Canonical form for observer construction

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$$\left\{\begin{array}{lcl} \dot{x}_1 & = & F_1(x_1, x_2, u) \\ \dot{x}_2 & = & F_2(x_1, x_2, x_3, u) \\ & \vdots & \\ \dot{x}_n & = & F_n(x, u) \end{array}\right. \qquad \begin{array}{l} \frac{\partial F_1}{\partial x_2} \neq 0 \\ \frac{\partial F_2}{\partial x_3} \neq 0 \end{array}$$

$$\xi_1 = y = x_1,\ \xi_2 = F_1(x_1, x_2, u)$$

$$\xi_3 = \frac{\partial F_1}{\partial x_2} F_2(x_1, x_2, u), \cdots$$

$$\xi_{i+1} = \frac{\partial F_1}{\partial x_2} \cdots \frac{\partial F_{i-1}}{\partial x_i} F_i\left(x_1, \ldots, x_{i+1}, u\right)$$

$$\downarrow$$

$$\left\{\begin{array}{lcl} \dot{\xi}_1 & = & \xi_2 \\ \dot{\xi}_2 & = & \xi_3 + \frac{\partial F_1}{\partial x_1}\dot{x}_1 + \frac{\partial F_1}{\partial u}\dot{u} \\ & \vdots & \\ \dot{\xi}_n & = & G(x, u, \dot{u}) \end{array}\right.$$

Ref. H. **Hammouri, M. Farza,** *Nonlinear observers for local uniform observable systems*

retour

Canonical form of observability

$$\begin{cases} \dfrac{dx}{dt} & = A(t)x + b(x,u) \\ y & = C(t)x \end{cases}$$

$$A(t) = \begin{pmatrix} 0 & a_2(t) & 0 & \cdots & 0 \\ & & a_3(t) & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ & & & & a_n(t) \\ 0 & & \cdots & & 0 \end{pmatrix}$$

$$C(t) = \begin{pmatrix} a_1(t) & 0 & \cdots & 0 \end{pmatrix}$$

$$0 < a_m \leq a_i(t) \leq a_M$$

$$b(x,u) = b_1(x_1,u)\frac{\partial}{\partial x_1} + b_2(x_1,x_2,u)\frac{\partial}{\partial x_2} + b_n(x_1,\ldots,x_n,u)\frac{\partial}{\partial x_n}$$

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Modified Extended Kalman filter

Le2i

$$
\begin{aligned}
\frac{dz}{dt} &= A(t)z + b(z,u) - S(t)^{-1}C(t)'r^{-1}(C(t)z - y(t)) \\
\frac{dS}{dt} &= -(A(t) + b^*(z,u))'S - S(A(t) + b^*(z,u)) \\
&\quad + C(t)'r^{-1}C(t) - SQ_\theta S
\end{aligned}
$$

$$
\Delta = \begin{pmatrix} 1 & & & \\ & \frac{1}{\theta} & & \\ & & \ddots & \\ & & & (\frac{1}{\theta})^{n-1} \end{pmatrix}
\qquad
Q_\theta = \theta^2 \Delta^{-1} Q \Delta^{-1}
$$

If θ is large, high-gain observer (HGEKF)

If $\theta \approx 1$, Classical Extended Kalman filter (EKF)

Theorem

There exist $\lambda_0 > 0$ such that for any $0 \leq \lambda \leq \lambda_0$, there exist θ_0 such that for any $\theta(0) > \theta_0$, for any $S(0) \geq c\ Id$, for any compact $K \subset \mathbf{R}^n$, for any $z(0) \in K$ then if we set $\varepsilon(t) = z(t) - x(t)$ for any $t \geq 0$

$$||\varepsilon(t)||^2 \leq R(\lambda, c) e^{-at} \Lambda(\theta(0), t, \lambda) ||\varepsilon(0)||^2 \quad (1)$$

where

$$\Lambda(\theta(0), t, \lambda) = \theta(0)^{2(n-1)+\frac{a}{\lambda}} e^{-\frac{a}{\lambda}\theta(0)(1-e^{-\lambda t})}$$

and a is a positive constant and $R(\lambda, c)$ is a decreasing function of c.

Passer la preuve

Proof

Change of variables $\begin{cases} \tilde{x} & = \Delta x \\ \tilde{P} & = \frac{1}{\theta}\Delta P \Delta \end{cases} \qquad \left(P = S^{-1}\right)$

$+$ time change $d\tau = \theta(t)\, dt$

We set $\varepsilon = z - x = error$ then we calculate ${}^{T}\varepsilon(\tau)\, S(\tau)\, \varepsilon(\tau)$.

Observability give us $\alpha I \leq S(\tau) \leq \beta I$ then

$$ {}^{T}\varepsilon(\tau)\, S(\tau)\, \varepsilon(\tau) \longrightarrow 0 \Longleftrightarrow \varepsilon(\tau) \longrightarrow 0 $$

When $\tau \leq T$

$$ \|\varepsilon(\tau)\|^2 \leq \theta(\tau)^{2(n-1)} H(c)\, e^{-(a_1\theta(T)-a_2)\tau} \|\varepsilon(0)\|^2 $$

Parallel high-gain and non-high-gain EKF

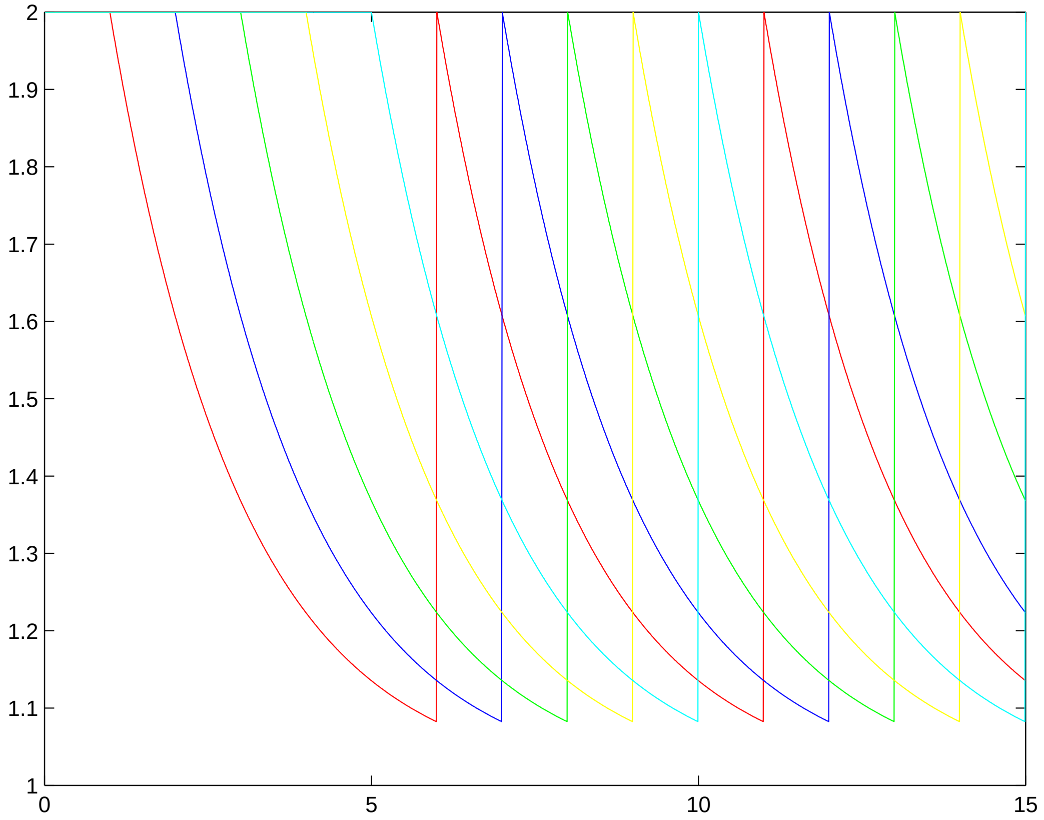
We use N observers in parallel. At times kT :

- a new observer is initialized with $\theta(kT) = \theta_0$,
- the older observer is killed.

Therefore, at any time t, we have N observers initialized at times kT, $(k-1)T \ldots (k-N+1)T$ where $k = \left\lfloor \frac{t}{T} \right\rfloor$.

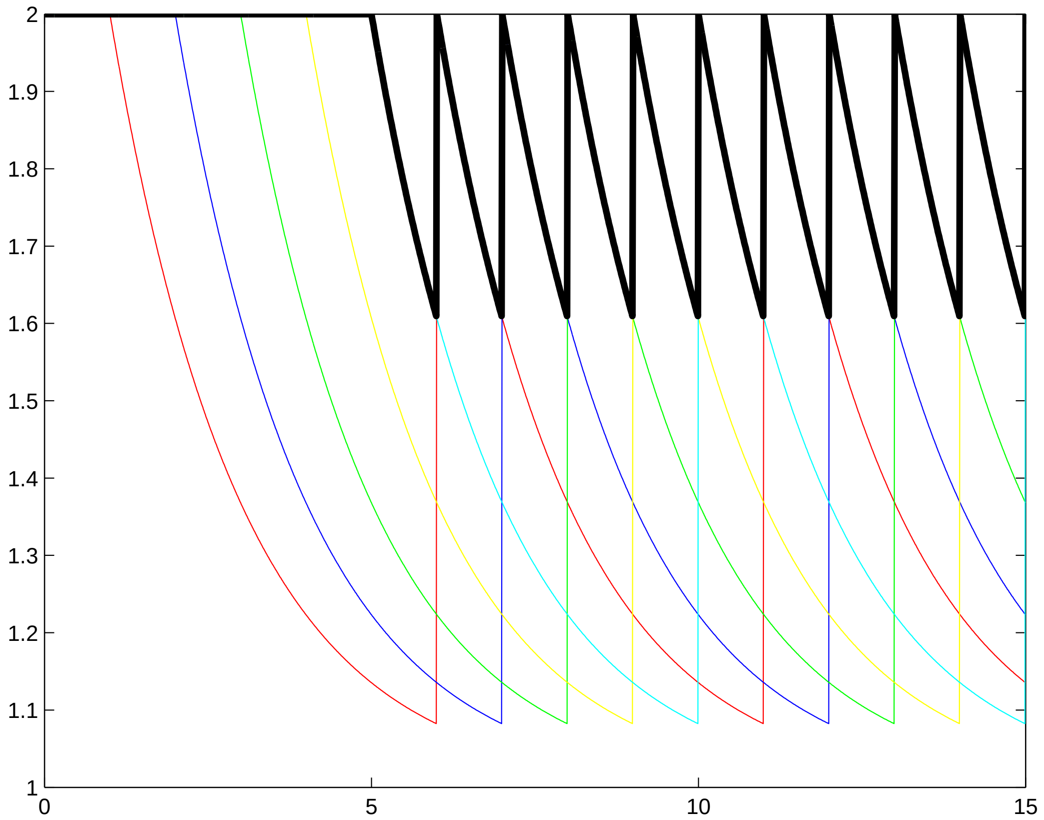
State estimation: the estimation given by the observer with smallest innovation $\|y - C\widehat{x}\|$.

Parallel Extended Kalman Filter



θ for 5 observers

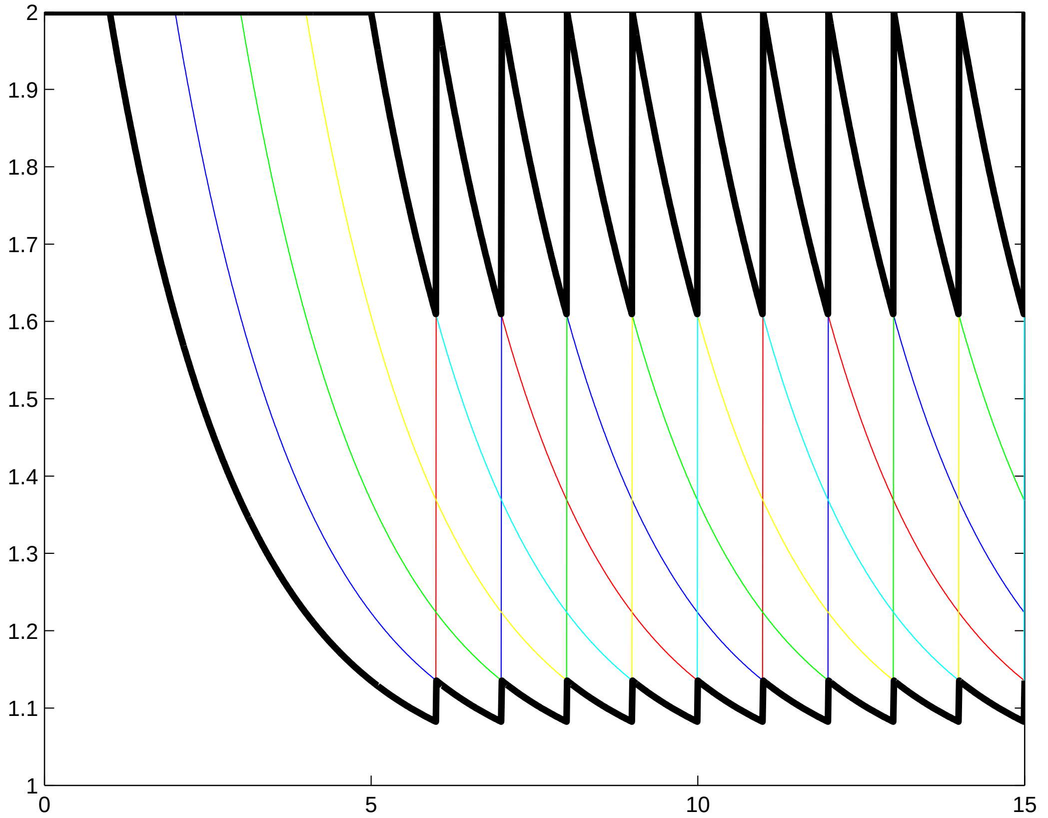
High-gain Extended Kalman Filter



θ for the youngest observer is

$$1 + e^{-\lambda(t-kT)}(\theta_0 - 1) \geq 1 + e^{-\lambda T}(\theta_0 - 1)$$

Standard Extended Kalman Filter



θ for the oldest observer is

$$1 + e^{-\lambda(t-kT+(N+1)T)}\,(\theta_0 - 1) \approx 1$$