Weighted Automata: Theory and Applications

WATA 2020/2021

20-23 April 2021

https://wata2020.lis-lab.fr

The workshop covers all aspects of weighted automata, ranging from the theory of weighted automata and quantitative logics to applications for real-time systems and natural language processing. The aim is to present tutorials and invited lectures by outstanding scientists in this area. Because of the Covid-19, WATA has been shifted from April 2020 to April 2021, and happens entirely online. There are more than 80 registered participants, coming from 16 countries around the world.

Organisers: Manfred Droste (Universität Leipzig), Paul Gastin (ENS Paris-Saclay, LMF), Pierre Guillon (Aix-Marseille Université, I2M), Benjamin Monmege (Aix-Marseille Université, LIS), Heiko Vogler (TU Dresden)

Sponsors: LIS, QuantLA, LMF, I2M, FRUMAM

Programme

Tuesday April 20

9:00-11:15 Tutorial - Andreas Maletti - Neural Networks and Weighted Automata

11:45-12:05 Uli Fahrenberg - Posets with Interfaces

12:05-12:25 Lena Schiffer - Combinatory Categorial Grammars as Acceptors of Weighted Tree Languages

14:00-14:45 Invited talk - Mikołaj Bojańczyk - Weighted languages, monads, and distributive laws

15:00-15:20 Frank Drewes - Can we preserve the polynomial parsability of restricted graph grammars when adding contextuality?

15:20-15:40 Stefan Stanimirović - Approximate simulation and bisimulation relations for fuzzy automata

15:40-... Discussion

Wednesday April 21

9:00-9:45 Invited talk - Guillermo Perez - Parameter Synthesis for One-Counter Automata

10:00-10:45 Invited talk - Laure Daviaud - Weighted automata and Cost-register Automata over the max-plus semiring

11:15-11:35 C. Aiswarya - Weighted Tiling Automata on Graphs: Evaluation Complexity

11:35-11:55 Gayathri Lakshmi M - Weighted Hexagonal Picture Automata

13:45-14:30 Invited talk - Karin Quaas - On the universality problem for unambiguous register automata

14:45-15:05 Kevin Stier - Disambiguation of Weighted Tree Automata

15:05-15:25 David Purser - The Big-O Problem for Weighted Automata

15:25-15:45 Frederic Dörband - Kleene and Büchi for Weighted Forest Languages over M-Monoids

15:45-... Discussion

Thursday April 22

9:00-11:15 **Tutorial - Anca Muscholl** - A View on String Transducers

11:45-12:05 Thomas Ruprecht - Statistical machine translation using streaming string transducers

12:05-12:25 Sven Dziadek - Greibach Normal Form and Simple Automata for Weighted ω-Context-Free Languages

14:00-14:45 **Invited talk - George Rahonis** - Logical description of (weighted) parametric component-based systems with (w)FOEIL

15:00-15:20 Aleksandar Stamenkovic - State reduction of weighted automata using certain equivalences

15:20-15:40 Gustav Grabolle - Weighted linear dynamic logic with two-sorted semantics

15:40-... Discussion

Friday April 23

9:00-9:45 **Invited talk - Glenn Merlet** - Separating words with weighted automata

10:00-10:45 **Invited talk - Jean-Eric Pin** - Polynomial functions and sequential products

11:15-11:35 Dávid Kószó - Decidability of crisp-determinization for weighted finite automata over past-finite monotonic strong bimonoids

11:35-11:55 Richard Mörbitz - Weighted Parsing for Grammar-Based Language Models over Multioperator Monoids

11:55-12:15 Miroslav Ćirić - Weakly linear systems for matrices over the max-plus quantale and their applications

13:45-14:30 **Invited talk - Filip Mazowiecki** - Copyless cost-register automata

14:45-15:05 Erik Paul - Finite Sequentiality of Max-Plus Tree Automata

15:05-15:25 Ivana Micic - Blockmodeling of fuzzy transition systems using approximate regular relations

15:25-15:45 Jelena Ignjatović - Weighted context-free grammars over a complete strong bimonoid

15:45-... Discussion

Neural Networks and Weighted Automata

Tutorial on Tuesday April 20, 9:00

Andreas Maletti

We carefully introduce (recurrent) neural networks from the perspective of weighted automata and present several variants that are used in practice. In addition, we investigate the expressive power and the decidability and complexity of several standard operations and compare those results to the corresponding results for weighted automata over the field of rationals or reals. Finally, we introduce a standard procedure for neural network training, which differs significantly from the standard learning techniques for weighted automata over fields

Weighted languages, monads, and distributive laws

Invited talk on Tuesday April 20, 14:00

Mikołaj Bojańczyk

I will discuss a perspective on weighted languages that is based on monads and distributive laws. Using this perspective, one can automatically get definitions of weighted languages, and polynomial time algorithms for their equivalence, in a wide variety of settings, such as words, trees, or graphs of bounded treewidth.

Parameter Synthesis for One-Counter Automata

Invited talk on Wednesday April 21, 9:00

Guillermo Perez

One-counter automata are obtained by extending classical finite-state automata with a counter whose value can range over non-negative integers and be tested for zero. The updates and tests applicable to the counter can further be made parametric by introducing a set of integer-valued variables called parameters. During this talk we will focus on parameter synthesis problems for such automata. That is, we ask whether there exists a valuation of the parameters such that all infinite runs of the automaton satisfy some omega-regular property. The problem has been shown to be encodable in a restricted one-alternation fragment of Presburger arithmetic with divisibility. We will see that said fragment, called EARPAD, is unfortunately undecidable. Fortunately, there is an alternative encoding of the problem into a decidable restriction of EARPAD. Finally, we will study an alternative solution going via alternating two-way automata. The latter gives us a polynomial-space algorithm for the special case of the problem where parameters can only be used in tests, and not updates, of the counter.

Weighted automata and Cost-register Automata over the max-plus semiring

Invited talk on Wednesday April 21, 10:00

Laure Daviaud

In this talk, I will review some concepts related to max-plus automata and cost-register automata, and in particular, look at the hierarchy of (some) classes that are defined using ambiguity, restrictions on register updates... and how these notions relate.

On the universality problem for unambiguous register automata

Invited talk on Wednesday April 21, 13:45

Karin Quaas

The talk gives an overview over recent results concerning the universality problem for unambiguous register automata.

A View on String Transducers

Invited talk on Thursday April 22, 9:00

Anca Muscholl

This tutorial will review recent results on string transducers, ranging from expressiveness questions to algorithmic ones. In particular, we will present two-way and streaming string transducers, with and without origin semantics, and we will discuss the equivalence problem.

Logical description of (weighted) parametric component-based systems with (w)FOEIL

Invited talk on Thursday April 22, 14:00

George Rahonis

One of the key aspects in component-based design is specifying the software architecture that characterizes the topology and the permissible interactions of the components of a system.

We present an extended propositional interaction logic and investigate its first-order level (FOEIL) which serves as a formal language for architectures applied on parametric component-based systems, i.e., systems that consist of an unknown number of instances of each component. Our logic achieves to encode the execution order of interactions, which is a main feature in several important architectures, as well as to model recursive interactions. We state the decidability of equivalence, satisfiability, and validity of FOEIL formulas.

Then, we introduce a weighted FOEIL (wFOEIL), over a commutative semiring, and describe parametric component-based systems and their architectures with quantitative features. We show decidability results for wFOEIL formulas over a (subsemiring of a) skew field.

This is a joint work with Maria Pittou.

Separating words with weighted automata

Invited talk on Friday April 23, 9:00

Glenn Merlet

The separating words problem is the problem of finding the smallest deterministic finite automaton that behaves differently on two given strings, meaning that it accepts one of the two strings and rejects the other string.

If the words have different length, one can compute this length modulo b for a well chosen b, which is logarithmic in the sum of the length.

If the lengths are equal, however, the problem is surprisingly difficult : the best lower bound on the number of states is logarithmic, while the upper bound is $O(n^2/5\ \ (\log n)^3/5)$. With weighted automata, one can imagine separating all words with finitely many automata. It happens for weights in $(\mathbb{R}, +, \times)$.

With Zur Izhakian, we inductively build pairs of words that can not be separated by any max-plus automaton of a given size. In this talk, I will present this problem in more detail, and the strategy of our proof, which is based on the analysis of ranks of large powers of tropical matrices.

Polynomial functions and sequential products

Invited talk on Friday April 23, 10:00

Jean-Éric Pin

Polynomial functions from **N** to **Z** are familiar to everyone. Is there a similar notion in a noncommutative setting, that is, for functions from a free monoid A^* to a free group $F(B)$? The aim of this lecture is to answer this question positively in two different ways. First, by using differential operators and next by using a new operation, the sequential product of functions.

Copyless cost-register automata

Invited talk on Friday April 23, 13:45

Filip Mazowiecki

Copyless cost-register automata are a recently defined fragment of weighted automata. In this talk I will discuss some of our recent results regarding their expressiveness and decidability properties. In particular I will sketch undecidability of the equivalence problem for linear copyless cost-register automata over the tropical semiring, disproving a conjecture of Alur et al. from 2012.

Contributed talks

By order of appearance

Posets with Interfaces
(Extended Abstract)

Uli Fahrenberg[1], Christian Johansen[2], Georg Struth[3], and Ratan Bahadur Thapa[2]

[1] École Polytechnique, Palaiseau, France
[2] University of Oslo, Norway
[3] University of Sheffield, UK

This work is inspired by Tony Hoare's programme of building graph models of concurrent Kleene algebra (CKA) [5] for real-world applications. CKA extends the sequential compositions, nondeterministic choices and unbounded finite iterations of imperative programs modelled by Kleene algebra into concurrency, adding operations of parallel composition and iteration, and a weak interchange law for the sequential-parallel interaction. Such algebras have a long history in concurrency theory, dating back at least to Winkowski [21].

CKA has both interleaving and true concurrency models, that is, shuffle as well as pomset languages. Series-parallel pomset languages, which are generated from singletons by finitary applications of sequential and parallel compositions, form free algebras in this class [10, 13] (at least when parallel iteration is ignored). The inherent compositionality of algebra is thus balanced by the generative properties of this model. Yet despite this and other theoretical work, applications of CKA remain rare.

One reason is that series-parallel pomsets are not expressive enough for many real-world applications: even simple producer-consumer examples cannot be modelled [14]. Tests, which are needed for the control structure of concurrent programs, and assertions are hard to capture in models of CKA (see [8] and its discussion in [9]). Finally, it remains unclear how modal operators could be defined over graph models akin to pomset languages, which is desirable for concurrent dynamic algebras and logics beyond alternating nondeterminism [3, 15].

A natural approach to generating more expressive pomset languages is to "cut across" pomsets in more general ways when (de)composing them. This can be achieved by (de)composing along interfaces, and this idea can be traced back again to Winkowski [21]. As a side effect, interfaces may yield notions of tests, assertions or modalities. When they consist of events, cutting across them presumes that they extend in time and thus form intervals. Interval orders [2, 20] of events with duration have been applied widely in partial order semantics of concurrent and distributed systems [6, 11, 12, 16–19] and the verification of weak memory models [4], yet generating them remains an open problem [7].

We propose a new class and algebra of posets with interfaces (*iposets*) based on these ideas. We introduce a new gluing composition that acts like standard serial po(m)set composition outside of interfaces, yet glues together interface events, thus composing events that did not end in one component with those that did not start in the other one. Our definitions are categorical so that isomorphism classes of posets are considered ab initio. We show that the hierarchy of gluing-parallel posets generated by finitary applications of this gluing composition and the standard parallel composition, starting from singleton iposets, contains both the series-parallel posets and the interval orders. Iposets thus retain the pleasant compositionality properties of series-parallel pomsets and the wide applicability of interval orders in concurrency and distributed computing.

This extended abstract is based on [1] which will be presented at the 18th International Conference on Relational and Algebraic Methods in Computer Science, RAMICS, in April 2020.

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Combinatory Categorial Grammars as Acceptors of Weighted Tree Languages

Lena Katharina Schiffer*
Institute of Computer Science, Universität Leipzig, Germany.
E-mail: `schiffer@informatik.uni-leipzig.de`

Combinatory Categorial Grammar (CCG) is an extension of categorial grammar that is well-established in computational linguistics. It is mildly context-sensitive, so it is efficiently parsable and reaches an expressiveness that is conjectured to be sufficient to describe natural languages. The basis for CCG is provided by a lexicon and a rule system. The lexicon assigns syntactic categories to the symbols of the input and the rule system describes how adjoining categories can be combined to eventually obtain a (binary) derivation tree. The seminal work by Vijay-Shanker and Weir showed weak equivalence of CCG, Tree-Adjoining Grammar (TAG), and Linear Indexed Grammar (LIG), i.e. they accept the same string languages.[1] However, in a linguistic setting we are also interested in the underlying structure of an input sentence in the form of a constituency parse tree. In our previous work we related three different variants of CCG to classes of tree languages.[2] Here we extend our results to a weighted scenario using arbitrary commutative semirings as a weight structure. The tree language accepted by a CCG is defined as a relabeling of its derivation trees. In the weighted case, each rule is associated with a weight. To obtain the weight of a given tree, we sum over the weights of all derivation trees that are relabeled to that tree, where the weight of a single derivation tree is the product of the weights of the applied rules. Our main result is that the weighted tree languages accepted by weighted CCGs are included in the weighted simple monadic context-free tree languages (wsCFTG). When restricting the rule system to application rules and composition rules of first degree, the accepted weighted tree languages are exactly the weighted regular tree languages. When only application rules are allowed, a proper subset of the weighted regular tree languages is accepted.

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Can we preserve the polynomial parsability of restricted graph grammars when adding contextuality?

Henrik Björklund, Johanna Björklund, Frank Drewes, Anna Jonsson

Department of Computing Science, Umeå University, Sweden

Abstract (describing work-in-progress)

The management of semantic representations lies at the heart of many high-level natural language tasks, e.g., question answering and knowledge mining. Motivated by the interest in graph-based semantic representations, we explore two current graph-generating formalisms with the goal of combining them to obtain a formalism that can express well-structured semantic graphs while at the same time being efficiently parsable. Specifically, the semantic representation that we aim to express is the *abstract meaning representation* (AMR) by Banarescu et al. [1].

The first formalism that we consider is the *order-preserving dag grammars* (OPDGs) presented by Björklund et al. [2], a type of graph grammar based on *hyperedge replacement grammars* (HRGs [3], see, e.g., [4] for a survey) that is efficiently parsable by imposing a number of structural restrictions. Intuitively, these restrictions combine to ensure that each generated graph can be uniquely represented by a term in a particular graph algebra. Under this lens, the graph language essentially becomes a tree language of terms. The second formalism is the *contextual HRG*, developed by Drewes and Hoffmann [5]. This formalism extends the ordinary HRGs with so-called *contextual* rules, which allow for isolated nodes in their left-hand-sides. Contextual rules can access previously generated nodes, that are not attached to the replaced nonterminal hyperedge, and add structure to them.

Despite their restrictions, OPDGs seem to be able to describe central structural properties of AMR, but their limitation lies in the modelling of reantrancies. We propose to distinguish between two types of reentrancies: we speak of (i) *structural* reentrancies when they are syntactically governed, e.g. by control or coordination, and of (ii) *non-structural* reentrancies when they represent coreferences, which can in principle refer to any antecedent, essentially disregarding the structure of the graph. An example of (i) could be subject control as in "They persuaded him to talk to her", where the person who does the talking must be the same person who was persuaded, whereas "[. . .], but she liked them" is an example of (ii) since antecedent of "them" may be picked from anywhere in "[. . .]" for the semantic representation to be a valid one. Type (i) can to a large extent be modelled using OPDGs, but modelling type (ii) cannot be done (except for in very limited cases) since it requires adding edges in an unstructured way, which cannot be achieved using only context-freeness.

We have previously shown that contextual HRGs can handle both structural and non-structural reentrancies [6]. Moreover, a parser generator has been provided by Drewes, Hoffmann, and Minas for contextual HRGs [7] that, when it succeeds in producing a parser, guarantees that the parser will run in quadratic (and in the common case, linear) time on its input. However, for some input HRGs the generator discovers a parsing conflict and fails to output a parser. It is unknown whether the class of contextual HRGs on which the parser generator succeeds is sufficient to cover all AMR languages, but it is known that these are incomparable to OPDGs. For these reasons, we are now looking to combine OPDGs with the contextual mechanism used in [7], so as to be able to efficiently parse graph languages that contain reentrancies of both type (i) and (ii).

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Approximate simulation and bisimulation relations for fuzzy automata

Stefan Stanimirović[1], Ivana Micić[2], Miroslav Ćirić[3]

University of Niš, Faculty of Sciences and Mathematics, Višegradska 33, 18000 Niš, Serbia
[1] *stefan.stanimirovic@pmf.edu.rs*
[2] *ivana.micic@pmf.ni.ac.rs*
[3] *miroslav.ciric@pmf.edu.rs*

Algorithms for checking and computing the behavioural equivalences of various systems have been investigated in many areas of mathematics and computer science under different names, most commonly as simulations and bisimulations. They have also taken an important place in the automata theory, or more precisely, in the context of nondeterministic [2, 5], weighted [1, 4], fuzzy [3] and probabilistic automata, etc.

Roughly speaking, two states of fuzzy automata are considered bisimilar if and only if they can perform the same actions to reach bisimilar states. As it turns out, this condition can be too rigorous even for some simple real time systems. The study of approximate bisimulation relations for fuzzy automata has lately received a wide attention [6, 7, 8, 9, 10].

In this talk, we introduce two types of λ-approximate simulations and four types of λ-approximate bisimulations for fuzzy automata, where λ is the degree taken from the underlying set of truth values, which is a complete Heyting algebra. We show that λ-approximate simulations between fuzzy automata imply that the degree of inclusion of the corresponding fuzzy languages is at least λ, while λ-approximate bisimulations imply that the degree of equality of the corresponding fuzzy languages is at least than λ. Moreover, we show that, for any λ-approximate (bi)simulation, the exact degree of fuzzy language similarity (or equality) can be computed. We investigate further properties of λ-approximate (bi)simulations, and in particular, we pay attention to those λ-approximate (bi)simulation which are uniform fuzzy relations. In the end, we show that λ-approximate (bi)simulations are superior in the state reduction of fuzzy automata when compared to exact (bi)simulations, in the sense that there are cases when no reduction can be made using fuzzy (bi)simulations, while it is possible to reduce the number of states of a given fuzzy automaton using λ-approximate (bi)simulations.

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Weighted Tiling Automata on Graphs: Evaluation Complexity

C. Aiswarya

Chennai Mathematical Institute, India
International Research Lab ReLaX, CNRS France
aiswarya@cmi.ac.in

Paul Gastin

LSV, ENS Paris-Saclay,
CNRS, Universite Paris-Saclay, France
paul.gastin@lsv.fr

Abstract

We consider weighted automata to represent functions from graphs to a commutative semiring such as Natural semiring or Tropical semiring. The automaton labels the nodes of a graph by its states, and check if the neighbourhood of every node belongs to a the set of permissible tiles, and assigns a weight accordingly. The weight of a labeling is the semiring-product of the weights assigned to the nodes, and the weight of the graph is the semiring-sum of the weights of labelings. We show that we can model interesting algorithmic questions using this formalism - like computing the clique number of a graph or computing the permanent of a matrix. We study the complexity of evaluation as a function problem, and the associated decision problems (threshold languages). The complexity varies between NP and PSPACE in the counting hierarchy for different semirings and problems. We give tight upper and lower bounds for each combination.

Weighted Hexagonal Picture Automata

Meenakshi Paramasivan[1], D. Gnanaraj Thomas[2], S. James Immanuel[3], and M. Gayathri Lakshmi[4]

[1] Institut für Informatik, Universität Leipzig, D-04009 Leipzig, Germany,
[2] Department of Applied Mathematics, Saveetha School of Engineering, SIMATS, Chennai - 602105, India,
gnanarajthomas@gmail.com
[3] Department of Mathematics, Sri Sairam Institute of Technology, Chennai 600044, India,
james_imch@yahoo.co.in
[4] Department of Mathematics, Saveetha Engineering College, Chennai - 602105, India,
kirthana306@gmail.com

Abstract. Two-dimensional picture languages generated by grammars or recognized by automata have been studied since 1970s for the complications arising in the framework of pattern recognition and image analysis. Rani Siromoney and her co-authors (1970) studied two-dimensional picture languages with applications.
K. Inoue and I. Nakamura (1977) introduced two-dimensional on-line tessellation automata (2OTA). D. Giammarresi and her co-authors worked on two-dimensional picture languages and investigated connections to Tiling systems, Domino systems (1997) by defining the family REC of *recognizable picture languages* and 2OTA. The family REC is robust and has been characterized by many different devices, for instance it has been shown that a set of pictures is recognized by a finite tiling system if and only if it is definable in existential monadic second-order logic (1996).
Bozapalidis and Grammatikopoulou (2005) introduced weighted picture automata (WPA). WPA operates (the unweighted version of a WPA characterizes precisely REC) in a natural way on pictures and whose transitions carry weights; the weights are taken as elements from a given commutative semiring. Ina Fichtner (2007, 2011) has investigated formal power series on pictures and proved the equivalence between recognizable picture languages and EMSO-definable picture languages in quantitative setting.
Two-dimensional hexagonal arrays on a triangular grid can be viewed or treated as the two-dimensional representations of three-dimensional rectangular parallelopipeds. R. Siromoney (1976) worked on hexagonal arrays and hexagonal patterns that are found in literature on picture processing and scene analysis. K. G. Subramanian and his co-authors (2005) defined local and hexagonal recognizable picture languages. D. G. Thomas and his co-authors (2007) defined three directions on-line tessellation automata (3OTA) to recognize hexagonal picture languages. In this paper, we introduce weighted three directions OTA (W3OTA) and study formal power series on hexagonal picture languages.

Disambiguation of Weighted Tree Automata

Kevin Stier, Markus Ulbricht

Faculty of Mathematics and Computer Science, Universität Leipzig, Germany

Abstract

We extend the disambiguation construction presented by Mohri and Riley [1] in two ways. First we change the underlying structure of their automata from words to trees and second we show that these results hold not only for the tropical semiring but also the arctic one.

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The Big-O Problem for Weighted Automata

Dmitry Chistikov
University of Warwick

Stefan Kiefer
University of Oxford

Andrzej S. Murawski
University of Oxford

David Purser
University of Warwick and MPI-SWS

Given two weighted automata $\mathcal{A}$, $\mathcal{B}$ over an algebraic structure $(\mathcal{S}, +, \times)$, the equivalence problem asks whether the two associated functions $f_{\mathcal{A}}, f_{\mathcal{B}} \colon \Sigma^* \to \mathcal{S}$ are equal: $f_{\mathcal{A}}(w) = f_{\mathcal{B}}(w)$ for all finite words w over the alphabet Σ. Over the ring $(\mathbb{Q}, +, \times)$, equivalence is decidable in polynomial time by the results of Schützenberger [9] and Tzeng [11].

Replacing $=$ with $\leq$ makes the problem harder: even for the ring $(\mathbb{Q}, +, \times)$ the question of whether $f_{\mathcal{A}}(w) \leq f_{\mathcal{B}}(w)$ for all $w \in \Sigma^*$ is undecidable—even if $f_{\mathcal{A}}$ is constant [7].

We introduce and study another natural problem, in which the ordering is relaxed from inequality to inequality to within a constant factor.

Definition. *Given two WA $\mathcal{A}$ and $\mathcal{B}$, is it true that there exists a constant $c > 0$ such that*

$$f_{\mathcal{A}}(w) \leq c \cdot f_{\mathcal{B}}(w) \qquad \textit{for all } w \in \Sigma^* \text{ ?}$$

Using standard mathematical notation, this condition asserts that $f_{\mathcal{A}}(w) = O(f_{\mathcal{B}}(w))$ as $|w| \to \infty$, and we refer to this problem as the *big-O* problem accordingly.

The *big-*Θ problem (which turns out to be computationally equivalent to the big-O problem), in line with the $\Theta(\cdot)$ notation in analysis of algorithms, asks whether $f_{\mathcal{A}} = O(f_{\mathcal{B}})$ and $f_{\mathcal{B}} = O(f_{\mathcal{A}})$.

We restrict our attention to the ring $(\mathbb{Q}, +, \times)$ and only consider *non-negative weighted automata*, i.e., those in which all transitions have non-negative weights. Our main findings are as follows:

- The big-O problem for non-negative WA is **undecidable in general**, by a reduction from nonemptiness for probabilistic automata. The result applies even for the special cases of labelled Markov chains and probabilistic automata.
- For **unambiguous automata**, (where every word has at most one accepting path) the big-O problem is decidable in polynomial time.
- In the **unary case**, i.e., if the input alphabet Σ is a singleton, the big-O problem is also decidable and, complete for the complexity class **coNP**. Our upper bound argument refines an analysis of growth of entries in powers of non-negative matrices by Friedland and Schneider [8], and the lower bound is by a reduction from unary NFA universality [10].
- In a more general **bounded case**, i.e., if the languages of all words positive weighted words are included in $w_1^* w_2^* \dots w_m^*$ for some finite words $w_1, \dots, w_m \in \Sigma^*$ the big-O problem is decidable subject to Schanuel's conjecture (a well-known conjecture in transcendental number theory [5], entailing that the first-order theory of the real numbers with the exponential function is decidable [6]).

Relation to total variation distances In the labelled Markov chain setting, the big-O problem can be reformulated as a boundedness problem for the following function. For two LMCs $\mathcal{A}$ and $\mathcal{B}$, define the (asymmetric) *ratio variation function* by

$$r(\mathcal{A}, \mathcal{B}) = \sup_{E \subseteq \Sigma^*} \frac{f_{\mathcal{A}}(E)}{f_{\mathcal{B}}(E)},$$

where $f_{\mathcal{A}}(E)$ and $f_{\mathcal{B}}(E)$ denote the total probability mass associated with an arbitrary set of finite words $E \subseteq \Sigma^*$ in $\mathcal{A}$ and $\mathcal{B}$, respectively. The supremum over $E \subseteq \Sigma^*$ can be replaced with supremum over $w \in \Sigma^*$. Consequently, the big-O problem for LMCs is equivalent to deciding whether $r(\mathcal{A}, \mathcal{B}) < \infty$.

Finding the value of r amounts to asking for the optimal (minimal) constant in the big-O notation.

r is a ratio-oriented variant of the classic *total variation distance tv*, defined by

$$tv(\mathcal{A}, \mathcal{B}) = \sup_{E \subseteq \Sigma^*} f_{\mathcal{A}}(E) - f_{\mathcal{B}}(E),$$

which is a well-established way of comparing two labelled Markov chains [1, 4]. We also consider the problem of approximating r to a given precision and the problem of comparing it with a given constant (threshold problem), showing that both are undecidable.

Application to privacy Consider a system $\mathcal{M}$, modelled by a single labelled Markov chain, where output words are observable to the environment but we want to protect the privacy of the starting configuration. We write f_s for the weighting function of $\mathcal{M}$ when executed from state s.

Definition. *[2, 3] Let $R \subseteq Q \times Q$ be a symmetric relation, which relates the starting configurations intended to remain indistinguishable. Given $\epsilon \geq 0$, $\mathcal{M}$ is ϵ-differentially private (with respect to R) if, for all pairs $s, s' \in Q$ such that $(s, s') \in R$, we have $f_s(E) \leq e^{\epsilon} \cdot f_{s'}(E)$ for every observable set of traces $E \subseteq \Sigma^*$.*

Note that there exists such an ϵ if and only if $r(s, s') < \infty$ for all $(s, s') \in R$ or, equivalently, (the LMC $\mathcal{M}$ executed from) the state s is big-O of (the LMC $\mathcal{M}$ executed from) the state s' for all $(s, s') \in R$. The minimal such ϵ satisfies $e^{\epsilon} = \max_{(s,s') \in R} r(s, s')$.

Hence, our results show that even deciding whether the ratio variation distance is finite or $+\infty$ is, in general, impossible. Likewise, it is undecidable whether a system modelled by a labelled Markov chain provides any degree of differential privacy, however low.

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Kleene and Büchi for Weighted Forest Languages over M-Monoids

Frederic Dörband[1]
Technische Universität Dresden, February 10, 2020

We recall forests as finite tuples of trees. This syntactic extension of trees already occurred in [2,3,4]. We define weighted forest automata (wfa) as an extension of weighted tree automata (wta). Our definition generalizes the unweighted forest automata introduced in [1] to the weighted case, while also generalizing the weighted forest automata introduced in [3] from commutative semirings to M-monoids. In comparison with [2] , our paper is a generalization from semirings to M-monoids, but also a restriction from hedge languages to forest languages.

A wfa is syntactically similar to a wta, however the semantics of wfa allow for entire forests to be processed (instead of single trees). The essence of our paper is a theorem stating that our automaton model generates only so-called rectangular weighted forest languages. That is, for every recognizable weighted forest language φ, there are recognizable weighted tree languages (the so-called rectangular components of φ), whose horizontal concatenation equals φ.

Using this rectangularity property, we then infer both, a Kleene-like result and a Büchi-like result for weighted forest languages. On the one hand, this generalizes the Kleene-like result for weighted tree automata from [5] to the case of forests and the Kleene-like result for unweighted forest automata [1] to the case of M-monoids. On the other hand, this generalizes the Büchi-like result for weighted tree automata from [6] to the case of forests.

For our Kleene-like result, we introduce rational forest expressions, which are horizontal products of rational tree expressions and for our Büchi-like result, we introduce forest M-expressions, which are horizontal products of tree M-expressions. We ultimately ask the question, whether the definitions for rational forest expressions and forest M-expressions can be done inductively. However, we see that straightforward inductive definitions are not possible due to ambiguity in the choice of rectangular components.

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Statistical machine translation using streaming string transducers

Thomas Ruprecht
Technische Universität Dresden
thomas.ruprecht@tu-dresden.de

Abstract

Nondeterministic streaming string transducers (*nsst*) [1] are string-to-string transducers and feature a finite set of states, as usual. Each transition reads exactly one input symbol and composes several output strings that are stored in registers; this may introduce new symbols and arbitrarily compose the strings previously stored in the registers, but not duplicate them. Since there are no further restrictions, the input language of each nsst is regular. The properties of the string-to-string transductions imposed by nsst were examined intensively by Alur and Deshmukh [1], yet – compared to well-established transducer formalisms, such as finite state transducers – this formalism is rather young. Apart from the transduction computed by nsst, Bojańczyk [2] also introduced origin semantics for this transducer model. They consider the relation between input and output positions with respect to which output positions were introduced when the input position was read. Bojańczyk, Daviaud, Guillon, and Penelle [3] characterized the class of these origin graphs for nsst and compared it to the classes of origin graphs defined transductions by two-way automata and monadic second-order logic.

Statistical machine translation (*smt*) deals with the automatic translation from sentences of one natural language into sentences of another natural language [6]. Typically, large bilingual corpora are used to automatically induce a finite set of probabilistic translation rules of some underlying formalism. This set of rules serves for the translation of previously unseen sentences. Recently Nederhof and Vogler [8] investigated the string-to-string transduction imposed by nsst whose input is restricted to a multiple context-free language, resulting in transductions of synchronous multiple context-free grammars. As synchronous multiple context-free grammars were successfully applied in smt [5], the question arises how nsst perform in this setting.

Here we consider *probabilistic nondeterministic streaming string transducers* (*pnsst*) as an underlying formalism for smt. We present an induction for pnsst from bilingual corpora with word alignments (which are treated as origin graphs). Using techniques known for the induction of multiple context-free grammars [7], we define nsst that recognize a given set of origin graphs. Weights for each transition are induced according to the maximum likelihood estimate [4] with respect to the set of origin graphs. Some properties of the induced pnsst are discussed in the context of inference, i.e. the prediction of the most probable origin graph for a fixed input.

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Greibach Normal Form and Simple Automata for Weighted ω-Context-Free Languages

Manfred Droste[1], Sven Dziadek[1], and Werner Kuich[2]

[1] *Institut für Informatik, Universität Leipzig, Germany*
[2] *Institut für Diskrete Mathematik und Geometrie, Technische Universität Wien, Austria*

In weighted automata theory, many classical results on formal languages have been extended into a quantitative setting. Here, we investigate weighted context-free languages of infinite words, a generalization of ω-context-free languages (Cohen, Gold 1977) and an extension of weighted context-free languages of finite words (Chomsky, Schützenberger 1963). As in the theory of formal grammars, these weighted context-free languages, or ω-algebraic series, can be represented as solutions of (mixed) ω-algebraic systems of equations and by weighted ω-pushdown automata.

In our first main result, we show that (mixed) ω-algebraic systems can be transformed into *Greibach normal form*. We use the Greibach normal form in our second main result to prove that *simple ω-reset pushdown automata* recognize all ω-algebraic series. Simple ω-reset automata do not use ϵ-transitions and can change the stack only by at most one symbol. These results generalize fundamental properties of context-free languages to weighted context-free languages.

This talk is based on [1] and its corresponding short version [2].

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State reduction of weighted automata using certain equivalences

Aleksandar Stamenković

Faculty of Sciences and Mathematics, University of Niš, Višegradska 33,
18000 Niš, Serbia

In this talk, for a given weighted automaton $\mathcal{A}$ over an alphabet X, we introduce $L-$weakly left and $L-$weakly right invariant equivalences corresponding to $\mathcal{A}$, where L is an arbitrary language over X. We give a simple procedure to build the resulting weighted automaton from the staring one using $L-$weakly left (resp. $L-$weakly right) invariant equivalence. Since resulting weighted automata have smaller or equal number of states than the starting ones, the above-mentioned equivalences can be used for state reduction. Moreover, we prove that starting and resulting weighted automata are $L-$equivalent in case L is suffix-closed (prefix closed) and using that result we obtain some additional consequences. We also provide a procedure for computing $L-$weakly left (resp. $L-$weakly right) invariant equivalences corresponding to a weighted automaton $\mathcal{A}$. Since, in general, given procedure has some drawbacks regarding its efficiency, we investigate some particular cases where these shortcomings are overcomed.

Weighted linear dynamic logic with two-sorted semantics

Gustav Grabolle

In 2013 Giuseppe De Giacomo and Moshe Vardi introduced linear dynamic logic. This logic uses the intuitive, two-sorted syntax of PDL, has good complexity properties, and extends the expressive power of LTL: De Giacomo and Vardi prove LDL and finite automata are equally expressive. This is done via alternating automata. In 2016 Manfred Droste and George Rahonis introduced a weighted version of LDL with weights taken from semirings.

In this talk, I will introduce a new adaptation of LDL into the weighted setting. While weighted LDL uses generalized rational expressions in the definition of its semantics, nLDL mirrors the two-sorted semantics of LDL. This is to obtain an intuitive syntax similar to LDL. Moreover, nLDL is more expressive than weighted LDL, and relates to weighted LTL as LDL relates to LTL. I will show that, similar to PDL, the depth of the question-mark operator entails a hierarchy on the expressive power of nLDL. To characterize nLDL, I will introduce n-level weighted finite automata which are a subclass of nested weighted automata and capture the expressive power of nLDL.

Decidability of crisp-determinization for weighted finite automata over past-finite monotonic strong bimonoids

M. Droste[a] Z. Fülöp[b1] D. Kószó[b2] H. Vogler[c]

[a] *Institute of Computer Science, University of Leipzig*
[b] *Department of Foundations of Computer Science, University of Szeged*
[c] *Faculty of Computer Science, Technische Universität Dresden*

Abstract: A general problem in the theory of automata is the question whether an automaton is determinizable, i.e., whether we can construct an equivalent deterministic automaton. While each classical finite automaton is determinizable, this is not true for weighted finite automata (wfa). In fact, there is a nondeterministic wfa over the field $\mathbb{Q}$ which is not determinizable (cf. [2]). Thus, it is natural to identify classes of wfa which have the following property: for each wfa $\mathcal{A}$ of that class it is decidable whether $\mathcal{A}$ is determinizable. If a class has this property (respectively, does not have this property), then we say that the determinization problem for this class is decidable (respectively, undecidable).

We consider the crisp-determinization problem for the class of wfa over strong bimonoids [3,5]. A wfa is crisp-deterministic if it is deterministic, and each of its transitions carries either the additive or multiplicative unit of the strong bimonoid; weights different from these units may only appear as initial and final weights. In [6] it was shown that the crisp-determinization problem for the class of wfa over strong bimonoids is undecidable.

In this presentation, we deal with wfa over past-finite monotonic strong bimonoids. Such algebras are based on monotonic semirings [1] and they share many properties with the semiring of natural numbers (strong kind of well-foundedness, monotonicity of operations, zero-sum and zero-divisor free, partial order on the carrier set), however distributivity is not required and also does not follow from the axioms. We obtain a general characterization of when trim wfa over past-finite monotonic strong bimonoids are crisp-determinizable. As a consequence, we show that the crisp-determinization problem for the class of (a) wfa over additively locally finite and past-finite monotonic strong bimonoids and (b) unambiguous wfa over past-finite monotonic strong bimonoids is decidable [4]. In this case, assuming the strong bimonoid to be computable, the equivalent crisp-deterministic wfa can be computed. Moreover, we show that for each wfa $\mathcal{A}$ over a past-finite monotonic strong bimonoid and a natural number k it is decidable whether the cardinality of the image of the semantics of $\mathcal{A}$ is bounded by k.

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Weighted Parsing for Grammar-Based Language Models over Multioperator Monoids

Richard Mörbitz Heiko Vogler

We develop a general framework for weighted parsing called *weighted RTG-based language models*, define the *M-monoid parsing problem*, and present an algorithm for its solution.

There is a variety of weighted parsing problems, e.g., probabilistic constituent parsing, where the task is to compute for each English sentence the most probable constituent tree (according to some formal grammar). We view each of these particular problems as a mapping parse: $\mathcal{L} \to \mathbb{K}$, where $\mathcal{L}$ is the set of syntactic objects (e.g., the set of English sentences) and $\mathbb{K}$ is the set of weights (e.g., the set of English constituent trees).

Our approach makes heavy use of universal algebra, in the sense that both $\mathcal{L}$ and $\mathbb{K}$ are identified with the carrier sets of algebras. We employ the initial algebra semantics[1] to model the syntactic structure of $\mathcal{L}$ using regular tree grammars (RTG) and we choose multioperator monoids (M-monoids)[2] to compute the weights in $\mathbb{K}$. The M-monoid parsing problem determines how, given a syntactic object from $\mathcal{L}$, the weight in $\mathbb{K}$ is to be computed using both algebras. Thus particular weighted parsing problems amount to particular choices of $\mathcal{L}$ and $\mathbb{K}$ and are therefore instances of the M-monoid parsing problem.

We show that, in general, the M-monoid parsing problem cannot be solved by a terminating algorithm. We determine a large class of weighted RTG-based language models for which this is still possible. This class is essentially a generalization of the graphs weighted with closed semirings from Mohri's "single source shortest distance" framework.[3] We prove that our algorithm terminates and is correct for the class of *closed weighted RTG-based language models*.

In the end we are also interested in application scenarios of our algorithm, i.e., which weighted parsing problems are covered by the class of closed weighted RTG-based language models. It turns out that our approach subsumes the previous approaches to weighted parsing, semiring parsing[4] and weighted deductive parsing,[5] and also covers weighted parsing problems which are outside the scope of both (e.g., parsing as intersection). Moreover, we can even solve the problems of algebraic dynamic programming.[6]

This work was presented at the 14th International Conference on Finite-State Methods and Natural Language Processing (FSMNLP 2019), September 23–25, 2019, Dresden, Germany.

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Weakly linear systems for matrices over the max-plus quantale and their applications

Miroslav Ćirić[1], Aleksandar Stamenković[1], and Dragan Djurdjanović[2]

[1]University of Niš, Faculty of Science, Department of Computer Science, Niš, Serbia
miroslav.ciric@pmf.edu.rs, aca@pmf.ni.ac.rs
[2]University of Texas, Department of Mechanical Engineering, Austin, TX, USA
dragand@me.utexas.edu

Weakly linear systems are a special kind of systems of relational or matrix inequations and equations that have recently emerged in the study of fuzzy automata, and have also been applied in social network analysis. They have been used to reduce the number of states of fuzzy automata [9–12], in the study of simulations and bisimulations for nondeterministic, fuzzy and weighted automata [1–4], as well as in the positional analysis of social networks [5, 6]. In addition, certain types of WLS have been used to improve determinization algorithms for fuzzy automata [8] and in the conflict analysis of fuzzy discrete event systems [12]. In all these cases, the underlying structure of membership values was a complete residuated lattice.

General properties of weakly linear systems of fuzzy relation inequations and equations, with a complete residuated lattice as the underlying structure of membership values, have been studied in [5–7]. The key role in this study is played by completeness and residuation in the structure of membership values, which ensure completeness and residuation in the corresponding lattices of fuzzy relations. The residuation reduces the problem of solving WLS to the problem of computing post-fixed points of suitable isotone functions on the lattice of fuzzy relations, and the completeness of this lattice enables to use the Knaster-Tarski Fixed Point Theorem, according to which solutions of a WLS form a complete lattice, and therefore, there exists the greatest solution. The greatest solutions of WLS are computed using a procedure which is a modification of the Kleene Fixed Point Theorem. This procedure does not necessarily terminate in a finite number of steps, and some sufficient conditions have been found when this happens.

Here we consider weakly linear systems in the context of matrices over the max-plus algebra. The complete max-plus algebra is very similar to complete residuated lattices[1], the only difference is that its multiplicative identity is not the greatest element. In fact, the complete max-plus algebra is a commutative unital quantale. We prove the existence of the greatest solution contained in a given matrix X_0, and present a procedure for its computation. In case of finite WLS when all finite entries of matrices forming this WLS and the matrix X_0 are integers, rationals or particular irrationals and a finite solution exists, the procedure finishes in a finite number of steps. If in that case an arbitrary finite

[1] we are talking about complete residuated lattices defined in the way that is the most common in the fuzzy set theory (cf. [2, 3, 10, 5–8, 11, 12]).

solution is given, a lower bound on number of computational steps is calculated. Otherwise, we use our algorithm to compute approximations to finite solutions.

We will also mention applications of WLS in the study of max-plus automata.

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Finite Sequentiality of Max-Plus Tree Automata

Erik Paul

Institute of Computer Science
Leipzig University, Germany

Abstract

We consider the finite sequentiality problem for max-plus tree automata and its decidability for unambiguous and finitely ambiguous max-plus tree automata. A max-plus tree automaton is a weighted tree automaton over the max-plus semiring. A max-plus tree automaton is called unambiguous if there is at most one accepting run on every tree and it is called finitely ambiguous if the number of accepting runs on every tree is bounded by a global constant. The finite sequentiality problem asks whether for a given max-plus tree automaton, there exist finitely many deterministic max-plus tree automata whose pointwise maximum is equivalent to the given automaton. We show that for unambiguous and finitely ambiguous max-plus tree automata, the finite sequentiality problem is decidable. This generalizes results by Bala and Koniński from words to trees.

Blockmodeling of fuzzy transition systems using approximate regular relations*

Ivana Micić*, Stefan Stanimirović, Zorana Jančić

University of Niš, Faculty of Sciences and Mathematics, Višegradska 33, 18000 Niš, Serbia

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Abstract

For a fuzzy automata over complete Heyting algebra, the λ-approximate bisimulations are recently defined and used for factorization of automata. These relations are shown to be very good means for constructing the models, which are similar at the degree λ, to the original automata, but with the much smaller size. Based on this methodology, in current work, we investigate the λ-approximate regular fuzzy relations on the fuzzy transition systems over complete residuated lattices. We provide an algorithm for determining the λ-approximate regular fuzzy relations which are preorders. Afterward, we use these preorders for reduction of fuzzy transition systems. At the end, we propose a method for determining all values λ, such that reduction by means of λ-approximate regular preorders produces the system of a different size.

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*Corresponding author. Tel.: +38118224492.

Email addresses: ivana.jancic@pmf.edu.rs (Ivana Micić), stefan.stanimirovic@pmf.edu.rs (Stefan Stanimirović), zorana.jancic@pmf.edu.rs (Zorana Jančić)

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Weighted context-free grammars over a complete strong bimonoid

Jelena Ignjatović[1], Miroslav Ćirić[1], and Manfred Droste[2]

[1]University of Niš, Faculty of Science, Department of Computer Science, Niš, Serbia
jelena.ignjatovic@pmf.edu.rs, miroslav.ciric@pmf.edu.rs
[2]University of Leipzig, Institute of Computer Science, Leipzig, Germany
droste@informatik.uni-leipzig.de

Weighted context-free grammars are classical context-free grammars in which the productions carry weights. Adding weights allows a quantitative view of derivations, which can be very useful in many applications. Traditionally, the weights are assumed to form the algebraic structure of a semiring, but recently other ways of weight computation are also considered. Shang, Lu and Lu [8] studied context-free grammars with weights taken from particular algebraic structures called lattice-ordered quantum MV algebras. Droste and Vogler [3] were dealing with CFGs over unital valuation monoids, whereas Jin and Li [5] and Rahonis and Torpari [7] have considered CFGs over bimonoids.

Here we are interested in weighted context-free grammars over complete strong bimonoids. Due to lack of distributivity we distinguish between two ways to define behaviour of such grammars: *depth-first semantics* and *breadth-first semantics*. In the depth-first semantics, we associate the weight of the derived word by summing weights of all derivations of that word, where the weight of a derivation is computed multiplying weights of the productions that form it. Contrary to that, in the breadth-first semantics, we compute weights of derivations of different lengths, gradually increasing the length. If the underlying weight-structure is a semiring, we prove that these two semantics coincide. Moreover, we show that two semantics coincide for every weighted context-free grammar G over K without multiple direct derivations if and only if the underlying complete strong bimonoid K is right distributive. When K is an additively idempotent complete strong bimonoid, we prove that two semantics coincide for every weighted context-free grammar G over K if and only if K is a semiring.

Restricting attention only to grammars with leftmost derivations, we define *leftmost depth-first semantics* and *leftmost breadth-first semantics* and we show that these two semantics coincide for very weighted context-free grammar G over K if and only if K is right distributive.

Similar issues will be considered in the context of weighted pushdown automata, along with the question of the equivalence of weighted context-free grammars and weighted pushdown automata.

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