Taylor Expansion is Game Semantics

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Following the work presented in [BCVA23] on the link between resource λ-calculus and game semantics, we show that the Taylor expansion of a λ-term is isomorphic to its interpretation in pointer concurrent games.

Resource λ-calculus arose from linear logic and differential λ-calculus, and has a strongly finitary behavior. Tsukada and Ong showed in [TO16] that (β-normal, η-long) resource terms correspond to plays of Hyland-Ong games (up to Mellies' homotopy equivalence). These results were extended in [BCVA23], using *pointer concurrent games*, a game model inspired from concurrent games which represents plays quotiented by homotopy. Normal, η-long resource terms are isomorphic to *augmentations* (canonical representatives of Hyland-Ong plays up to homotopy), and that isomorphism is compatible with the β reduction, giving us the following diagram:

$$\begin{tikzcd} s \arrow[r, mapsto, "\mathcal{N}"] \arrow[d, mapsto, "\llbracket - \rrbracket"'] \arrow[dr, phantom, "(\star)"] & \mathcal{N}(s) \arrow[d, phantom, "\wr"] \\ \llbracket s \rrbracket \arrow[r, phantom, "="] & \llbracket \mathcal{N}(s) \rrbracket \end{tikzcd}$$

where s is an η-long resource term, $\mathcal{N}$ is the normalisation, and $\llbracket - \rrbracket$ is the interpretation in pointer concurrent games.

Taylor expansion translates a λ-term (with possibly infinite behavior) to an infinite sum of resource λ-terms. In this work in progress, we extend the previous isomorphism to show that game semantics is compatible with Taylor expansion in the following sense:

$$\begin{tikzcd} M \arrow[r, mapsto, "\mathcal{T}"] \arrow[d, mapsto, "\llbracket - \rrbracket"'] \arrow[dr, phantom, "(1)"] & \mathcal{T}(M) \arrow[r, mapsto, "\mathcal{N}"] \arrow[d, mapsto, "\llbracket - \rrbracket"] \arrow[dr, phantom, "(2)"] & \mathcal{N}(\mathcal{T}(M)) \arrow[d, phantom, "\wr"] \\ \llbracket M \rrbracket \arrow[r, phantom, "="] & \llbracket \mathcal{T}(M) \rrbracket \arrow[r, phantom, "="] & \llbracket \mathcal{N}(\mathcal{T}(M)) \rrbracket \end{tikzcd}$$

where M is a λ-term and $\mathcal{T}$ is the Taylor expansion. To do so, we define a Taylor expansion sending simply-typed λ-terms to terms of the simply-typed, η-long resource calculus, and we show that (1) commutes – then (2) is obtained from $(\star)$.

References

[BCVA23] Lison Blondeau-Patissier, Pierre Clairambault, and Lionel Vaux Auclair. Strategies as Resource Terms, and Their Categorical Semantics. In Marco Gaboardi and Femke van Raamsdonk, editors, *8th International Conference on Formal Structures for Computation and Deduction (FSCD 2023)*, volume 260 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 13:1–13:22, Dagstuhl, Germany, 2023. Schloss Dagstuhl – Leibniz-Zentrum für Informatik.

[TO16] Takeshi Tsukada and C.-H. Luke Ong. Plays as resource terms via non-idempotent intersection types. In Martin Grohe, Eric Koskinen, and Natarajan Shankar, editors, *Proceedings of the 31st Annual ACM/IEEE Symposium on Logic in Computer Science, LICS '16, New York, NY, USA, July 5-8, 2016*, pages 237–246. ACM, 2016.