CIRM winter school - Discrete mathematics and logic: between mathematics and the computer science

Algebraic approach to register minimization in streaming string transducers

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LABORATOIRE D'INFORMATIQUE & SYSTÈMES UMR 7020

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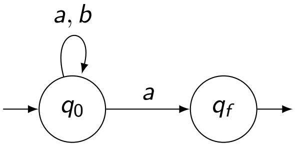
Regular languages

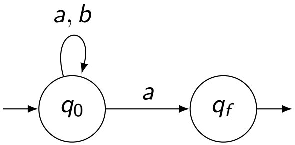


Automaton recognizing $(a+b)^*a$

Other characterizations:

- Algebra : syntactic congruence
- Regular expressions
- Logic : MSO

Regular languages



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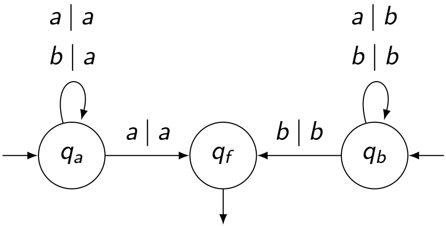
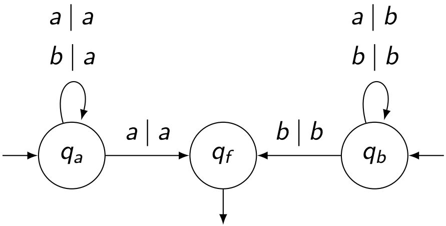
Transducers



Transducer realizing the function *last*

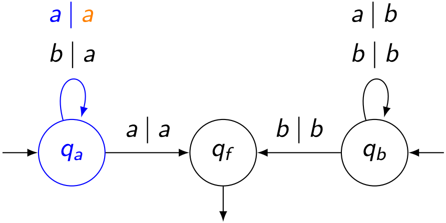
Transducers



Transducer realizing the function *last*

aba

Transducers



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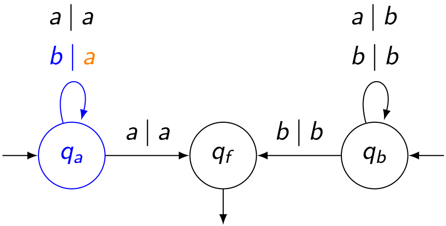
$$aba \longmapsto a$$

Transducers



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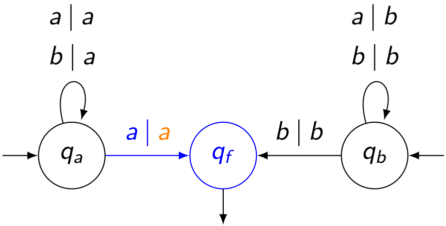

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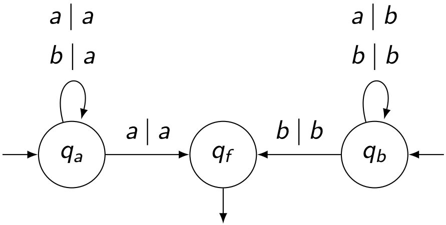


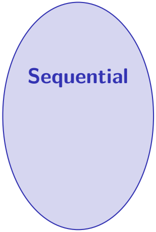
Transducer realizing the function *last*

$$last(aba) = aaa$$

Classes of functions

Sequential functions

Deterministic underlying automaton



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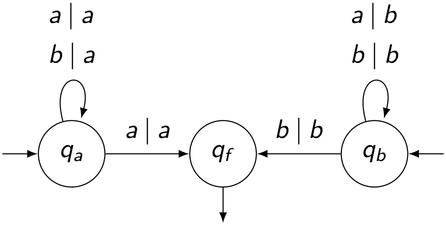

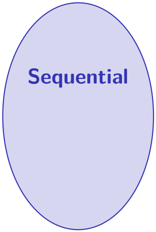

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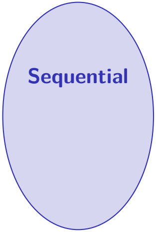

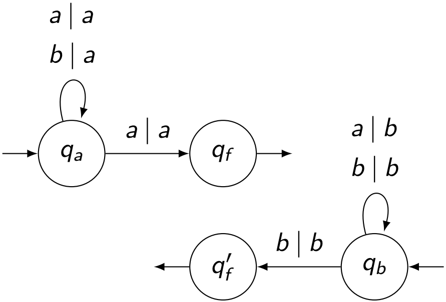

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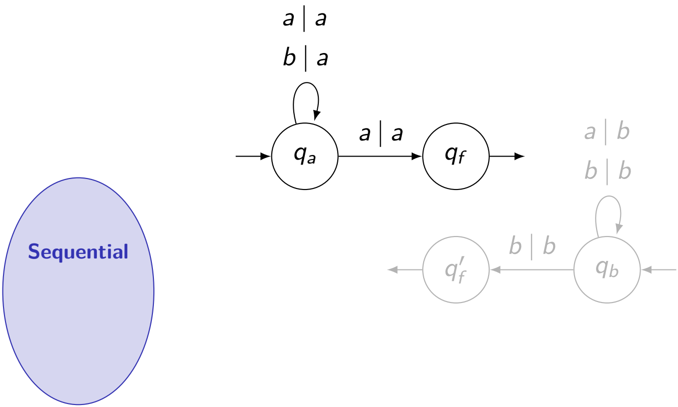

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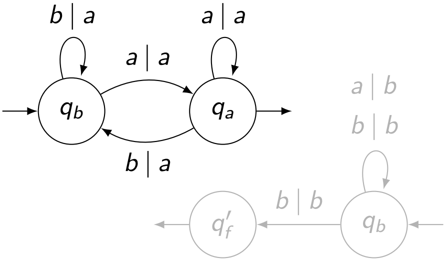

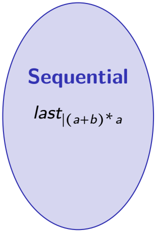

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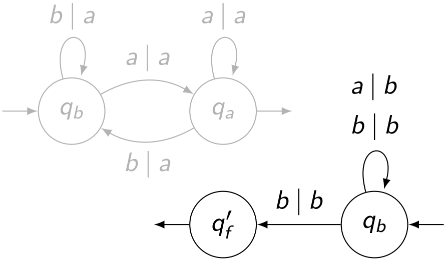

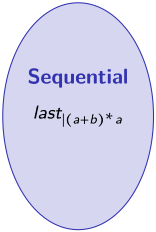

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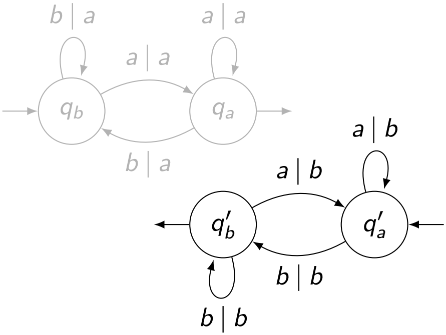


Sequential

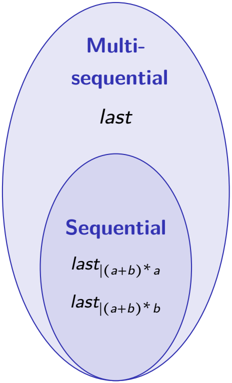
$last_{|(a+b)^*a}$

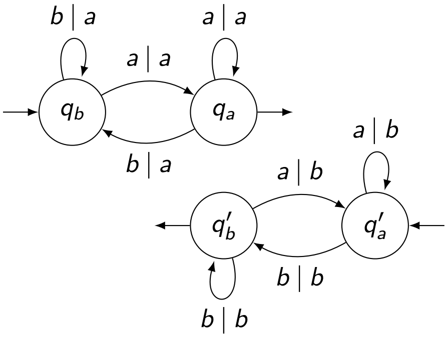
$last_{|(a+b)^*b}$

Classes of functions

Multi-sequential functions

Finite union of sequential transducers





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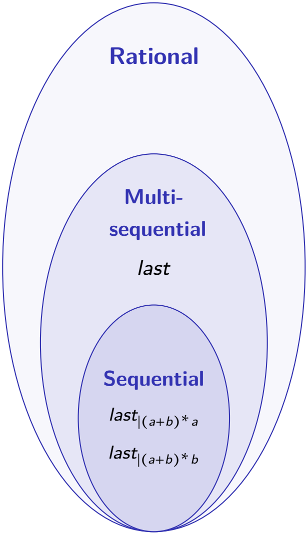
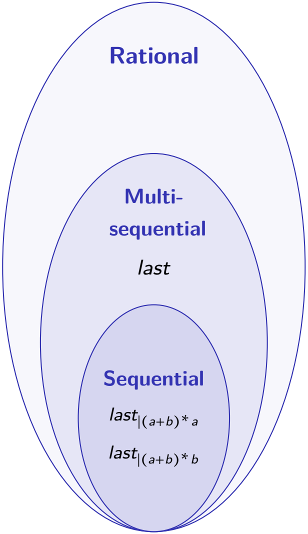


Rational functions

Any finite-state transducer

Classes of functions



Rational functions

Any finite-state transducer

$$last^* :$$

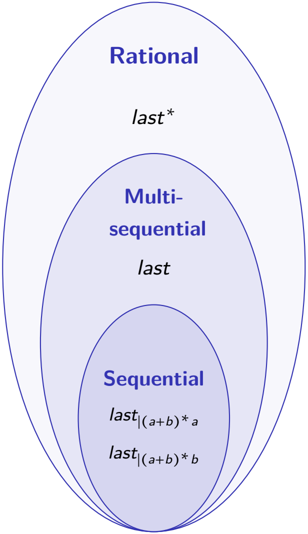
$$w_1 \# w_2 \# \dots \# w_n$$

$$\downarrow$$

$$last(w_1) \# last(w_2) \# \dots \# last(w_n)$$

any $n \in \mathbb{N}^*$

Classes of functions



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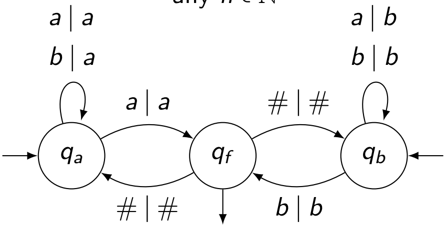

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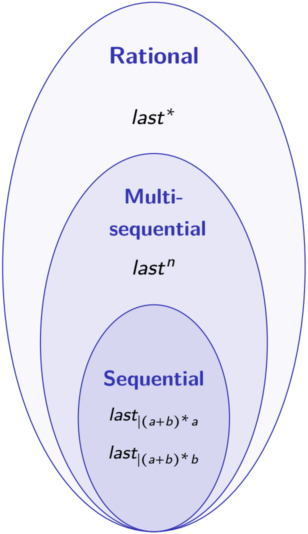

Rational functions

Any finite-state transducer

$last^n$:

$$w_1 \# w_2 \# \dots \# w_n$$
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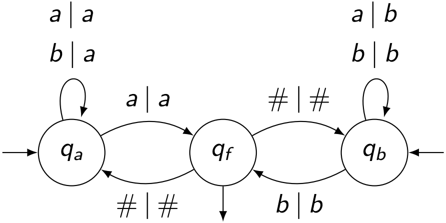
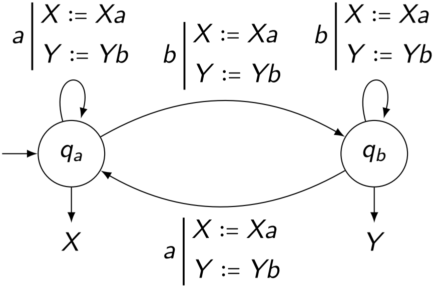
fixed $n \in \mathbb{N}^*$



Streaming String Transducers[1] (SST)



SST realizing *last*

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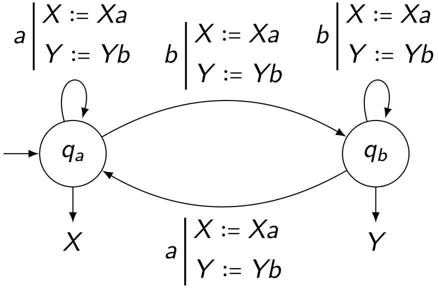

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$$\begin{cases} X = \epsilon \\ Y = \epsilon \end{cases} \qquad aba \mapsto$$

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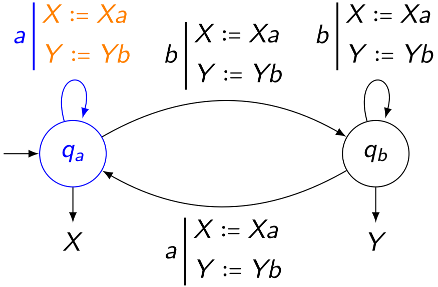

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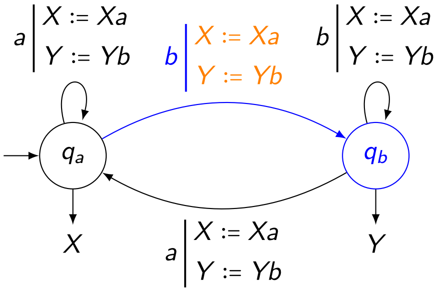

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$$\begin{cases} X = aa \\ Y = bb \end{cases} \qquad aba \mapsto$$

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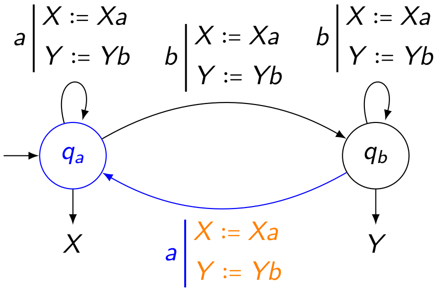

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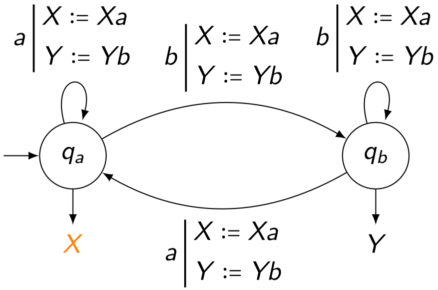

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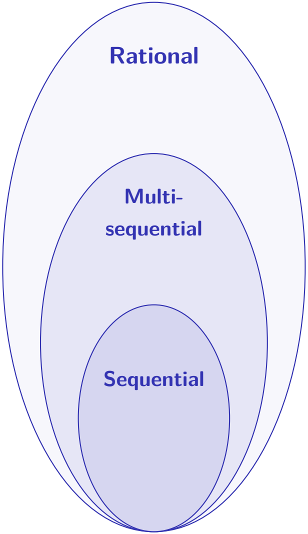

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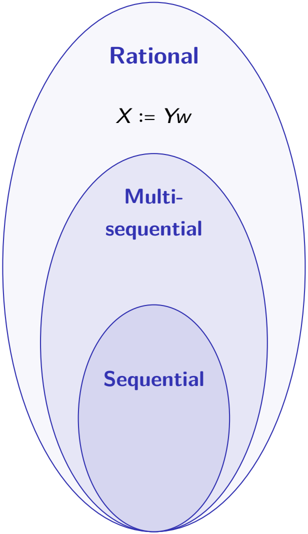

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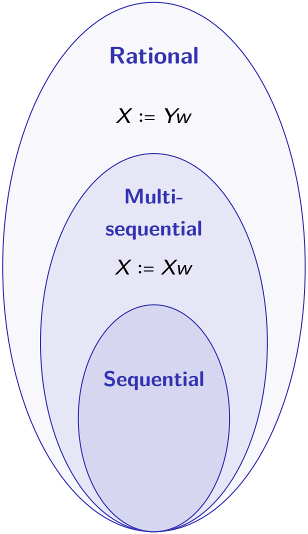

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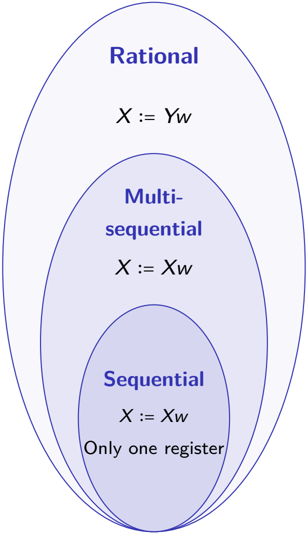

Register minimization

Problem

Input : A rational function f realized by an SST $\mathcal{T}$, an integer k

Question : $\exists$? an SST $\mathcal{T}'$, with k registers, that is equivalent to $\mathcal{T}$ (i.e. realizing f).

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For rational functions :

- Daviaud, L., Reynier, P., & Talbot, J. (2016).A generalised twinning property for minimisation of cost register automata. *LICS 2016*

For multi-sequential functions :

- Daviaud, L., Jecker, I., Reynier, P., & Villevalois, D. (2017).Degree of sequentiality of weighted automata. *FOSSACS 2017*

Algebraic characterization

Let Σ be a finite alphabet and $L \subseteq \Sigma^*$ a language.

Definition

An equivalence relation $\sim$ over Σ^* is a right *congruence* if

$$\forall u, v \in \Sigma^* \; \forall \sigma \in \Sigma \qquad u \sim v \Rightarrow u\sigma \sim v\sigma$$

Definition

(right) *syntactic congruence* of L:

$$u \sim_L v \Leftrightarrow \forall w \in \Sigma^*, uw \in L \Leftrightarrow vw \in L$$

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Theorem (Myhill-Nerode)

L is a regular language iff $\sim_L$ has a finite index (i.e. $|\Sigma^* / \sim_L| < \infty$)

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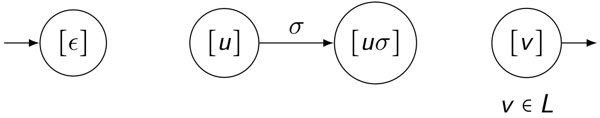

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- Allows to define the minimal automaton of L
- Allows to decide if a language belongs to a subclass (ex: L is definable in first-order logic iff $\sim_L$ is aperiodic)
- Used in learning algorithms (Angluin, 1987)
- ...

Sequential functions : Syntactic congruence

Let Σ be a finite alphabet and $f : \Sigma^* \to \Sigma^*$ be a function.

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$$u \sim_f v \Leftrightarrow \begin{cases} \end{cases}$$

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f is a sequential function iff $\sim_f$ has a finite index

[1] Choffrut, C. (1977). Une caracterisation des fonctions sequentielles et des fonctions sous-sequentielles en tant que relations rationnelles. *Theor. Comput. Sci.*

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- Allows to define the minimal sequential transducer of f

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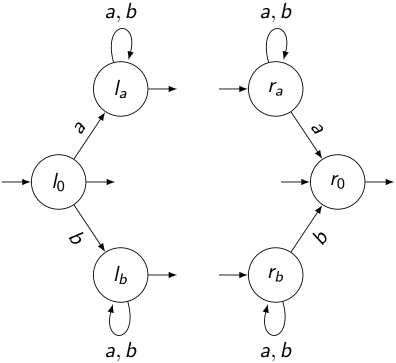
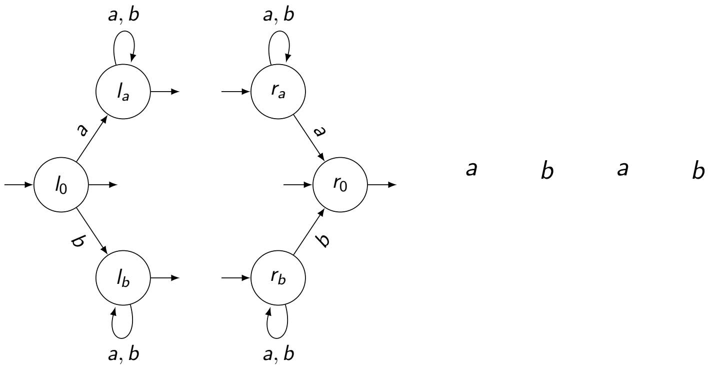
Rational functions : Bimachines



$$\omega(l_0, \sigma, r_\tau) = \omega(l_\tau, \sigma, r_0) = \tau$$
$$\omega(l, \sigma, r) = \sigma \text{ for any other } (l, \sigma, r)$$

Bimachine realizing *swap*
$(swap(\sigma w \tau) = \tau w \sigma)$

Rational functions : Bimachines



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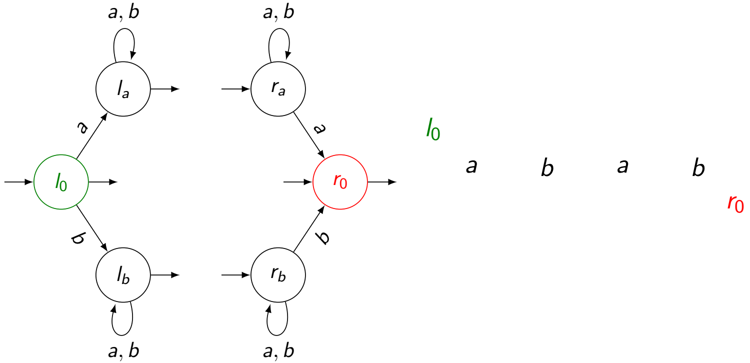

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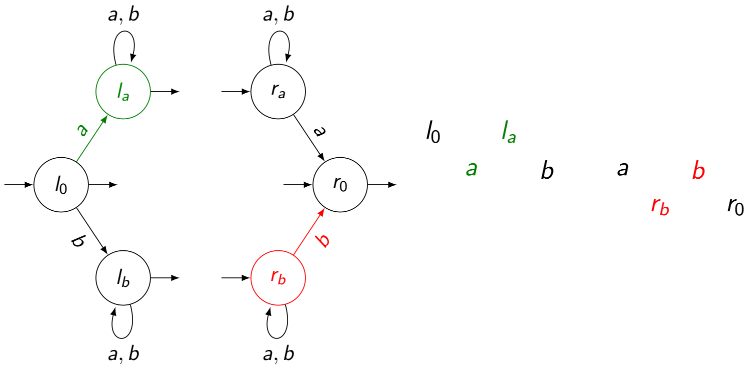

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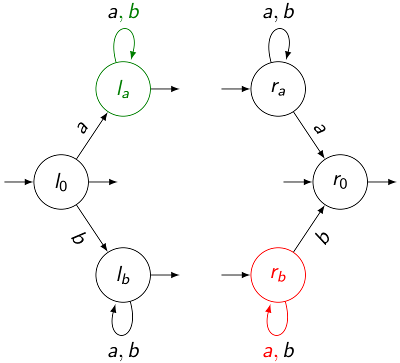

l_0 l_a l_a

a b a b

r_b r_b r_0

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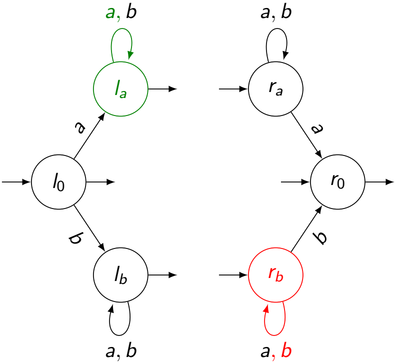
Bimachine realizing *swap*
$(swap(\sigma w \tau) = \tau w \sigma)$

Rational functions : Bimachines



l_0		l_a		l_a		l_a		
	a		b		a		b	
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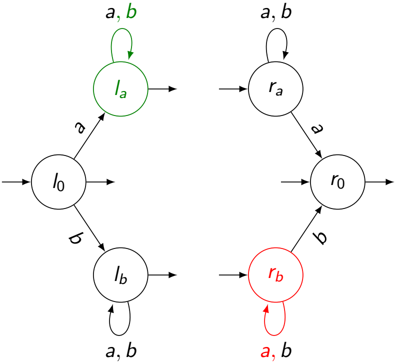

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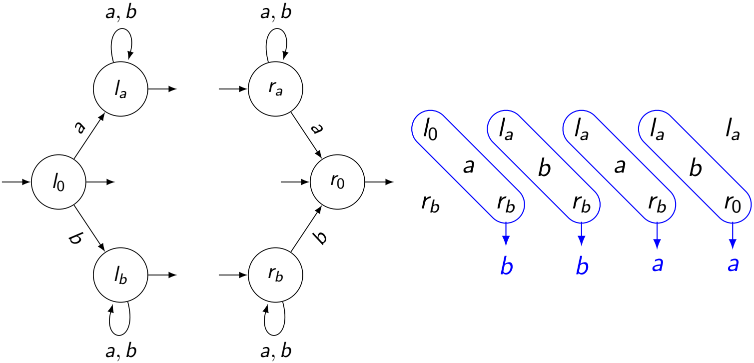

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Definition

Left syntactic congruence of f:

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Theorem (Reutenauer and Schützenberger[1])

If f is a rational function then $\backsim_f$ has a finite index

[1] Reutenauer, C., & Schützenberger, M. P. (1991). Minimization of rational word functions. *SIAM J. Comput.*

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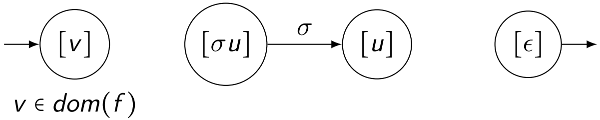
Theorem (Reutenauer and Schützenberger)

If f is a rational function then $\leftharpoondown_f$ has a finite index

- Allows to define the minimal right automaton of f



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Right syntactic congruence of f:

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Theorem (Reutenauer and Schützenberger)

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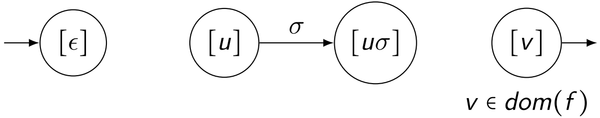
- Allows to define the minimal left automaton of f



Register minimization

Known results

For rational functions :

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Based on finding patterns in the transducer realizing the function

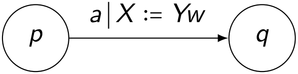
Objective : Solving the register minimization problem for rational functions using their algebraic characterization

Register flows

If $X := Yw$ we say that the register Y **flows** to X

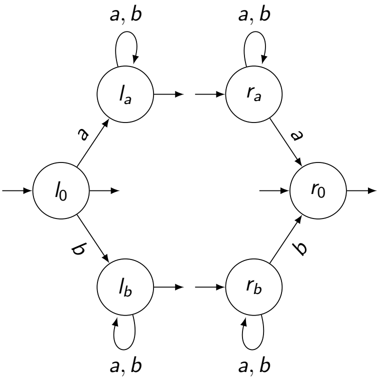
Independent flows

If



then Y depends only on X and a

Bimachine ≃ SST with independent flows



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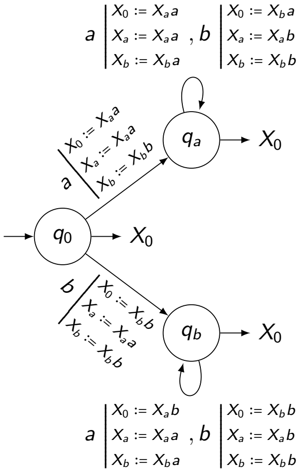
Bimachine

$\simeq$



SST

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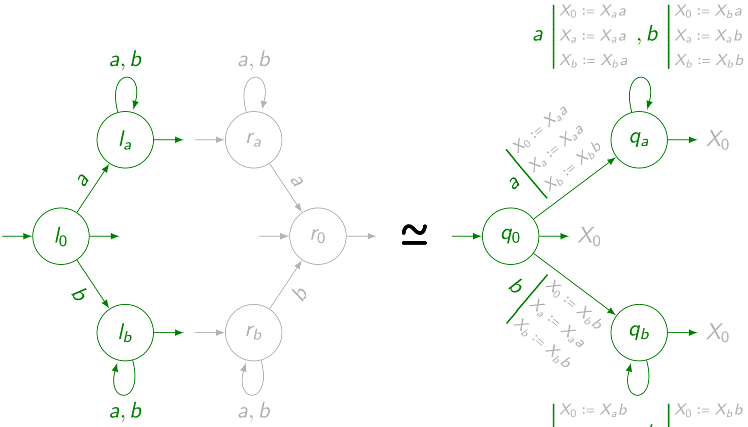

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Bimachine **SST**

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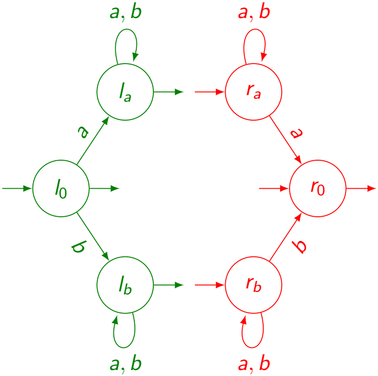

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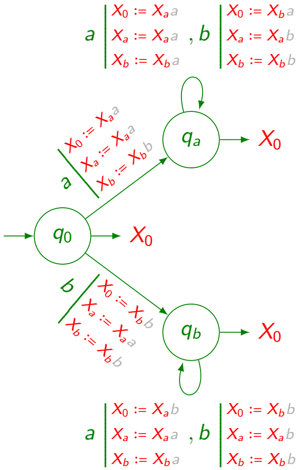

SST

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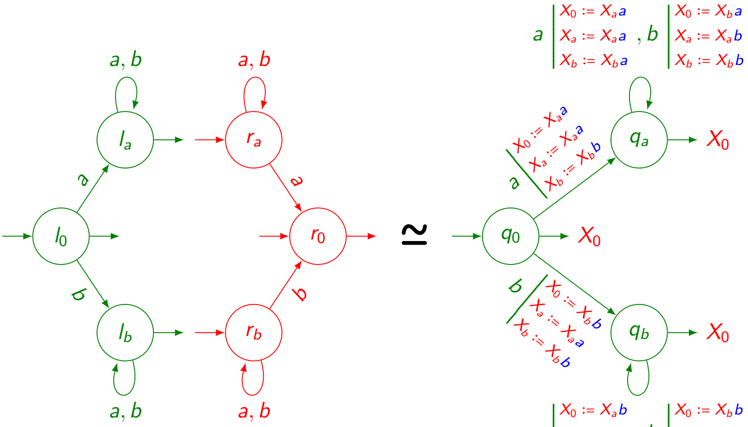

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Bimachine **SST**

Bimachine minimization

Filiot, E., Gauwin, O., & Lhote, N. (2019). Logical and algebraic characterizations of rational transductions. *Log. Methods Comput. Sci.*

- Bimachines can be minimized in PTIME
- Left / right minimization → Tradeoff between the size of the left and the right automaton

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- Bimachines can be minimized in PTIME
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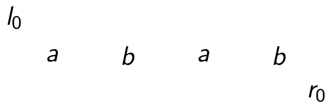
Corollary

For SST with independent flows :

⇒ Minimal number of registers = Number of states of the minimal right automaton

⇒ Tradeoff between the size of the underlying automaton and the number of registers

Asynchronous bimachines



Execution of a bimachine realizing *swap*

Asynchronous bimachines

l_0 l_a

a b a b

r_b r_0

Execution of a bimachine realizing *swap*

Asynchronous bimachines

l_0 l_a l_a

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r_b r_b r_0

Execution of a bimachine realizing *swap*

Aix*Marseille université Socialement engagée

Asynchronous bimachines

$$\begin{array}{ccccccccc} l_0 & & l_a & & l_a & & l_a & & \\ & a & & b & & a & & b & \\ & & r_b & & r_b & & r_b & & r_0 \end{array}$$

Execution of a bimachine realizing *swap*

Asynchronous bimachines

l_0		l_a		l_a		l_a		l_a
	a		b		a		b	
r_b		r_b		r_b		r_b		r_0

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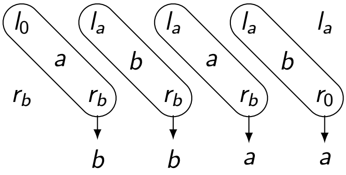
Asynchronous bimachines



Execution of a bimachine realizing ***swap***

Asynchronous bimachines

a b a b

Execution of an asynchronous bimachine realizing *swap*

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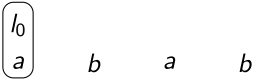

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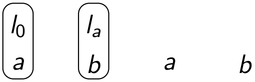
Asynchronous bimachines



Execution of an asynchronous bimachine realizing *swap*

Asynchronous bimachines

Yahia Idriss BENALIOUA

l_0	l_a	l_a	
a	b	a	b

Execution of an asynchronous bimachine realizing *swap*

Asynchronous bimachines

$\begin{pmatrix} l_0 \\ a \end{pmatrix}$ $\begin{pmatrix} l_a \\ b \end{pmatrix}$ $\begin{pmatrix} l_a \\ a \end{pmatrix}$ $\begin{pmatrix} l_a \\ b \end{pmatrix}$

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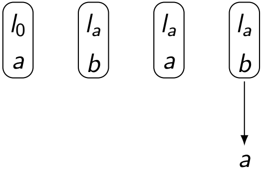

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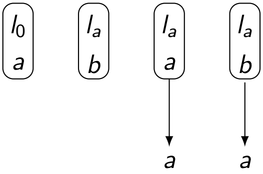

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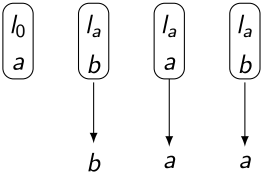

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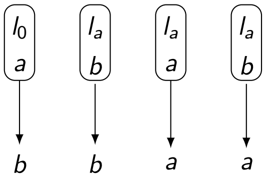

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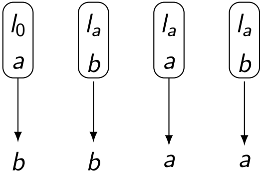

Execution of an asynchronous bimachine realizing *swap*

Asynchronous bimachines ≃ SST

Minimization of asynchronous bimachines

(ongoing work)

$$a^n \mapsto \begin{cases} a^n & \text{if } n \equiv 0 \mod 3 \\ b^n & \text{otherwise} \end{cases}$$

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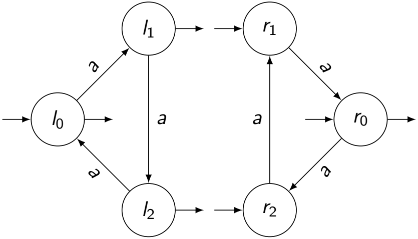

ω	l_0	l_1	l_2
r_0	b	b	a
r_1	b	a	b
r_2	a	b	b

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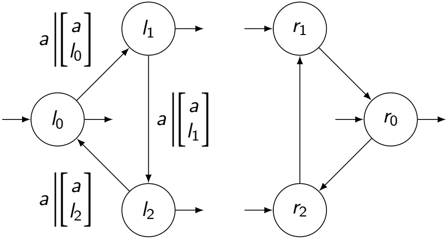

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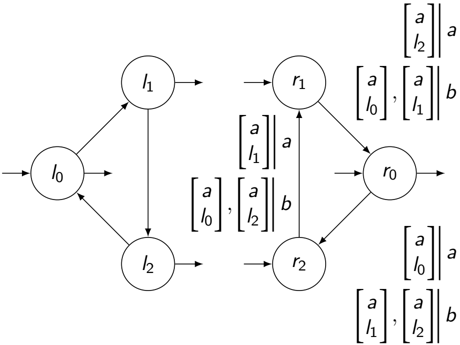

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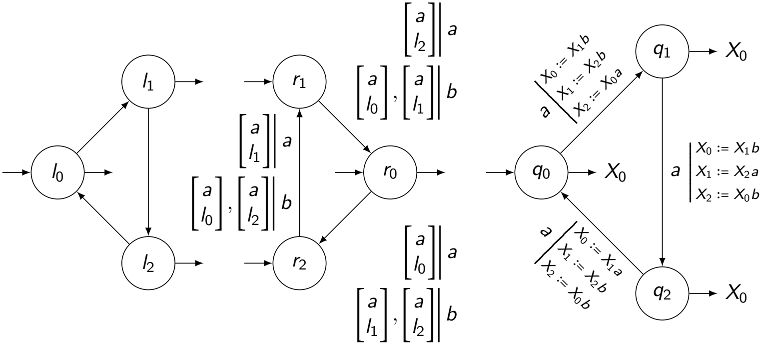

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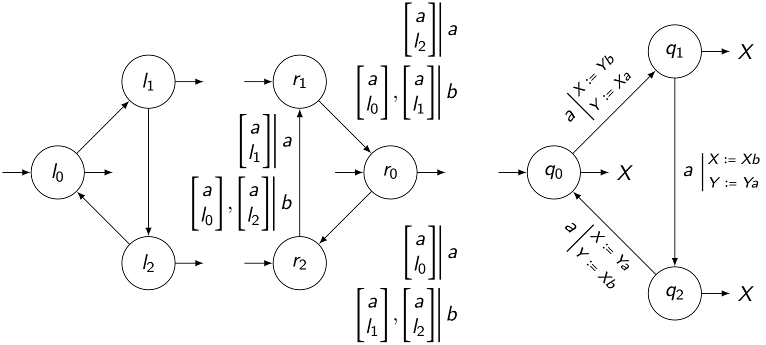

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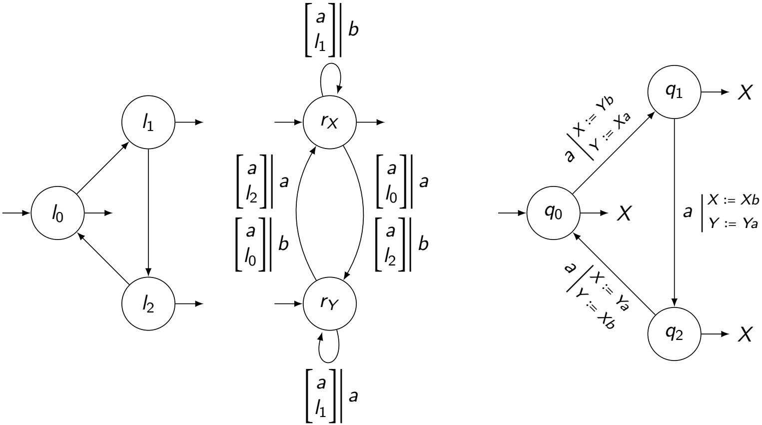

Minimization of asynchronous bimachines

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Problem

Input : A partial sequential function f, an integer k

Question : $\exists$? a sequential transducer, with k states, realizing an extension of f.

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- NP-complete for letter-to-letter transducers

Pfleeger, C. P. (1973). State reduction in incompletely specified finite-state machines. *IEEE Trans. Computers*

Conclusion

Principal results:

- Bimachine ≃ SST with independent flows → register minimization for this class
- Asynchronous bimachine ≃ SST

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Perspectives:

- Minimization of asynchronous bimachines
- Links with other problems

Thank you for your attention