An observer to estimate, what else ?

- Control design
  - Stabilization
  - Feedback
  - Optimal control
- Optimization
  - Static optimization
  - Economic variables
  - Monitoring
- Safety
  - Failure detection
  - Sensor failure
- Giveaway

An observer is
a soft sensor

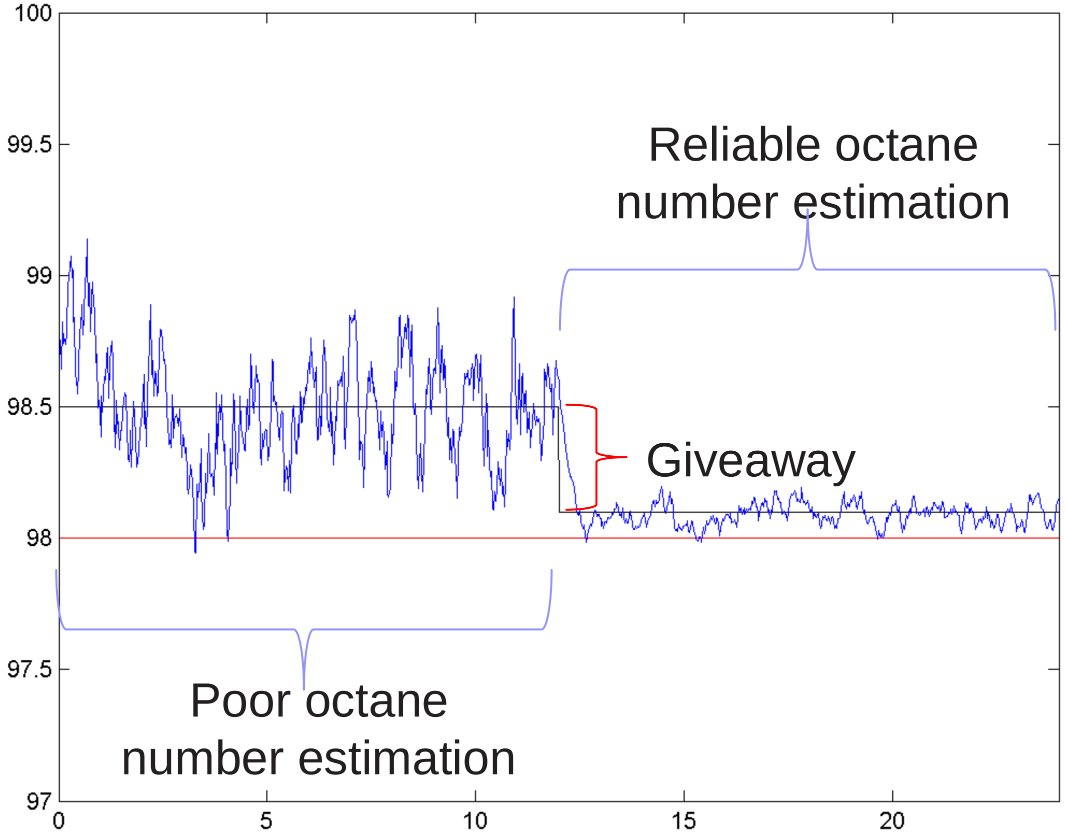
Giveaway

Constraint:

Octane number > 98

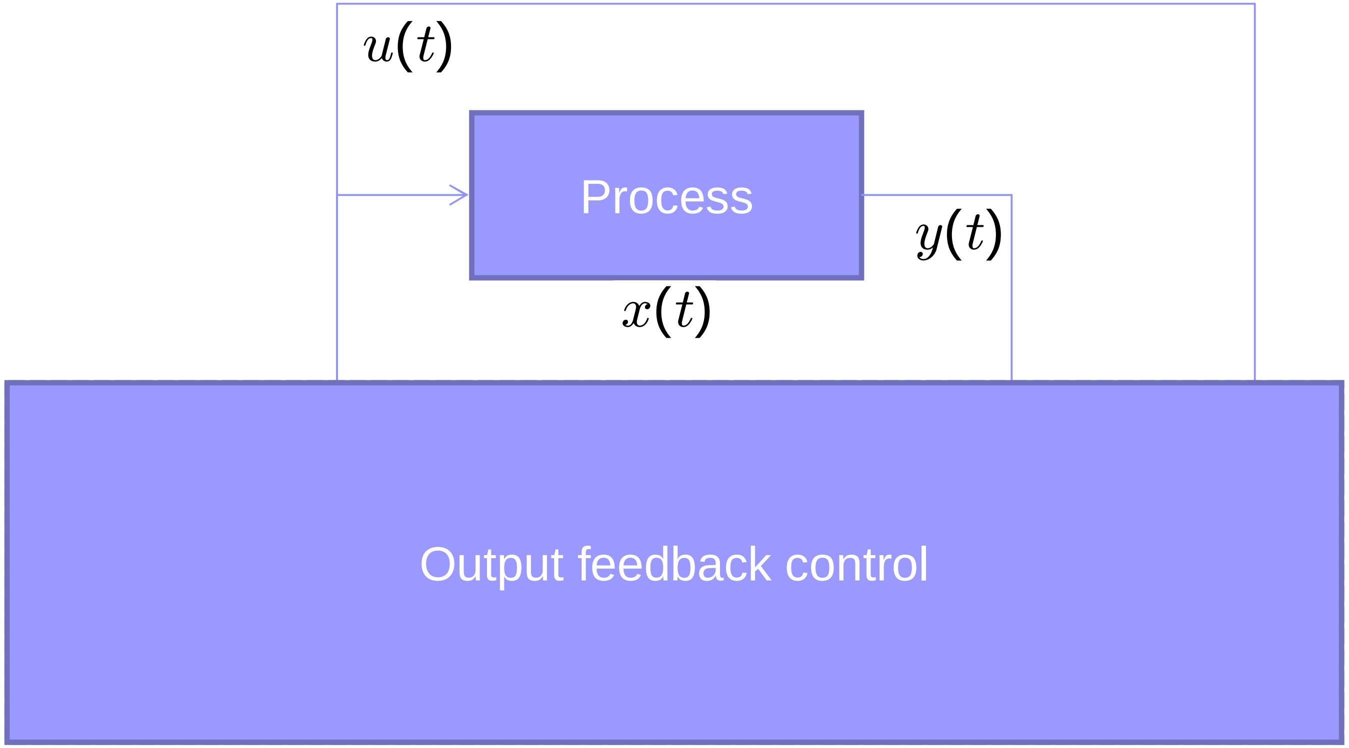
Poor observer:
Control target 98.5

Good observer:
Control target 98.1

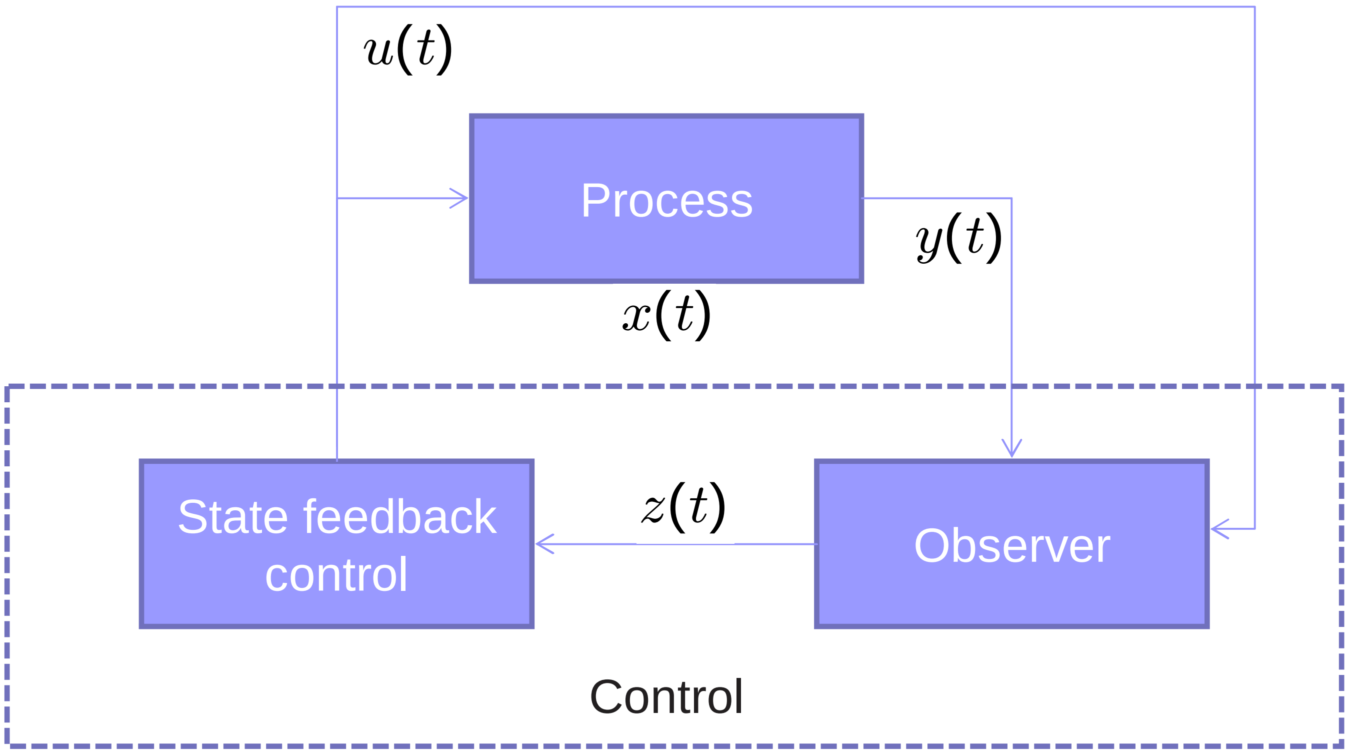


Same remark for a good controller…

Control design



Control design

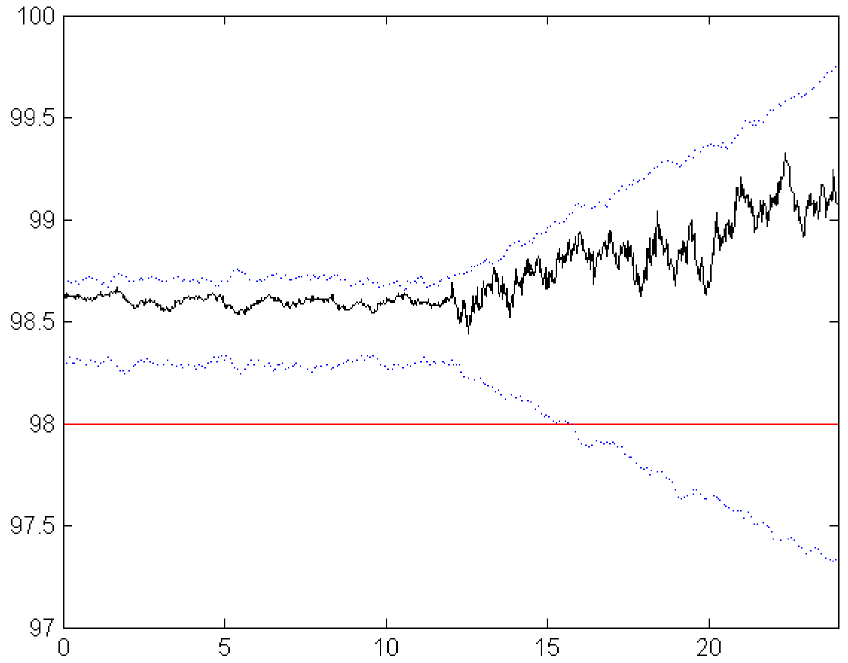


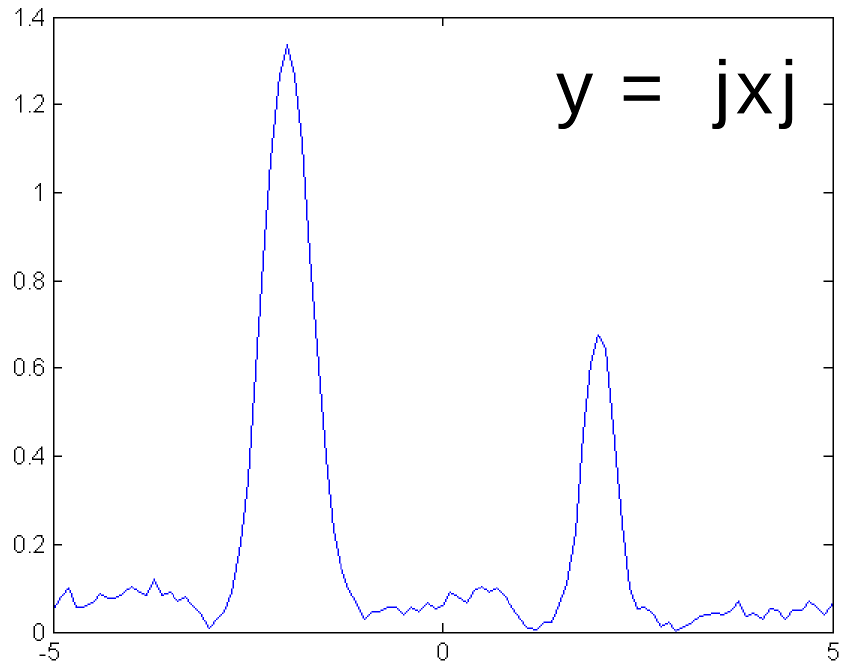
$u(x(t))$ has been replaced by $u(z(t))$ since $x(t)$ is unknown

Monitoring, confidence intervals,…

… any measurement of observer accuracy, as a physical sensor.

- Interval observers (Gouze, Rapaport, …)
- Kalman filtering (Kalman, Bucy, …)
- Nonlinear filtering (DMZ, Clark, Davis, Pardoux, …)
- …





Interval observers I

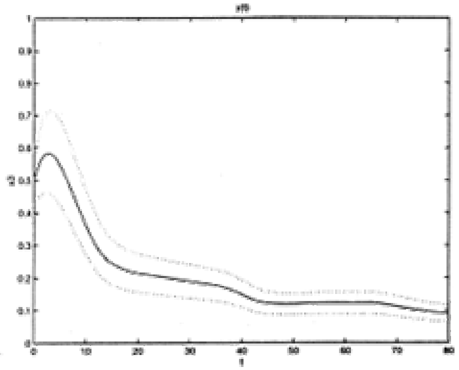
Population of larvae (biomass x_1), juveniles (x_2) and adults (x_3).

$$\begin{cases} \dot{x}_1 &= -\alpha_1 x_1 - m_1 x_1 + \frac{a(t)x_3}{b+x_3} \\ \dot{x}_2 &= \alpha_1 x_1 - \alpha_2 x_2 - m_2 x_2 \\ \dot{x}_3 &= \alpha_2 x_2 - m_3 x_3 - c\left(t\right) x_3 \\ y &= x_3 \end{cases}$$

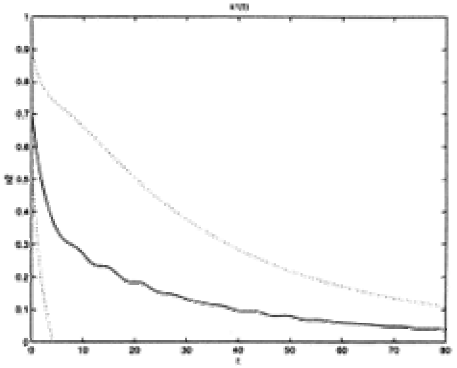
$c(t)$ represents an harvesting effort on the adult population.

α_i, m_i, a, b, c poorly known: just some intervals...

Interval observers II



(a) Measured variable x_3

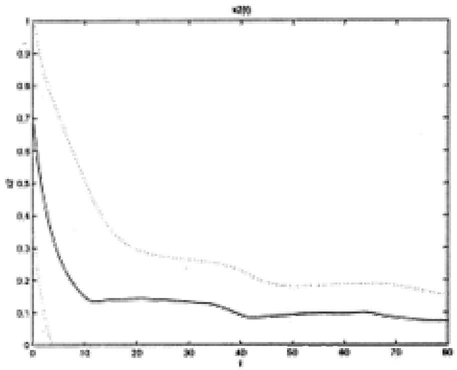


(b) Interval estimates of x_1

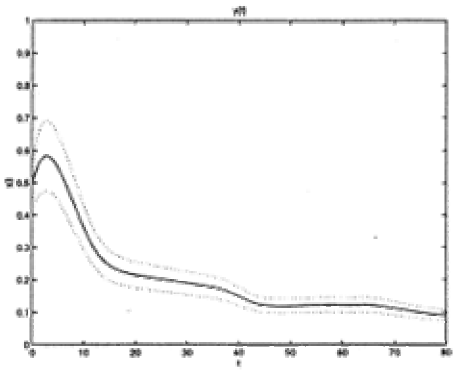
Interval observer

=

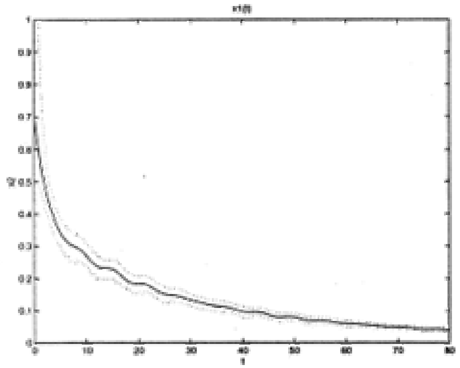
Worst case observer



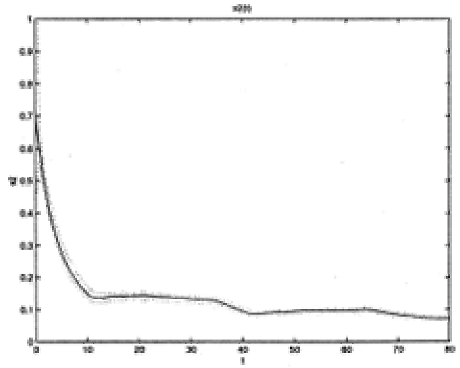
(c) Interval estimates of x_2



(d) Measured variable x_3, higher gains



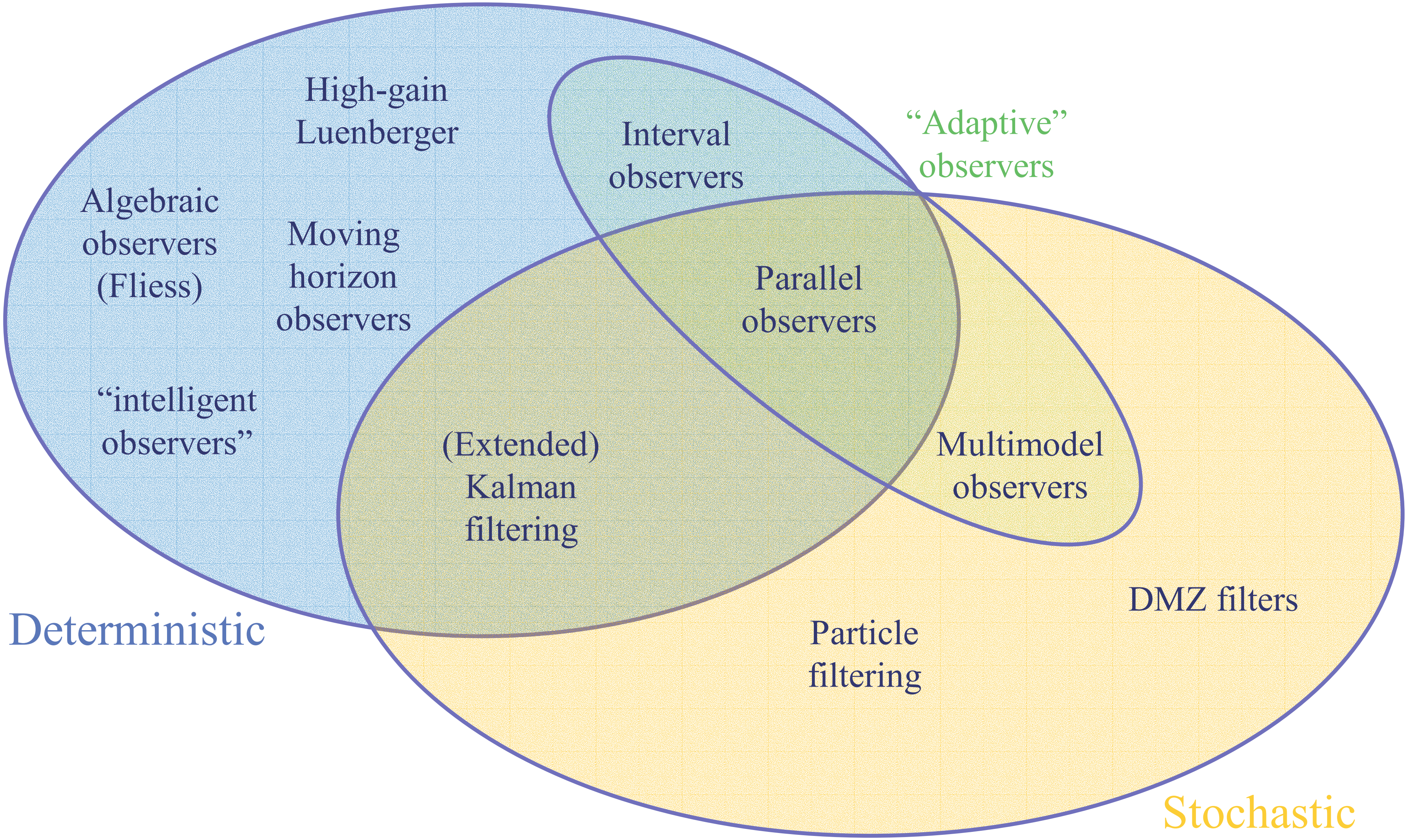
(e) Interval estimates of x_1, higher gains



(f) Interval estimates of x_2, higher gains

Gouze–Rapaport–Hadj-Sadok, "Interval observers for uncertain biological systems".

Different kind of observers



Nonlinear filtering

$$\begin{cases} \frac{dx}{dt} & = & f(x,u) \\ y & = & h(x,u) \end{cases}$$

Deterministic system and observer

$\frac{dz}{dt} = \cdots$ with $\| z(t) - x(t) \| \to 0$ as t goes to infinity.

Stochastic system and filter

$$\begin{cases} dX(t) & = & f(X(t), u)\, dt + Q^{\frac{1}{2}} dW(t) \\ dY(t) & = & h(X(t), u)\, dt + R^{\frac{1}{2}} dV(t) \end{cases}$$

Probability law of $X(t)$ knowing $Y(s)$ for s from 0 to t.

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Fokker-Planck equation

$$dX(t) = f(X(t), u)\,dt + Q^{\frac{1}{2}} dW(t)$$

$$\frac{d}{dt} p(t,x) = L^* p(t,x)\,dt$$

where

$$L = \frac{1}{2} \sum_{i,j=1}^{n} Q_{i,j} \frac{\partial^2}{\partial x_i \partial x_j} + \sum_{i=1}^{n} f_i(x,u) \frac{\partial}{\partial x_i}$$

Then

$$E\left[\phi(X(t))\right] = \frac{\int \phi(x) p(t,x)\,dx}{\int p(t,x)\,dx}$$

Duncan-Mortensen-Zakaï equation

$$\begin{cases} dX(t) & = & f(X(t), u)\,dt + Q^{\frac{1}{2}} dW(t) \\ dY(t) & = & h(X(t), u)\,dt + R^{\frac{1}{2}} dV(t) \end{cases}$$

$$\frac{d}{dt} p(t,x) = L^* p(t,x) dt + h(x,u) p(t,x) dY(t)$$

where

$$L = \frac{1}{2} \sum_{i,j=1}^{n} Q_{i,j} \frac{\partial^2}{\partial x_i \partial x_j} + \sum_{i=1}^{n} f_i(x,u) \frac{\partial}{\partial x_i}$$

Then

$$E\left[\phi(X(t)) \mid \mathcal{F}_t^Y\right] = \frac{\int \phi(x) p(t,x) dx}{\int p(t,x) dx}$$

DMZ equation for discrete-time systems

$$\begin{cases} dX(t) = f(X(t), u)\,dt + Q^{\frac{1}{2}} dW(t) \\ Y_k = C(u) X(t_k) + R^{\frac{1}{2}} V(k) \end{cases}$$

Un-normalized discrete-time DMZ equation:

$$q(t_k, x) = f_{Y_k}^{X(t_k)=x}(y_k) \int f_{X(t_k)}^{X(t_{k-1})=\xi}(x)\, p(t_{k-1}, \xi)\, d\xi$$

where

$$p(t_k, x) = \frac{1}{f_{Y_k}^{Y^{k-1}=y^{k-1}}(y_k)} q(t_k, x) = \frac{q(t_k, x)}{\int q(t_k, \xi)\, d\xi}$$

Particle filtering I

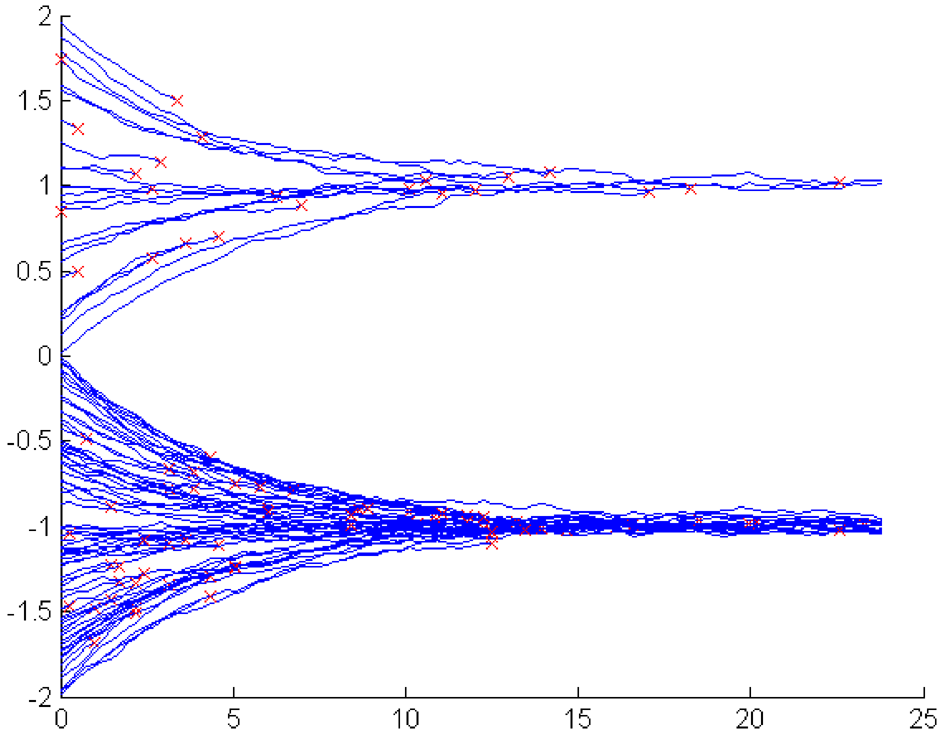
DMZ equation $\Leftrightarrow$ Reject–composition

Dirac approximation

$$q\left(t,x\right)\simeq\sum_{i=1}^{N(k)}\delta_{Z_i(t)}\left(x\right)$$

where $Z_i(t)$ denotes particules.

Particle filtering II



Filtering

=

Killing

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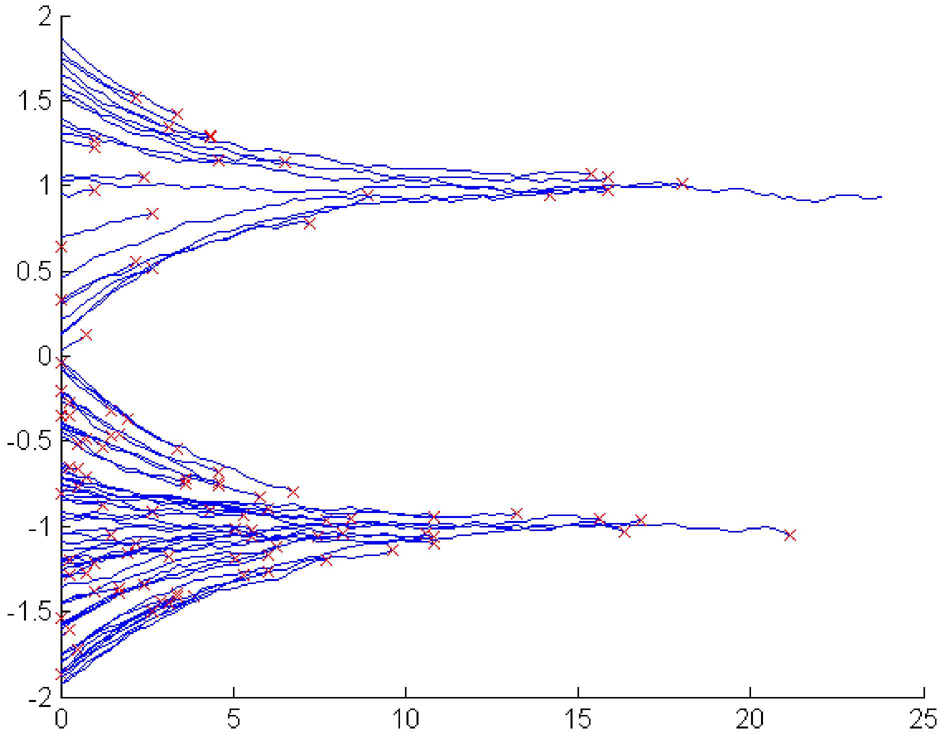
Particle filtering II



Filtering

=

Killing

But sometimes,
no survivals !

Particle filtering III

Weighted sum of Dirac approximation

$$q(t,x) \simeq \sum_{i=1}^{N(k)} b_i(t)\delta_{Z_i(t)}(x)$$

where

- $Z_i(t)$ denotes particules
- $b_i(t)$ denotes weights

Particle filtering IV

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$$P\left(\left\{\exists i, \frac{b_i(t)}{\sum b_j(t)} > 1 - \varepsilon\right\}\right) \longrightarrow 1 \text{ as } t \to \infty$$