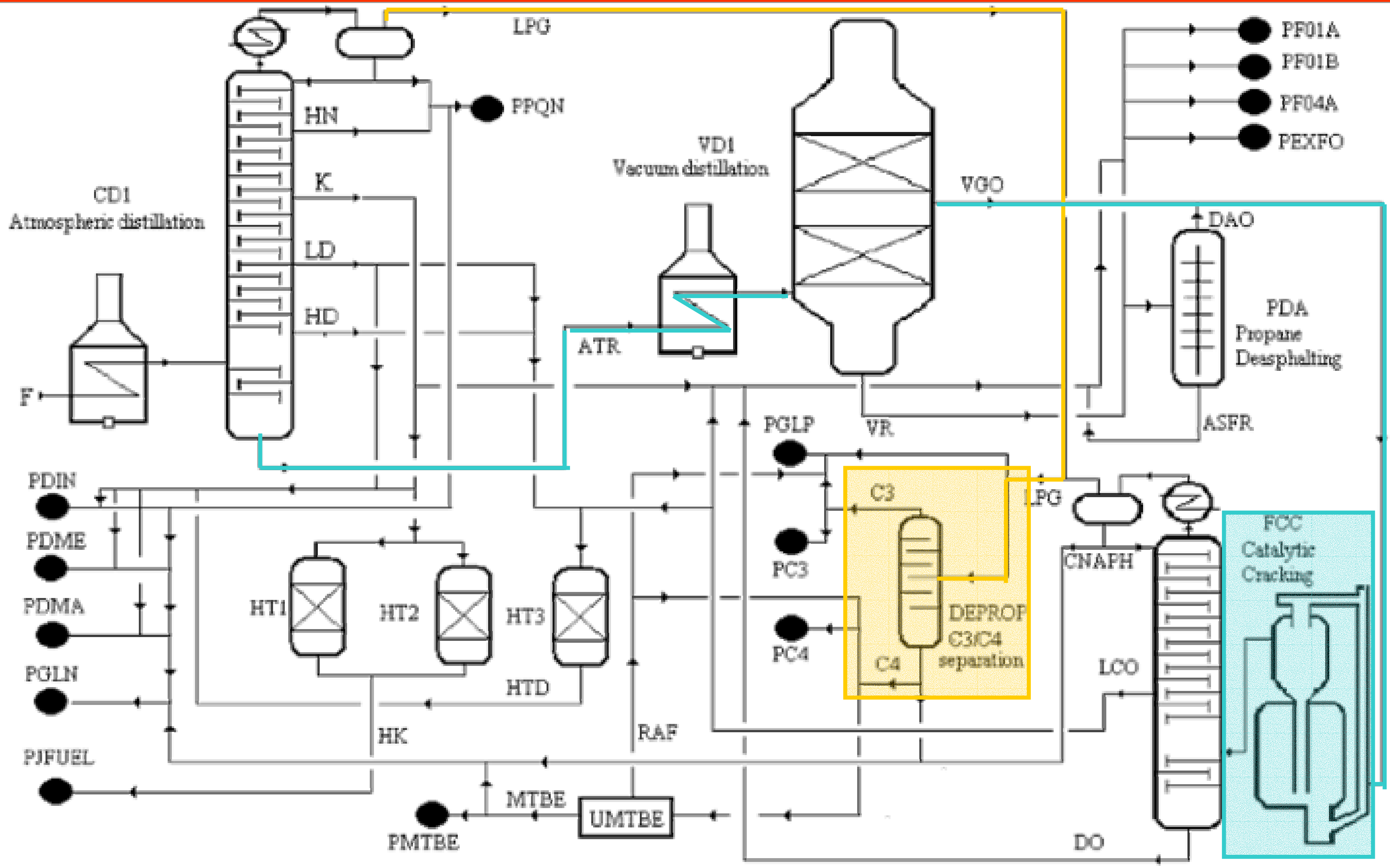
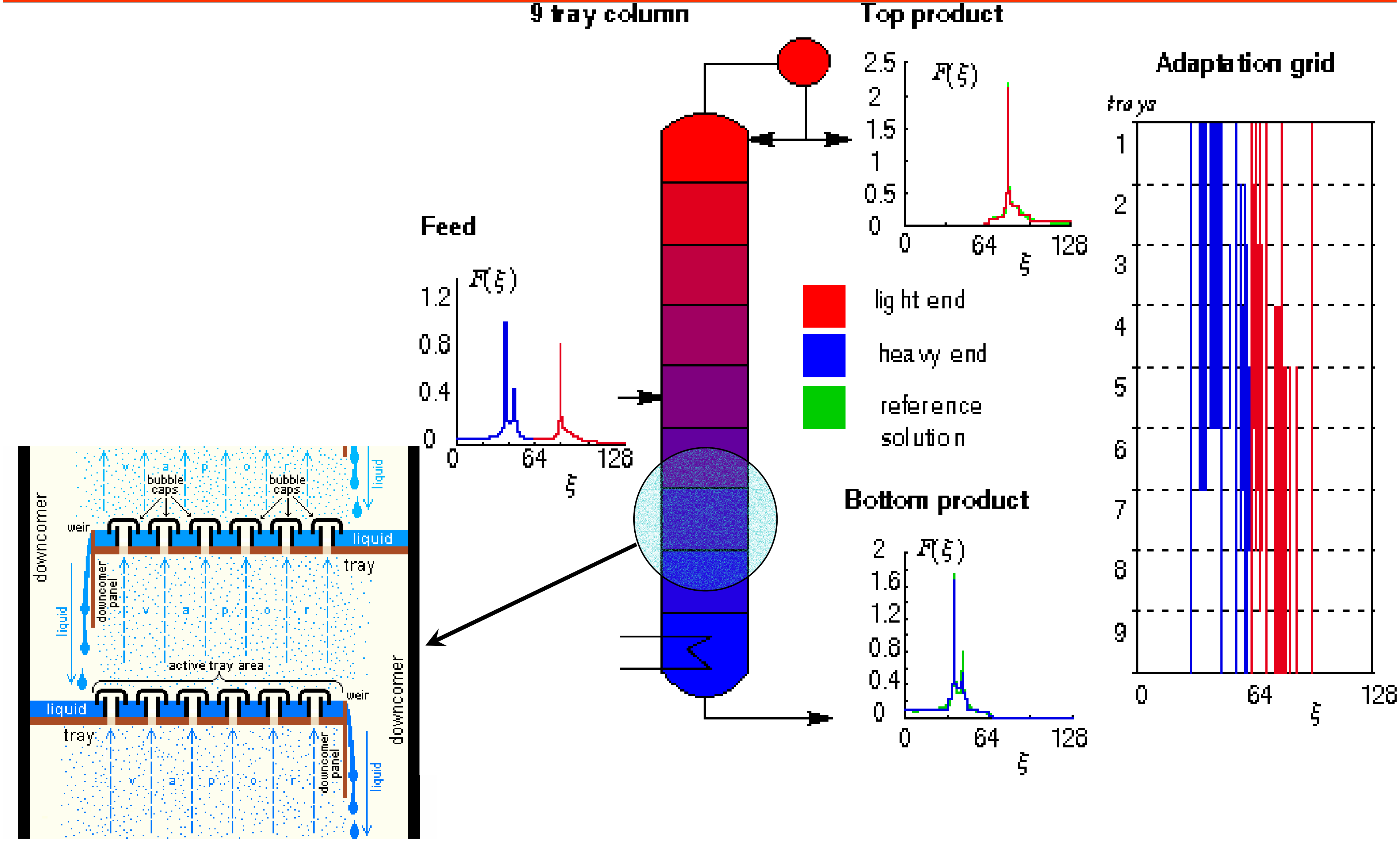
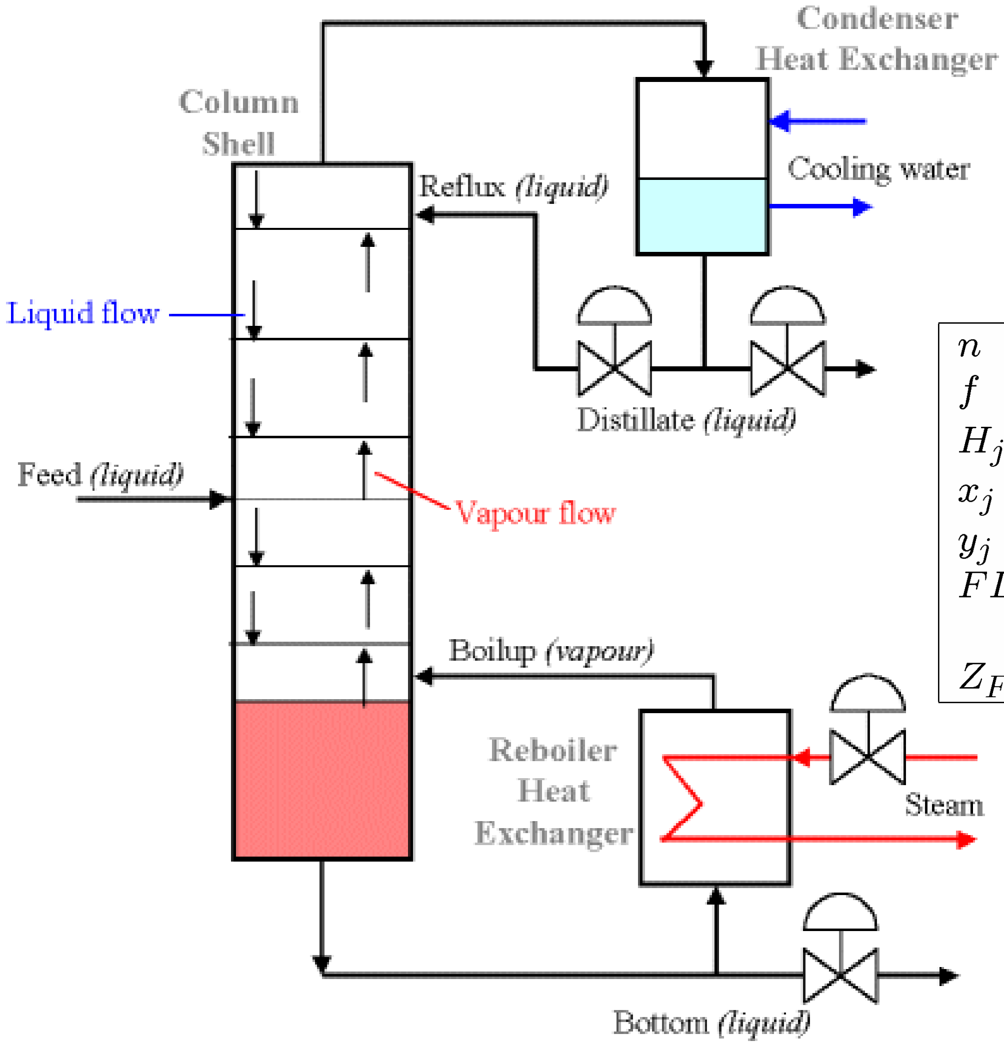
Refinery scheme



Binary distillation column scheme



CMO model of a binary column



Constant molar overflow

Liquid-vapor equilibrium

Constant pressure

n	number of trays,
f	index of the feed tray,
H_j	liquid hold up on the j^{th} tray$^{(*)}$
x_j	liquid composition on the j^{th} tray
y_j	vapor composition on the j^{th} tray
FL, FV, L, V	feed (liquid and vapor), reflux and vapor flow
Z_F	feed composition

(*) Tray number 1 is the condenser
Tray number 2 is the top (theoretical) tray
⋮
Tray number n is the bottom

CMO model I

Total condenser:

$$H_1 \frac{dx_1}{dt} = (V + FV)(y_2 - x_1).$$

Bottom of the column:

$$H_n \frac{dx_n}{dt} = (L + FL)(x_{n-1} - x_n) + V(x_n - y_n).$$

$$\begin{aligned} y_i &= k(x_i) \\ &= \tfrac{\alpha x_i}{1+(\alpha-1)x_i} \end{aligned}$$

$\alpha > 1$ relative volatility.

CMO model II

Rectifying section, $j = 2, \cdots, f-1$:

$$H_j \frac{dx_j}{dt} = L(x_{j-1} - x_j) + (V + FV)(y_{j+1} - y_j).$$

Stripping section, $j = f+1, \cdots, n-1$:

$$H_j \frac{dx_j}{dt} = (L + FL)(x_{j-1} - x_j) + V(y_{j+1} - y_j).$$

Feed tray:

$$\begin{aligned} H_f \tfrac{dx_f}{dt} \; &= FL(Z_F - x_f) + FV(k(Z_F) - y_f) \\ &\quad + L(x_{f-1} - x_f) + V(y_{f+1} - y_f). \end{aligned}$$

CMO model summary

$$\begin{cases} H_1 \dfrac{dx_1}{dt} &=& (V+FV)(y_2 - x_1) \\ H_j \dfrac{dx_j}{dt} &=& L(x_{j-1} - x_j) + (V+FV)(y_{j+1} - y_j) \\ H_f \dfrac{dx_f}{dt} &=& FL(Z_F - x_f) + FV(k(Z_F) - y_f) \\ & & +L(x_{f-1} - x_f) + V(y_{f+1} - y_f) \\ H_j \dfrac{dx_j}{dt} &=& (L+FL)(x_{j-1} - x_j) + V(y_{j+1} - y_j) \\ H_n \dfrac{dx_n}{dt} &=& (L+FL)(x_{n-1} - x_n) + V(x_n - y_n) \end{cases}$$

$$y_i = k(x_i)$$

Open loop stability

$$\dot{x} = f(x) + L\, g_L(x) + V\, g_V(x)$$

with $[0,1]^n$ positively invariant for $L, V \geq 0$. Denote x^* the objective, $u_L = L - L^*$ and $u_V = V - V^*$, $u_L, u_V \subset K$.

$$\dot{x} = f(x) + u_L\, g_L(x) + u_V\, g_V(x)$$

$V_0(x) = \sum_{i=1}^{n} |f_i(x)| \geq 0$ is s. t. $V_0(x) = 0 \Leftrightarrow x = 0$ and

$$D^+ V_0(x) \leq 0$$

so (Lasalle's invariance principle) the CMO model is g.a.s.

State feedback control, first attempt

State feedback:

$$
\begin{aligned}
u_L(x) &= -\text{sign}\left(D^+(V_0, g_L)\right) \inf\left(r\left|D^+(V_0, g_L)\right|, c\right) \\
u_V(x) &= -\text{sign}\left(D^+(V_0, g_V)\right) \inf\left(r\left|D^+(V_0, g_V)\right|, c\right)
\end{aligned}
$$

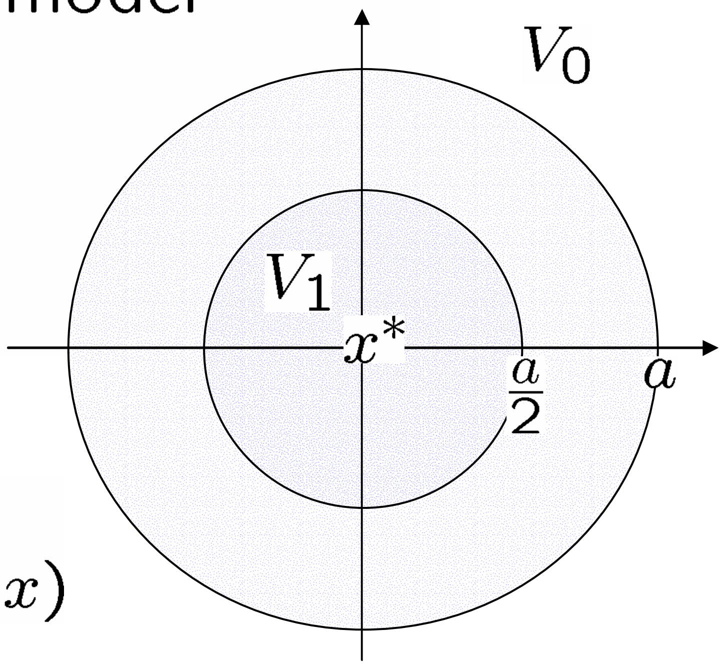
where $D^+(V_0, g) = \sum_{j=1}^{n} \alpha_j L_g f_j$,

$$
\alpha_j = \begin{cases} +1 & \text{if } f_j > 0 \text{ or } f_j = 0 \text{ and } \dot{f}_j > 0 \\ -1 & \text{otherwise} \end{cases}
$$

+ Makes the system g.a.s.
- Not good practical results

State feedback control, solution

$V_1(x) = x' M x$, $M > 0$, being a quadratic Lyapunov function of the linearized model



$$\beta_a(v) = \begin{cases} 0 & \text{if } v \leq \frac{a}{2} \\ 1 & \text{if } v \geq a \\ \text{is } C^\infty \text{ over } [0, +\infty[& \end{cases}$$

$$\begin{aligned} \bar{V}_0(x) &= \rho\, \beta_a(V_1(x))\, V_0(x) \\ &\quad + (1 - \beta_a(V_1(x)))\, V_1(x) \end{aligned}$$

$\bar{V}_0(z)$ makes the closed loop system g.a.s.

(and it works well)