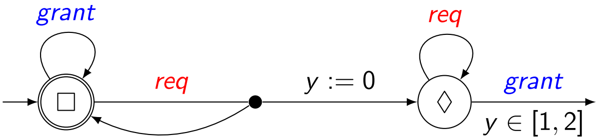

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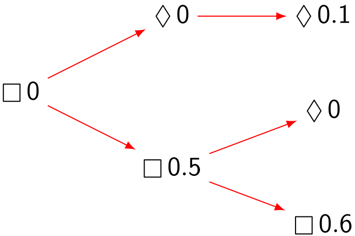

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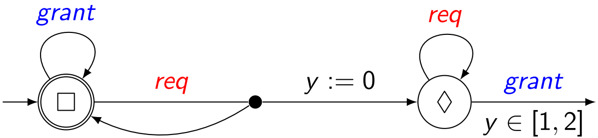

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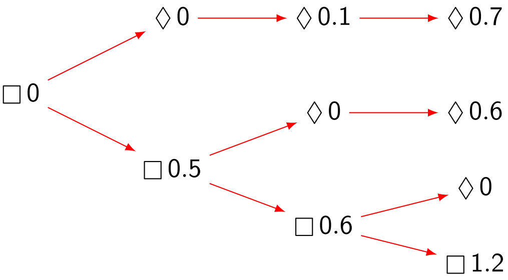

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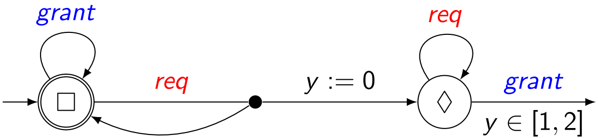

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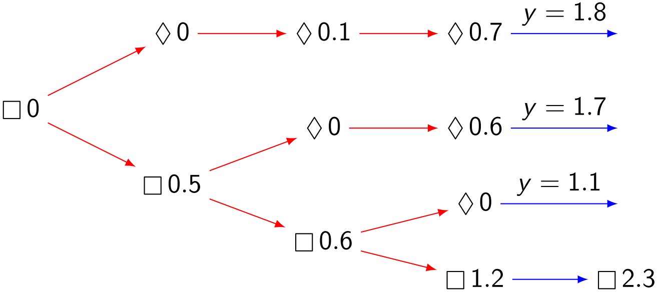

From MTL to OCATA

- Translation from MTL to OCATA is structural: the OCATA has **one state per subformula**
- **One clock** in the **syntax** of the automaton but... many clocks in the semantics!

Bounded-Ressources Reactive Synthesis for MTL

- Plant: $\mathcal{P}$, Specification: φ in MTL, Ressources: (X, m, K)
- **Convert** the MTL formula $\neg\varphi$ into an **OCATA** $\mathcal{A}$
- **Cast** the control problem into a **timed game** played on a tree
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  - each $\lambda_i \subseteq 2^{(X_{\mathcal{P}} \cup X \cup Q_{\mathcal{A}}) \times \mathrm{REG}_{m,K}}$: region associated to all clocks

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- Action (a, g, R)
  - a: letter of $\Sigma_c \cup \Sigma_e$
  - g: guard over clocks of X and $X_{\mathcal{P}}$
  - R: resets of clocks of X

$$\begin{tikzcd} & \boxed{(q, \{H_1, H_2, \ldots, H_n\})} \arrow[dl, "{a, g, R}"] \\ \boxed{(q', \{H'_1, H'_2, \ldots, H'_{n'}\})} & \end{tikzcd}$$

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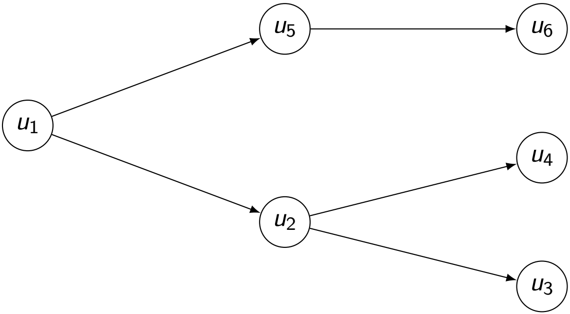
- If tree finite and winning strategy: we have a (finite) controller $\mathcal{T}$

Make the tree finite

For MTL specifications [Bouyer, Bozzelli, and Chevalier, 2006]: stop the computation with a well-quasi order $\sqsubseteq$ on the labels of the nodes



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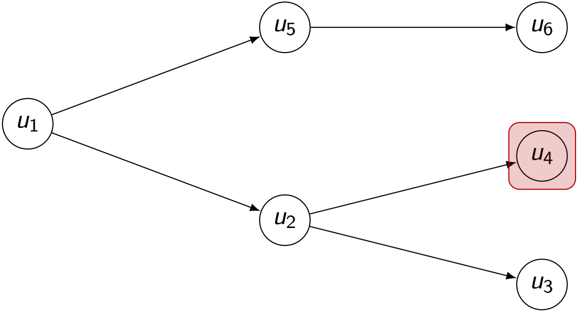

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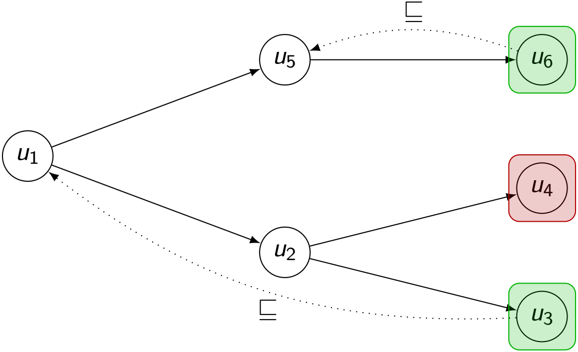

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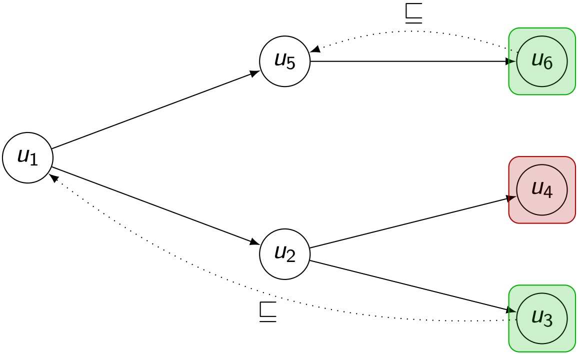

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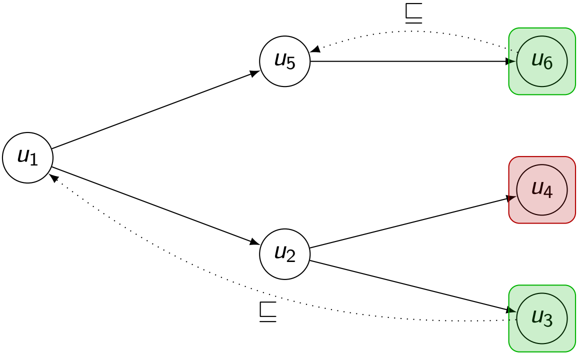

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- Complexity: **non-primitive recursive** due to well-quasi orderings

For MITL: interval semantics for OCATA

New semantics for OCATA [Brihaye, Estiévenart, and Geeraerts, 2013]:

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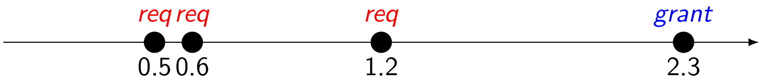

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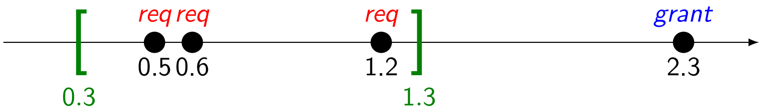

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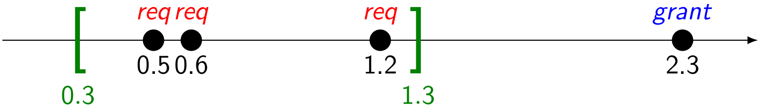
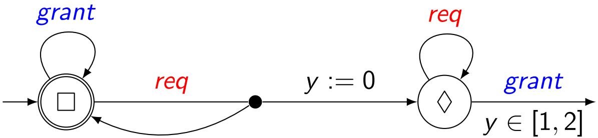
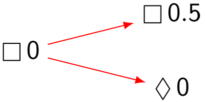
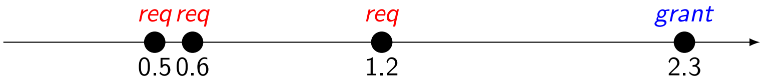


To check that this timed word satisfies φ, we **do not need to remember** the exact timestamp of each *req*

Example run with the interval semantics







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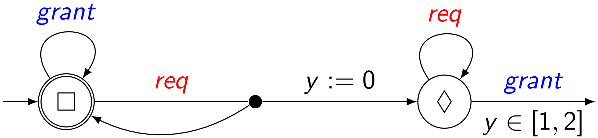

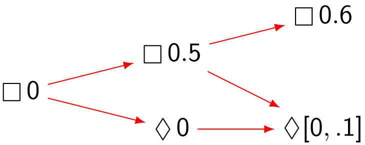

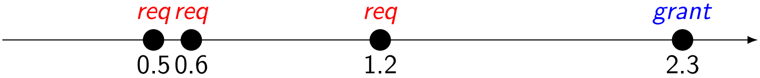

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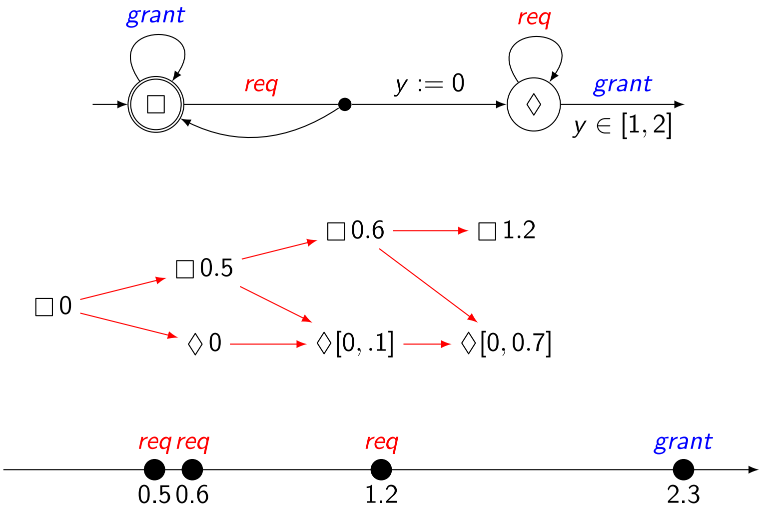

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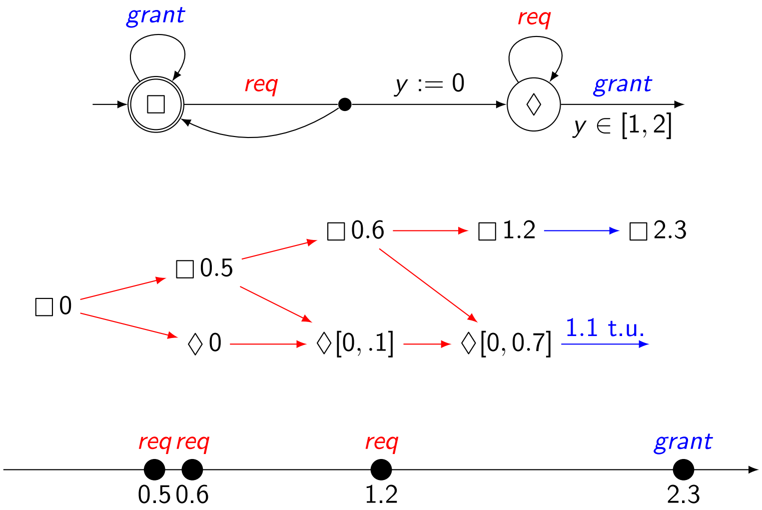

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Tree construction of [Bouyer, Bozzelli, and Chevalier, 2006]

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- **Zone**-based implementation doable: future work!
- **Heuristics**

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- Stop branches earlier using well-quasi-order $\sqsubseteq$ of [Bouyer, Bozzelli, and Chevalier, 2006]:
  - still valid, even though we do not use it for termination

What else?

Bounded-ress. reactive synthesis

- Decidable in 3-EXPTIME for complement of timed automata
- Undecidable for nd timed automata
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Trying to push further the **undecidability boundaries**?

Undecidability of IRS for MTL [Bouyer, Bozzelli, and Chevalier, 2006]

Reduction of the halting problem of a **deterministic channel machine** with

- single halting state s_{halt} with no outgoing transition
- no cycle with only **write actions** m!
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$\Rightarrow$: if machine does not halt, a controller would need to cheat or to play an infinite computation that requires infinite number of clocks (because of the unboundedness of the channel)

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Theorem:

Implementable Reactive Synthesis for MITL specifications over finite words is undecidable.

Results for MITL

	RS	IRS	BRessRS
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Infinite	Undecidable [Doyen, Geeraerts, Raskin, and Reichert, 2009]	Undecidable	3-EXPTIME [D'Souza and Madhusudan, 2002]

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Bounded-precision reactive synthesis problem (BPrecRS): *find a* **finite set of clocks** X, *and a td STS* $\mathcal{T}$ *of controller with* X *as clocks, and* **a given precision** (m, K) *such that every possible play accepted by the plant verifies the specification*

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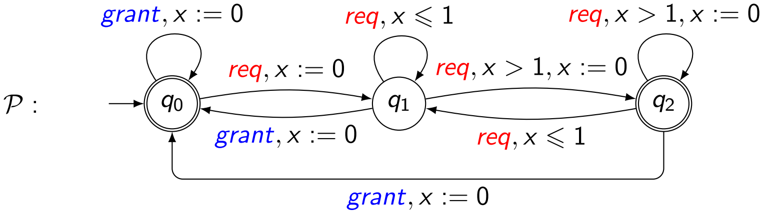
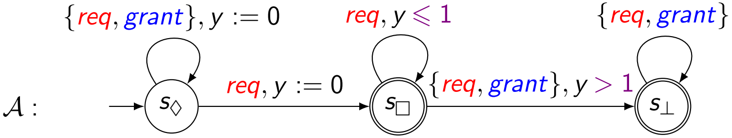
- But also **undecidable** via the previous proof!!

Running example

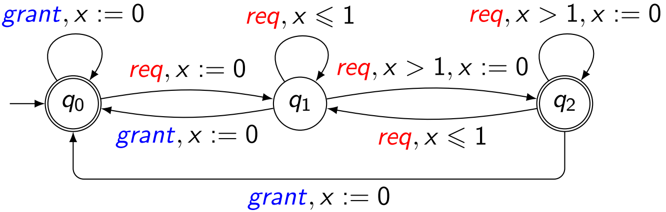


Specification: $\Box(req \Rightarrow \Diamond_{\leqslant 1}\, grant)$ equivalent to complement of



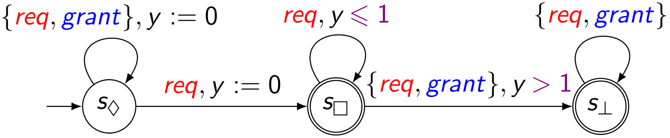
Running example

$\mathcal{P}$:



Specification: $\Box(req \Rightarrow \Diamond_{\leqslant 1}\, grant)$ equivalent to complement of

$\mathcal{A}$:



Question: find a controller $\mathcal{T}$ with precision $(m = 1, K = 1)$ such that "$(\mathcal{P} \| \mathcal{T}) \cap \mathcal{A} = \emptyset$"
Warning: set of clocks X for the controller not fixed a priori

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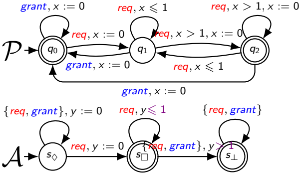
- Cut some useless branches with an order $\widetilde{\sqsubseteq}$ (that is not a wqo)

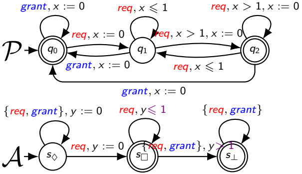
Running example: finite tree

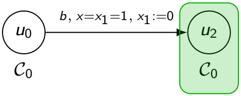


$\mathcal{C}_0$

$$\mathcal{C}_0 = \left(q_0, \left\{\left(s_\Diamond, \left\{\langle x_1, \{0\}\rangle, \langle x, \{0\}\rangle, \langle y, \{0\}\rangle\right\}\right)\right\}\right)$$

Running example: finite tree





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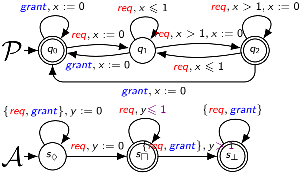
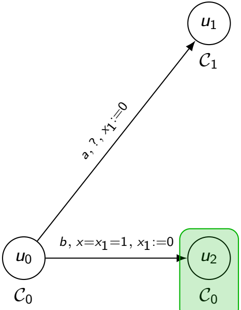
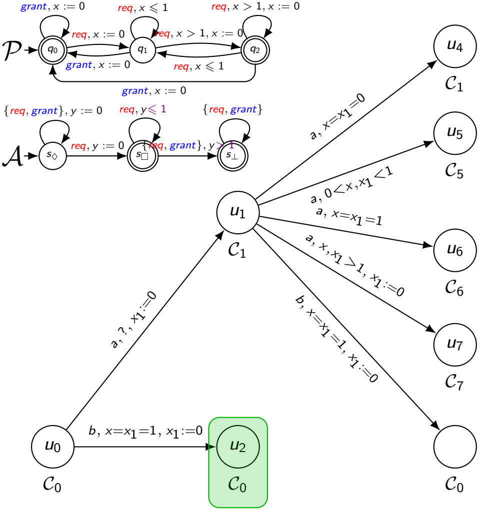




$$\mathcal{C}_0 = \big(q_0, \big\{\big(s_\Diamond, \{\langle x_1, \{0\}\rangle, \langle x, \{0\}\rangle, \langle y, \{0\}\rangle\}\big)\big\}\big)$$

$$\mathcal{C}_1 = \big(q_1, \big\{\big(s_\Diamond, \{\langle x_1, \{0\}\rangle, \langle x, \{0\}\rangle, \langle y, \{0\}\rangle\}\big), \big(s_\Box, \{\langle x_1, \{0\}\rangle, \langle x, \{0\}\rangle, \langle y, \{0\}\rangle\}\big)\big\}\big)$$

Running example: finite tree



$$\mathcal{C}_0 = \left(q_0, \left\{\left(s_\diamondsuit, \left\{\langle x_1, \{0\}\rangle, \langle x, \{0\}\rangle, \langle y, \{0\}\rangle\right\}\right)\right\}\right)$$

$$\mathcal{C}_1 = \left(q_1, \left\{\left(s_\diamondsuit, \left\{\langle x_1, \{0\}\rangle, \langle x, \{0\}\rangle, \langle y, \{0\}\rangle\right\}\right), \left(s_\square, \left\{\langle x_1, \{0\}\rangle, \langle x, \{0\}\rangle, \langle y, \{0\}\rangle\right\}\right)\right\}\right)$$

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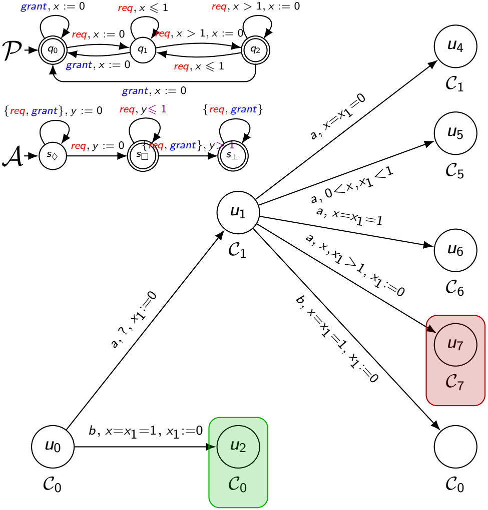

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$$\begin{aligned}\mathcal{C}_1 = (q_1, \{&(s_\Diamond, \{\langle x_1, \{0\}\rangle, \langle x, \{0\}\rangle, \langle y, \{0\}\rangle\}),\\ &(s_\Box, \{\langle x_1, \{0\}\rangle, \langle x, \{0\}\rangle, \langle y, \{0\}\rangle\})\})\end{aligned}$$

$$\begin{aligned}\mathcal{C}_7 = (q_2, \{&(s_\Diamond, \{\langle x_1, \{0\}\rangle, \langle x, \{0\}\rangle, \langle y, \{0\}\rangle\}),\\ &(s_\Box, \{\langle x_1, \{0\}\rangle, \langle x, \{0\}\rangle, \langle y, \{0\}\rangle\})\ldots\})\end{aligned}$$

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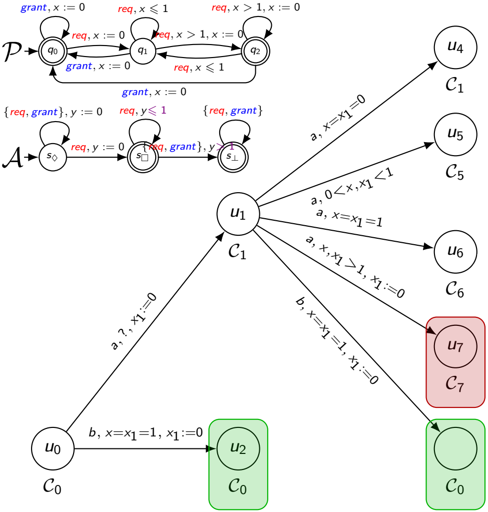

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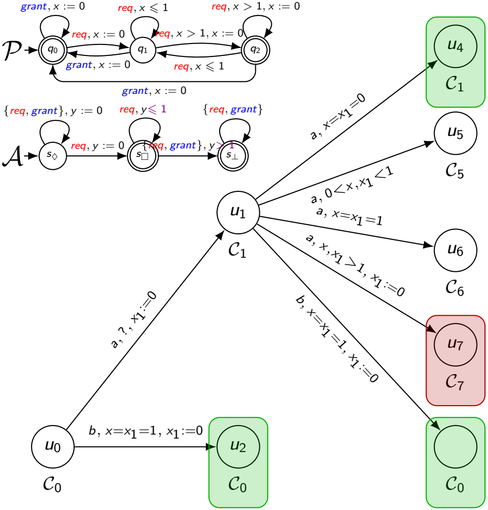

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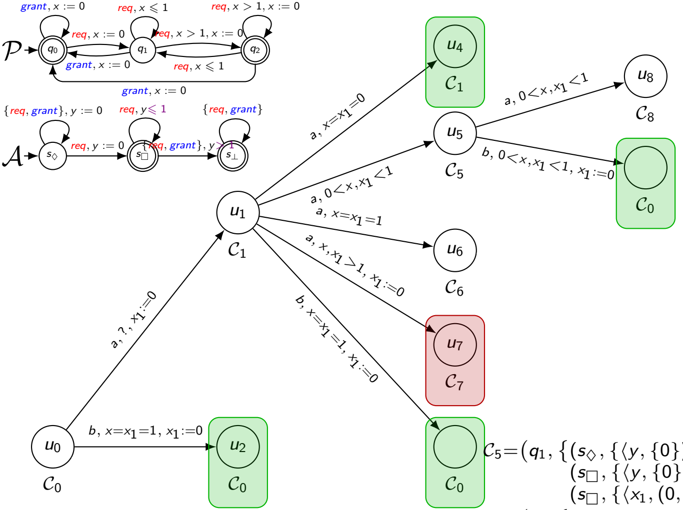

$$\mathcal{C}_5 = \Big(q_1, \big\{(s_\Diamond, \{\langle y, \{0\}\rangle\}\{\langle x_1, (0,1)\rangle, \langle x, (0,1)\rangle\}),\\ (s_\Box, \{\langle y, \{0\}\rangle\}\{\langle x_1, (0,1)\rangle, \langle x, (0,1)\rangle\}),\\ (s_\Box, \{\langle x_1, (0,1)\rangle, \langle x, (0,1)\rangle, \langle y, (0,1)\rangle\})\big\}\Big)$$

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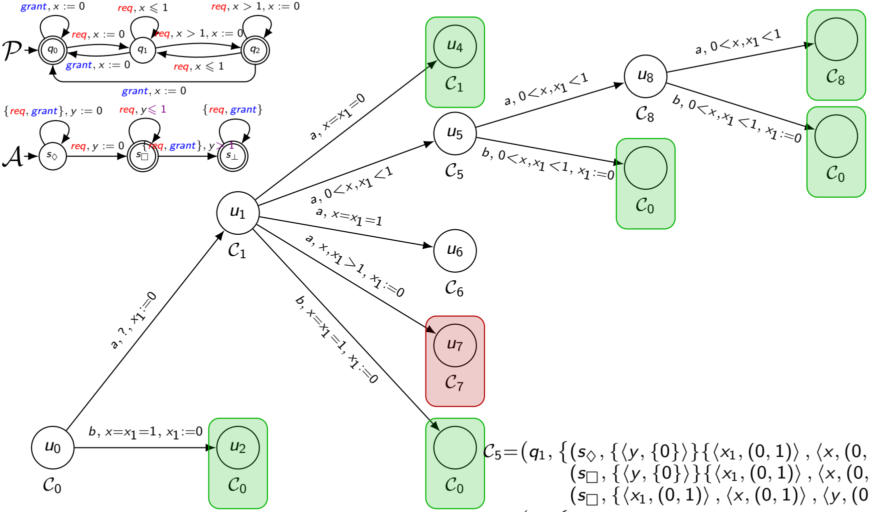

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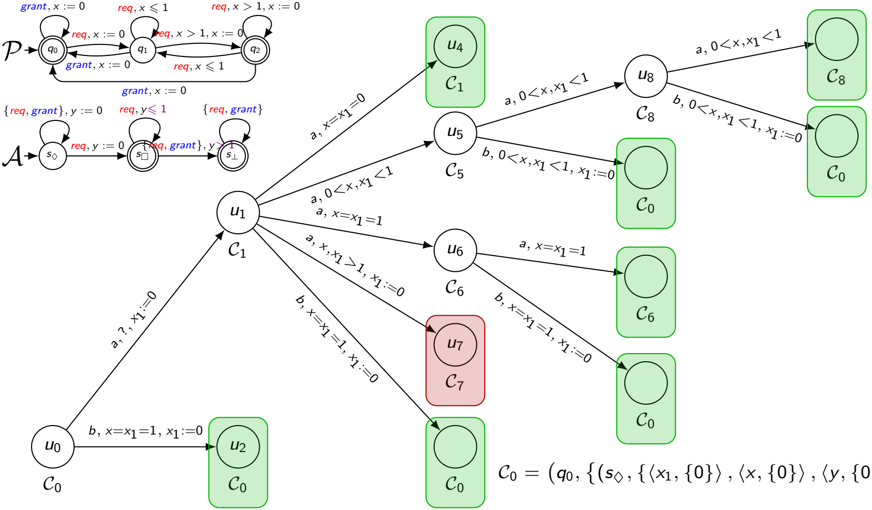

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$$\begin{aligned}\mathcal{C}_6 = (q_1, \{&(s_\Diamond, \{\langle y, \{0\}\rangle, \langle x_1, \{1\}\rangle, \langle x, \{1\}\rangle\}),\\ &(s_\Box, \{\langle y, \{0\}\rangle, \langle x_1, \{1\}\rangle, \langle x, \{1\}\rangle\}),\\ &(s_\Box, \{\langle x_1, \{1\}\rangle, \langle x, \{1\}\rangle, \langle y, \{1\}\rangle\})\})\end{aligned}$$

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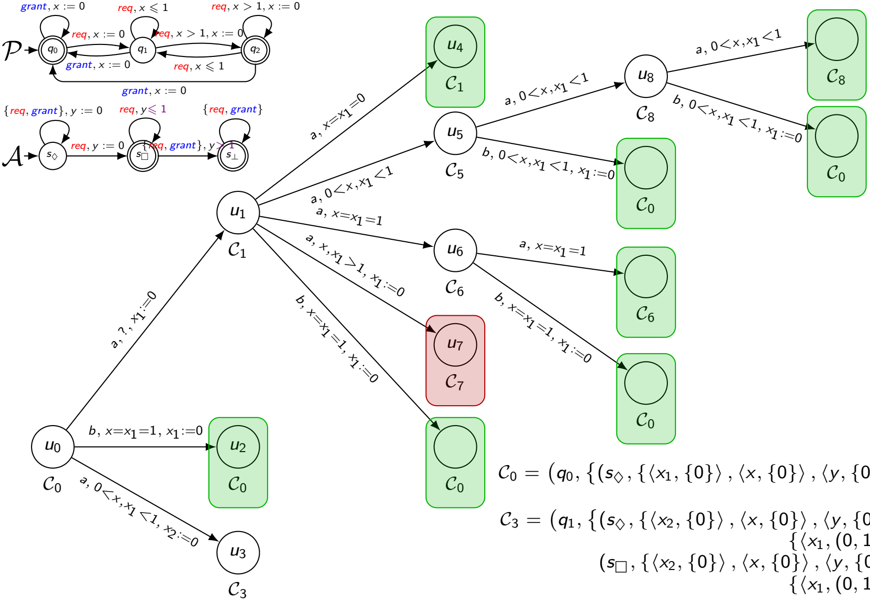

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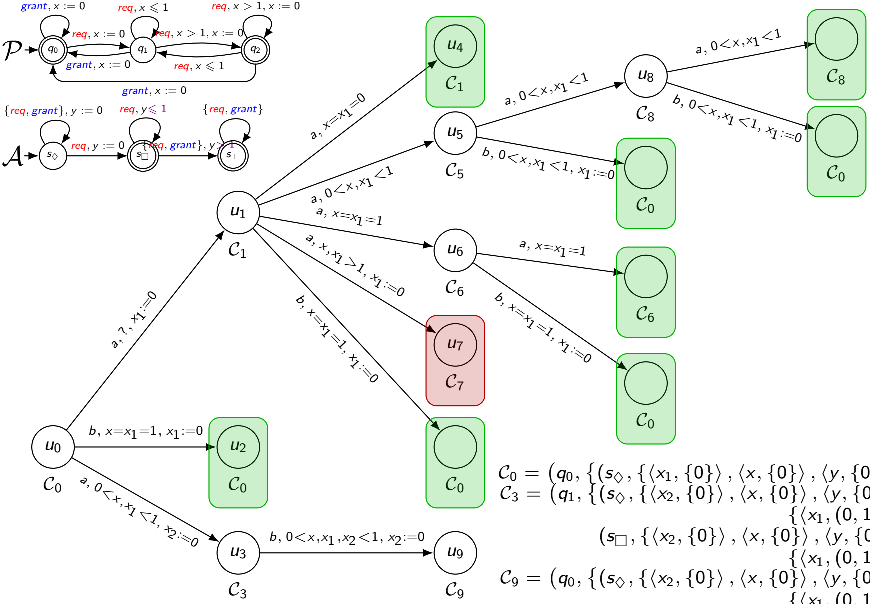

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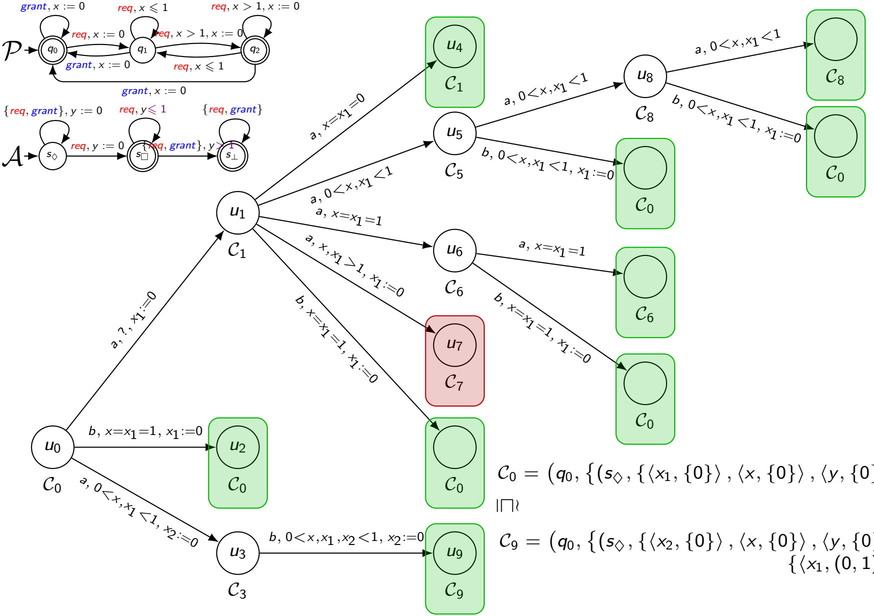

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$$\sqsubseteq$$

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Reactive synthesis with plant for MITL specifications

	RS	IRS	BPrecRS	BRessRS
Finite	...Ackerman-hard...	Undecidable	Undecidable + semi-algo	**on-the-fly** 3-EXPTIME
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Thank you for your attention

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