Tiling groups
Some results about tilings of groups

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Let’s start with Grids
Subshifts of Finite Type – Configurations

Finite alphabet: \( \mathcal{A} = \{ \text{\textcolor{blue}{blue}}, \text{\textcolor{green}{green}}, \text{\textcolor{magenta}{magenta}}, \text{\textcolor{orange}{orange}} \} \)

Configuration: \( x \in \mathcal{A}^\mathbb{Z}^d \)
Subshifts of Finite Type

Finite alphabet: \( \mathcal{A} = \{ \text{\color{blue} \text{\textbullet}}, \text{\color{green} \text{\textbullet}}, \text{\color{magenta} \text{\textbullet}}, \text{\color{orange} \text{\textbullet}} \} \)

Set of forbidden patterns: \( F = \{ \text{\color{blue} \text{\textbullet}} \} \)

Subshift: \( \mathcal{X}_F = \{ x \in \mathcal{A}^{\mathbb{Z}^d} \mid \forall m \in F, m \text{ does not appear in } x \} \)
Subshifts of Finite Type

Finite alphabet: $\mathcal{A} = \{\text{\footnotesize \textcolor{cyan}{\text{blue}}} \text{\footnotesize \textcolor{green}{green}} \text{\footnotesize \textcolor{brown}{brown}} \text{\footnotesize \textcolor{orange}{orange}}\}$

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Subshift:

$X_F = \{x \in \mathcal{A}^{\mathbb{Z}^d} \mid \forall m \in F, m \text{ does not appear in } x\}$
Subshifts of Finite Type

Finite alphabet: \( \mathcal{A} = \{ \text{\color{blue} \text{\#}}, \text{\color{green} \text{\#}}, \text{\color{purple} \text{\#}}, \text{\color{orange} \text{\#}} \} \)

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Subshifts of Finite Type

Finite alphabet: $\mathcal{A} = \{\text{\[image\]}}\}

Set of forbidden patterns: $F = \{\text{\[image\]}}\}

Subshift $X_F = \{x \in \mathcal{A}^\mathbb{Z}^d \mid \forall m \in F, m \text{ does not appear in } x\}$
Subshifts of Finite Type

Finite alphabet: \( \mathcal{A} = \{ \text{blue, green, purple, orange} \} \)

Set of forbidden patterns: \( F = \{ \text{blue, orange, purple, green} \} \)

Subshift: \( X_F = \{ x \in \mathcal{A}^\mathbb{Z}^d \mid \forall m \in F, m \text{ does not appear in } x \} \)
Subshifts of Finite Type

Finite alphabet: \( A = \{ \text{square1, square2, square3, square4} \} \)

Finite Set of forbidden patterns: \( F = \{ \text{square5, square6, square7, square8} \} \)

Subshift of Finite Type (SFT):

\[ X_F = \{ x \in A^{\mathbb{Z}^d} \mid \forall m \in F, m \text{ does not appear in } x \} \]
Periodicity (1D)

\[ x \in \mathcal{A}^\mathbb{Z} \text{ is:} \]

- **periodic** \( \exists u, \forall v, \ x_{v-u} = x_v \)
Periodicity (1D)

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**Theorem**

*If let \( X \subset \mathcal{A}^\mathbb{Z} \) be an SFT.*

\[ X \neq \emptyset \Rightarrow \exists c \in X, c \text{ periodic} \]
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**Theorem**

If let \( X \subset \mathcal{A}^\mathbb{Z} \) be an SFT.

\[ X \neq \emptyset \Rightarrow \exists c \in X, \ c \text{ periodic} \]

There are no **aperiodic** 1D SFT
Periodic Configuration (2D)

$x \in A^\mathbb{Z}^2$ is:

- **1-periodic / weakly periodic**: $\exists u, \forall v, \ x_{v-u} = x_v$

- **2-periodic / strongly periodic**: $x$ is 1-periodic along $u_1, u_2$, not colinear $\Rightarrow$ finitely many different translations of $x$
Periodic Configuration (2D)

\( x \in A^{\mathbb{Z}^2} \) is:

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Aperiodic Subshift (2D)

\( X \neq \emptyset \) is:

- Weakly aperiodic: \( X \) contains no 2-periodic configuration
- Strongly aperiodic: \( X \) contains no 1-periodic configuration (no periodic configurations at all)
Aperiodic Subshift (2D)

\( X \neq \emptyset \) is:
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Theorem (folklore)

\text{strongly aperiodic SFT} \Rightarrow \text{weakly aperiodic SFT}
Aperiodic Subshift (2D)

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In 2D, strongly aperiodic SFT \( \iff \) weakly aperiodic SFT
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**Theorem (folklore)**
In 2D, strongly aperiodic SFT \( \Leftrightarrow \) weakly aperiodic SFT

**Theorem (Berger, 1966)**
In 2D, there are aperiodic SFTs
The Domino Problem

**Domino Problem**

Given a finite alphabet $\mathcal{A}$ and a finite set of forbidden patterns $F$, is $X_F$ non-empty?
The Domino Problem

Domino Problem
Given a finite alphabet $A$ and a finite set of forbidden patterns $F$, is $X_F$ non-empty?

Domino Problem 1D
In 1D, the domino problem is decidable

Domino Problem 2D (Berger, 1966)
In 2D, the domino problem is undecidable
Now the interesting part:
Groups
Infinite Structures
Cayley Graph of a Group

\[ G = \langle a, b \rangle \]
Cayley Graph of a Group

\[ G = \langle a, b \mid aba^{-1}b^{-1} = 1_G \rangle \cong \mathbb{Z}^2 \]
Tiling the Cayley Graph of a Group

**Subshifts:**
- Every vertex has a color
- Forbid a set of patterns to appear in the colorings
Periodicity, Aperiodicity

- Orbit: $\mathcal{O}(x) = \{ g \cdot x \mid g \in G \} \subseteq A^G$

A configuration $x \in A^G$ is:

- **weakly periodic:** $\exists g \in G, g \cdot x = x$
- **strongly periodic:** $|\mathcal{O}(x)| < \infty$
Periodicity, Aperiodicity

- Orbit: $\mathcal{O}(x) = \{g \cdot x \mid g \in G\} \subseteq A^G$

A configuration $x \in A^G$ is:
- weakly periodic: $\exists g \in G, g \cdot x = x$
- strongly periodic: $|\mathcal{O}(x)| < \infty$

A subshift $X \neq \emptyset$ is:
- weakly aperiodic: $X$ contains no strongly periodic configuration
- strongly aperiodic: $X$ contains no weakly periodic configuration
Domino Problem for Groups

Domino Problem (DP)

Given a finite alphabet $\mathcal{A}$ and a finite set of forbidden patterns $F$, is $X_F \subseteq \mathcal{A}^G$ non-empty?
Questions

Take $G$ your favorite (infinite) group

**Periodicity:**
- Is there a weakly aperiodic SFT?
- Is there a strongly aperiodic SFT?
- Are the two definitions equivalent?

**Domino Problem:**
- Is it decidable?
Many Questions

More generally, characterize groups $G$ for which

**Periodicity:**
- There is a weakly aperiodic SFT
- There is a strongly aperiodic SFT
- The two definitions equivalent

**Domino Problem:**
- Is (un)decidable
So Many Questions

- What is the link between weak aperiodicity and (un)decidability of DP?
- What is the link between strong aperiodicity and (un)decidability of DP?
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- What is the link between weak aperiodicity and (un)decidability of DP?
- What is the link between strong aperiodicity and (un)decidability of DP?

Usually, we find an aperiodic SFT before knowing that DP is undecidable.
Sometimes weak, sometimes strong.
(un)Decidability of the domino problem
DP undecidable

- Groups with undecidable Word Problem
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- $\mathbb{Z}^d$, $d \geq 2$ [Berger 1966]
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Origin-constrained DP:
- Lamplighter group [Bartholdi, Salo 2020]
DP decidable

- Virtually free groups
DP decidable

- Virtually free groups

Conjecture [Ballier, Stein 2013]

\[ G \text{ has decidable domino problem } \iff G \text{ is virtually free} \]
Inheritance properties

(Everything is finitely generated)

Subgroup [Jeandel 2015]

If $H$ is a subgroup of $G$ and $H$ has undecidable DP, $G$ has undecidable DP
Inheritance properties

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Subgroup of finite index [Aubrun, Barbieri, Jeandel 2018]
\( H \) subgroup of finite index of \( G \).
\( H \) has undecidable DP \( \Leftrightarrow \) \( G \) has undecidable DP
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$H$ subgroup of finite index of $G$.

$H$ has undecidable DP $\iff$ $G$ has undecidable DP

**Quotient [Aubrun, Barbieri, Jeandel 2018]**
$N$ a normal subgroup of $G$.

$G/N$ has undecidable DP $\Rightarrow$ $G$ has undecidable DP
"Finding” $\mathbb{Z}^2$

Most techniques rely on "finding” $\mathbb{Z}^2$ in the group

\[ \mathbb{Z}^2 \]

\[ \mathbb{Z}^d \quad G_1 \times G_2 \quad \text{Non-virtually } \mathbb{Z} \text{ polycyclic} \quad \text{Lamplighter} \]
For $\mathbb{H}^2$, Kari does not encode directly a Turing machine. It encodes computation in a hyperbolic space.

\[ BS(m, n) \quad \text{surface groups} \]
(a) Periodicity
Where it is impossible to make aperiodic SFTs

No weakly nor strongly aperiodic SFT:

- virtually $\mathbb{Z}$

Conjecture [Carroll, Penland 2015]

A finitely generated group has no weakly aperiodic SFT

$\iff$

it is finite or virtually $\mathbb{Z}$
Where it is impossible to make aperiodic SFTs

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**Conjecture [Carroll, Penland 2015]**

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$\iff$

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No strongly but $\exists$ weakly aperiodic SFTs:

- Other virtually free groups [Piantadosi 2008]
Groups with weakly aperiodic SFTs

- $BS(m,n)$ [Abrun, Kari 2013]
Groups with weakly aperiodic SFTs

- BS(m,n) [Abrun, Kari 2013]
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Groups with weakly aperiodic SFTs

- BS(m,n) [Abrun, Kari 2013]
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- Lamplighter group [Cohen 2020]
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We don’t know if they can have strongly aperiodic SFTs!
Groups with strongly aperiodic SFTs

**Theorem (Jeandel 2015)**

*If G have a strongly aperiodic SFT, then its Word Problem is decidable.*
Groups with strongly aperiodic SFTs

**Theorem (Jeandel 2015)**

If $G$ have a strongly aperiodic SFT, then its Word Problem is decidable.

**Theorem (Cohen 2017)**

Groups with $\geq 2$ ends cannot have strongly aperiodic SFT.
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- \( \mathbb{Z}^2 \rtimes G \) [Barbieri, Sablik 2018]
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- Hyperbolic groups [Cohen, Goodman-Strauss, Rieck 2017]
- \( \mathbb{Z}^2 \times G \) [Barbieri, Sablik 2018]
- \( G_1 \times G_2 \times G_3 \) [Barbieri 2019]
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- $\mathbb{Z}^2 \rtimes G$ [Barbieri, Sablik 2018]
- $G_1 \times G_2 \times G_3$ [Barbieri 2019]
- $BS(1, n), BS(n, n)$ [Esnay, M. 2020]
Inheritance properties (1)

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If $H$ is a subgroup of $G$ and $H$ has a weakly aperiodic SFT, $G$ has a weakly aperiodic SFT.

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$H$ subgroup of finite index of $G$.
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Inheritance properties (2)

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**Quotient, weak** [Caroll, Penland 2015]

N a normal subgroup of $G$.

$G/N$ has weakly aperiodic SFT $\Rightarrow G$ has weakly aperiodic SFT

**Quotient, strong** [Jeandel 2015]

N a normal subgroup of $G$.

N and $G/N$ have a strongly aperiodic SFT

$\Rightarrow$

$G$ has a strongly aperiodic SFT
Inheritance properties (3)

(Everything is finitely generated)

Direct product [Jeandel 2015]

If $G_1$ and $G_2$ have strongly aperiodic SFT, $G_1 \times G_2$ have strongly aperiodic SFT
Mixing the two !
# A nice table to finish

<table>
<thead>
<tr>
<th>DP</th>
<th>Aperiocity</th>
<th>( \exists ) Strongly aperiodic SFT</th>
<th>( \exists ) Weakly aperiodic SFT</th>
<th>( \nexists ) Strongly aperiodic SFT</th>
<th>( \nexists ) Weakly aperiodic SFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decidable DP</td>
<td>( ? )</td>
<td>Virtually free groups [Pia08]</td>
<td>Virtually ( \mathbb{Z} ) groups</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decidable WP</td>
<td>(virtually)</td>
<td>( \mathbb{Z}^d, d \geq 2 )</td>
<td>( \mathbb{H}^2 ) [GS05][Kar08]</td>
<td>( \mathbb{H}^2 \otimes H ) [BS19]</td>
<td></td>
</tr>
<tr>
<td>Undecidable DP</td>
<td>( G_1 \times G_2 \times G_3 ) [Jea15][Bar19]</td>
<td>Surface groups [CG17][ABM19]</td>
<td>( BS(m, n) )? Lamplighter Group ?</td>
<td></td>
<td></td>
</tr>
<tr>
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</table>

*(must have one end) [Coh17]*
Next steps?

It would be exciting to show:

- DP undecidable for hyperbolic groups
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- $BS(m, n); m \neq n \neq 1$ do not have strongly aperiodic SFTs
Next steps?

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- Lamplighter have undecidable (general) DP

Thank you! Especially Julien Esnay for the initial table and the nice Master's thesis with many results already gathered.
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