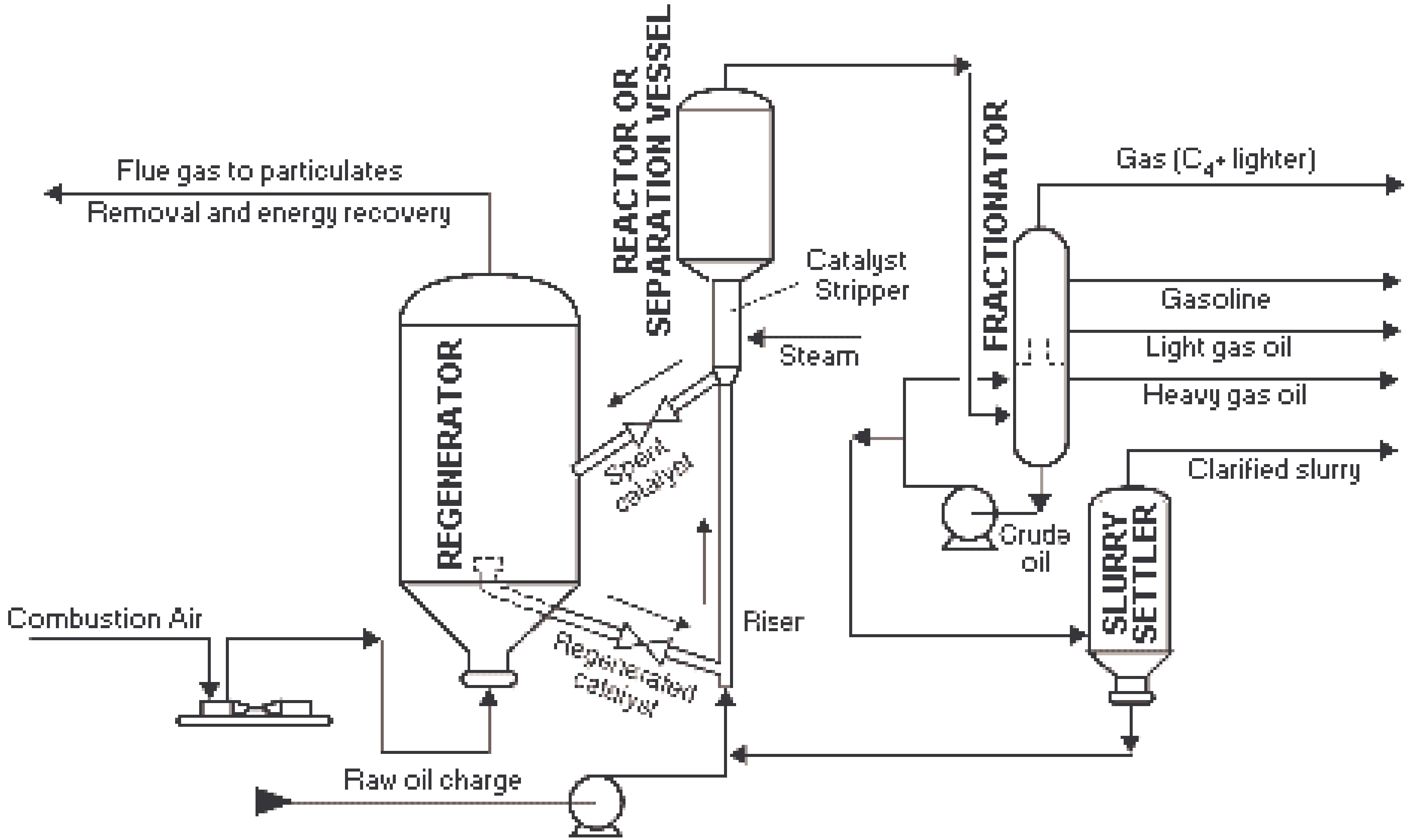
Application: Fluid Catalytic Cracker (FCC)



Reactor model

$$S_c H_{ra} \dot{T}_{ra} = S_c R_c \left(T_{rg} - T_{ra}\right) + S_{tf} R_{tf} \left(T_{tf} - T_{ra}\right) - \Delta H_{fv} R_{tf} - \Delta H_{cr} R_{tf} C_{tf}$$

$$C_{tf} = \frac{1}{1 + \frac{R_{tf}}{R_{cr}}} \qquad R_{cr} = K_{cr} P_{ra} H_{ra}$$

$$C_{cat}^2 = \frac{100 P_{ra} H_{ra}}{R_c C_{rc}^{0.06}} k_{cc} \exp\left(-\frac{A_{cc}}{R T_{ra}}\right) \qquad K_{cr} = \frac{k_{cr}}{C_{cat} C_{rc}^{0.15}} \exp\left(-\frac{A_{cr}}{R T_{ra}}\right)$$

$$H_{ra} \dot{C}_{sc} = R_c \left(C_{rc} - C_{sc}\right) + R_{cf}$$

$$R_{cf} = R_{cc} + R_{ad} \qquad R_{ad} = F_{cf} R_{tf} \qquad R_{cc} = K_{cc} P_{ra} H_{ra}$$

$$K_{cc} = \frac{k_{cc}}{C_{cat} C_{rc}^{0.06}} \exp\left(-\frac{A_{cc}}{R T_{ra}}\right)$$

Regenerator model

$$S_c H_{rg} \dot{T}_{rg} = S_c R_c \left(T_{ra} - T_{rg}\right) + S_a R_{ai} \left(T_{ai} - T_{rg}\right) + \Delta H_{rg} R_{cb}$$

$$R_{cb} = \frac{R_{ai}}{242}\left(21 - O_{fg}\right) \qquad O_{fg} = 21 \exp\left(\frac{-\frac{P_{rg} H_{rg}}{R_{ai}}}{\frac{1}{K_{od}} + \frac{1}{K_{or} C_{rc}}}\right)$$

K_{or} =unknown function of T_{rg}.

$K_{od} = 6.34\ 10^{-9} R_{ai}^2$

$$H_{rg} \dot{C}_{rc} = R_c \left(C_{sc} - C_{rc}\right) - R_{cb}$$

New form of the system

$$\begin{aligned}\varphi(x;u) &= \varphi\left(T_{rg}, T_{ra}, C_{rc}, C_{sc}, F_{cf}; R_{ai}\right)\\ &= \left(T_{rg}, T_{ra}, C_{tf}\left(C_{rc}, T_{ra}\right), \frac{C_{sc}}{C_{rc}}, \frac{F_{cf}}{C_{rc}}\right) = \xi\end{aligned}$$

$$\left\{\begin{array}{lllll}\dot{x}_1 & = \dot{T}_{rg} & = & \psi\left(x, \varphi\left(x_1\right), u\right) \\ \dot{x}_2 & = \dot{T}_{ra} & = & a_3(t)x_3 + f_2\left(x_1, x_2\right) \\ \dot{x}_3 & \simeq \dot{C}_{rc} & = & a_4(t)x_4 \\ & & & +f_3\left(x_1, x_2, x_3, \psi\left(x, \varphi\left(x_1\right), u\right), u, \dot{u}\right) \\ \dot{x}_4 & \simeq \dot{C}_{sc} & = & a_5(t)x_5 + f_4\left(x_1, x_2, x_3, x_4\right) \\ \dot{x}_5 & \simeq \dot{F}_{cf} & = & F\left(x\right)\end{array}\right.$$

Type 2

Here, $\psi = R_{cb}$, $\varphi = K_{or}$ and $\pi(x) = T_{rg} = x_1$.

$u = (R_{ai}, P_{ra})$

Time dependant change of coordinates

$$\frac{dx}{dt} = F(x, u)$$

Our *diffeomorphism* $\xi = \varphi(x, u)$ depend on u supposed to be smooth, hence:

$$\begin{aligned} \frac{d\xi}{dt} &= D_{\varphi}\left(\varphi_u^{-1}(\xi)\right) f\left(\varphi_u^{-1}(\xi), u\right) + \frac{\partial\varphi\left(\varphi_u^{-1}(\xi), u\right)}{\partial u}\dot{u} \\ &= F(\xi, u, \dot{u}) \end{aligned}$$

$$\begin{cases} \frac{d\widehat{\xi}}{dt} &= F\left(\widehat{\xi}, u, \dot{u}\right) + PC^T R^{-1}\left(y - C\widehat{\xi}\right) \\ \frac{dP}{dt} &= F^*\left(\widehat{\xi}, u, \dot{u}\right) P + PF^*\left(\widehat{\xi}, u, \dot{u}\right) + Q_\theta - PC^T R^{-1} CP \end{cases}$$

In the original coordinates

Since $C\varphi_u(x) = Cx$, equations are those of a modified extended Kalman filter

$$\begin{cases} \frac{d\widehat{x}}{dt} & = f(\widehat{x},u) + pC^T R^{-1}(y - C\widehat{x}) \\ \frac{dp}{dt} & = f^*(\widehat{x},u)\,p + pf^*(\widehat{x},u)^T + q_\theta(\widehat{x}) \\ & -ph^*(\widehat{x},u)^T R^{-1} h^*(\widehat{x},u)\,p \\ & +D_{\psi_u}^{-1}(\widehat{x})\,D_{\psi_u}^2 \cdot \left(ph^*(\widehat{x},u)^T R^{-1}(h(\widehat{x},u) - y)\right) p \\ & +pD_{\psi_u}^2 \cdot \left(ph^*(\widehat{x},u)^T R^{-1}(h(\widehat{x},u) - y)\right) D_{\psi_u}^{-1}(\widehat{x})^T \end{cases}$$

where $q_\theta(\widehat{x}) = D_{\varphi_u}(\widehat{x})^{-1} Q_\theta \left(D_{\varphi_u}(\widehat{x})^{-1}\right)^T$

The two last lines (transposed) correspond to the change of coordinate.

Tuning

We use a second order system to estimate K_{or}

i.e. $\dfrac{d^3 K_{or}}{dt^3} = 0$

We use three parallel extended Kalman filters such that

- $\theta_0 = 3$ (starting value for each observers)
- $\theta_{HG} = 2$ (minimal value of θ ensuring high-gain)
- Time between two consecutive initializations: 2 hours

EKF with two outputs

At last,

$$\xi = \left(T_{rg}, R_{cb}, \dot{K}_{or}, \ddot{K}_{or}, T_{ra}, C_{tf}, \frac{C_{sc}}{C_{rc}}, \frac{F_{cf}}{C_{rc}}\right)$$

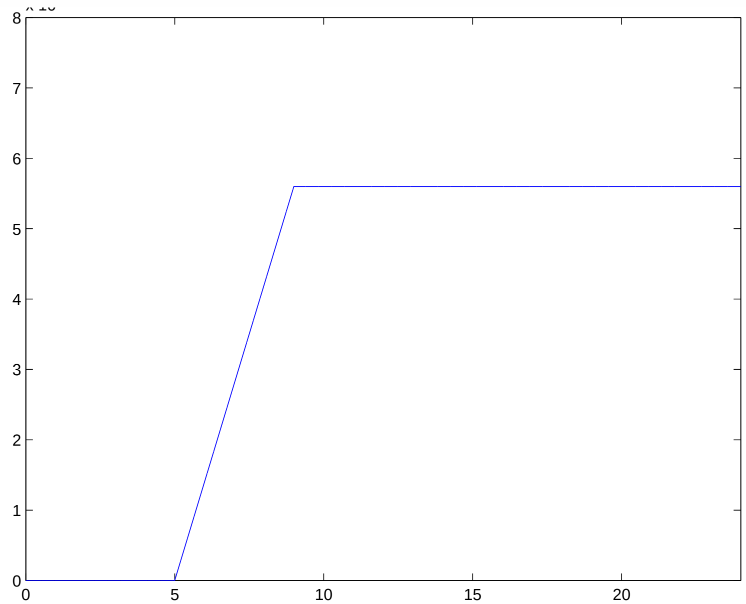
and

$$\Delta^{-1} = \mathrm{diag}\left(1, \theta, \theta^2, \theta^3, 1, \theta, \theta^2, \theta^3\right)$$

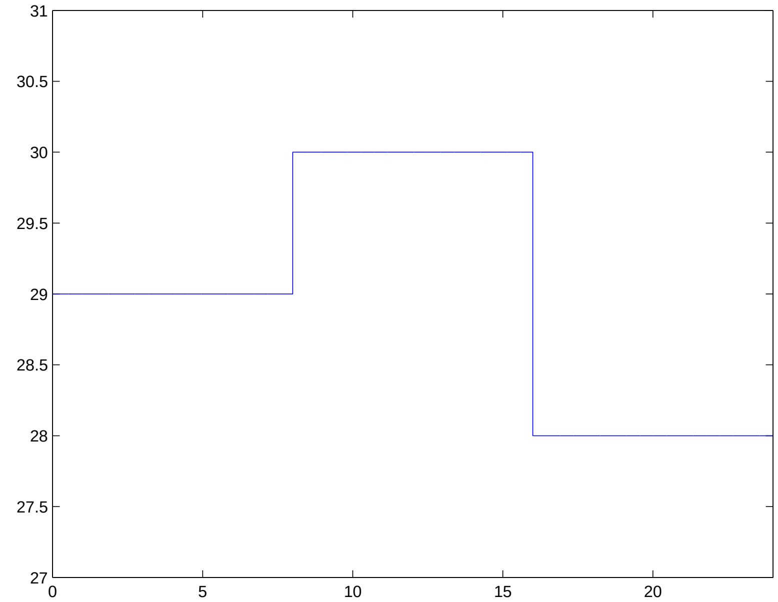
with $Q_\theta = \theta^2 \Delta^{-1} Q \Delta^{-1}$
and $R_\theta = \left(C\Delta^{-1}C'\right) R \left(C\Delta^{-1}C'\right)$

Simulations

Colored noise (Ornstein–Uhlenbeck process) on both measured variables T_{rg} and T_{ra}.



Unknown parameter F_{cf}



Control variable R_{ai}

$$K_{or} = 1.16\ 10^{-5} \exp\left(\frac{A_{or}}{R\left(\frac{1}{866.7} - \frac{1}{T_{rg}}\right)}\right)$$

Constants of the model

Reactor operating conditions

$$H_{ra} = 1.85\ 10^{-4},\ P_{ra} = 211.7,$$

Feed properties

$$R_{tf} = 41,\ T_{tf} = 492.8,\ S_{tf} = 3140,$$

Cat.recirculation

$$R_c = 290,\ S_c = 1047,$$

Heat constants

$$\Delta H_{cr} = 4.65\ 10^5,\ \Delta H_{fv} = 1.7410^5,$$

$$\Delta H_{rg} = 3.02\ 10^7,\ R = 8.314$$

$$k_{cr} = 8.31\ 10^{-2},\ A_{cr} = 6.28\ 10^4,$$

$$k_{cc} = 2.66\ 10^{-4},\ A_{cc} = 4.18\ 10^4$$

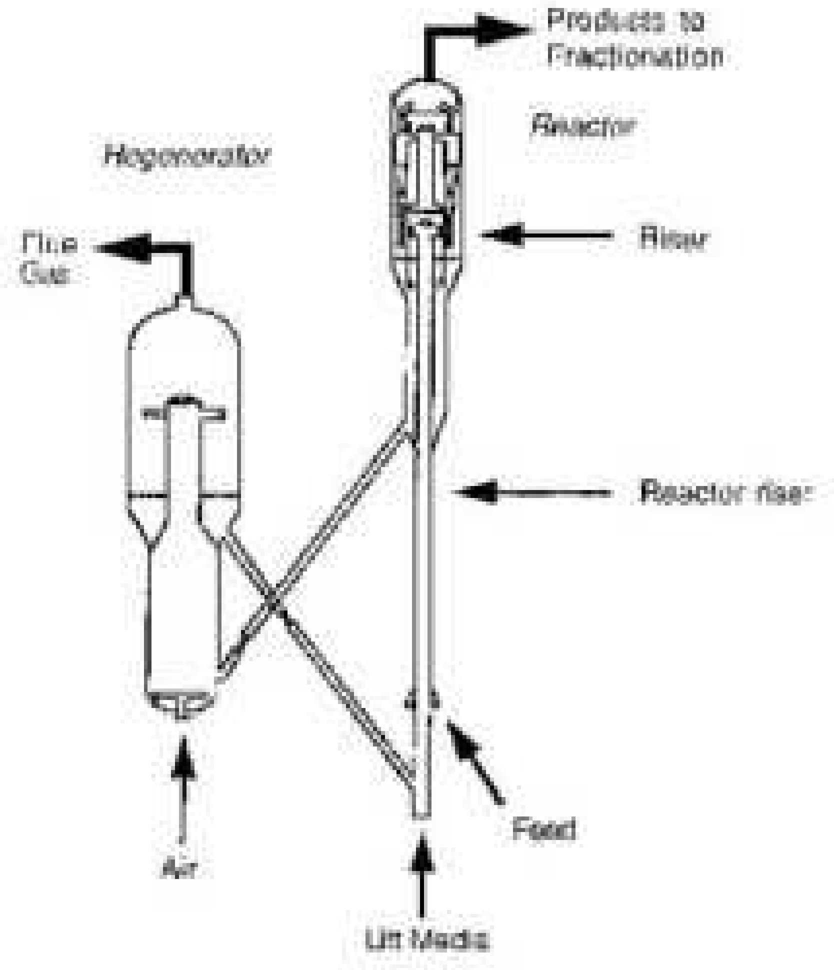
Regenerator operating conditions

$$H_{rg} = 1.53\ 10^5,\ P_{rg} = 254.4,$$

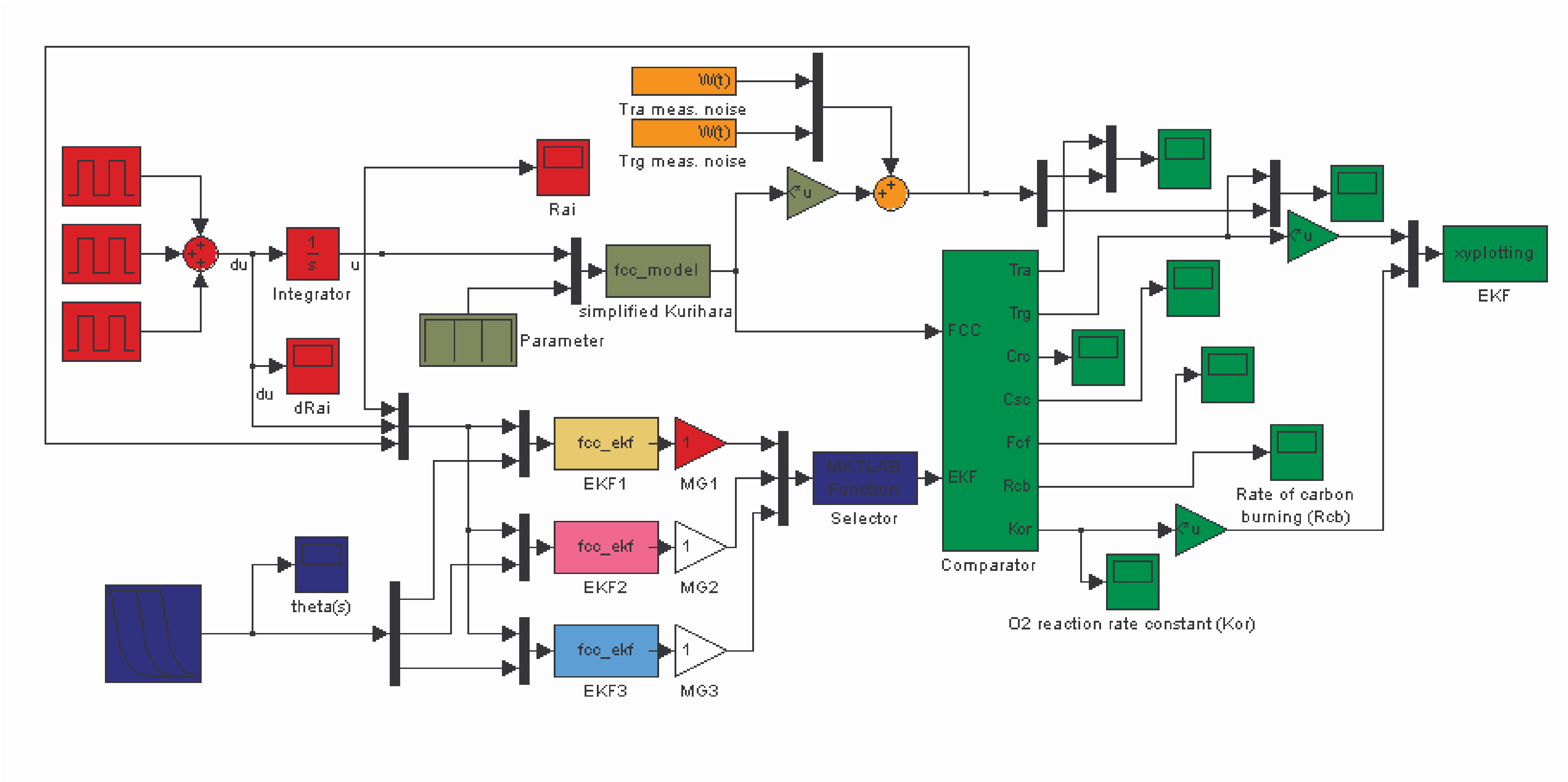
Air properties

$$R_{ai} = 26,\ T_{ai} = 394,\ S_{ai} = 1130$$

$$A_{or} = 1.47\ 10^5$$

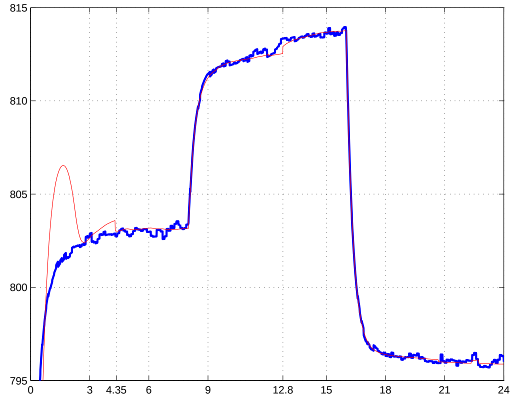


Simulation using Matlab/Simulink



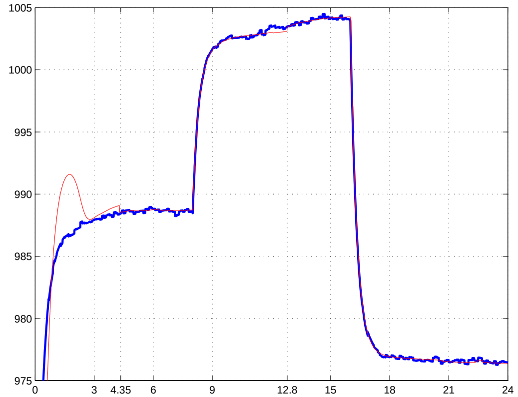
to be continued...

Observer: State estimation



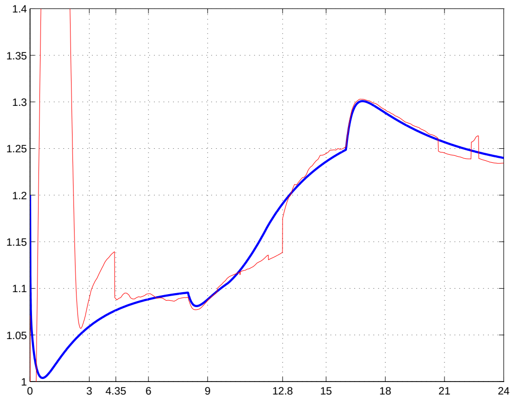
T_{ra}

T_{rg}



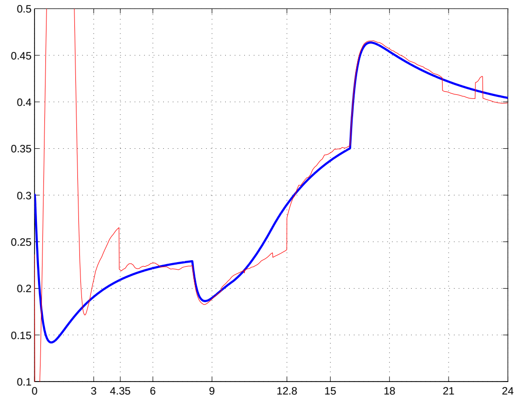
Reactor

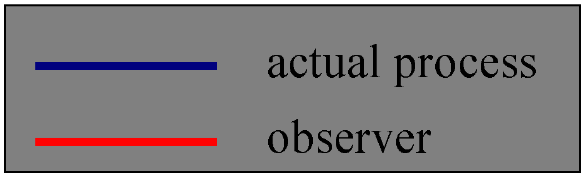
Regenerator



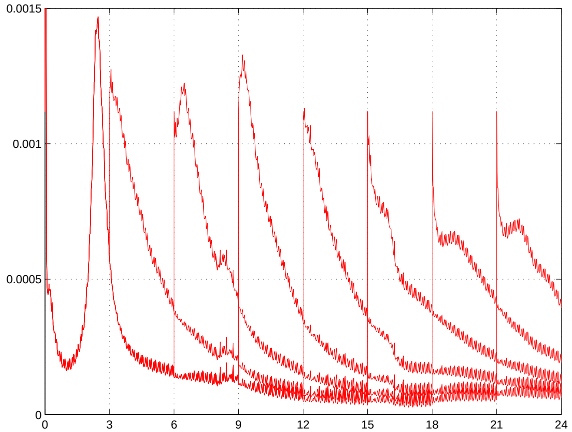
C_{sc}

C_{rc}

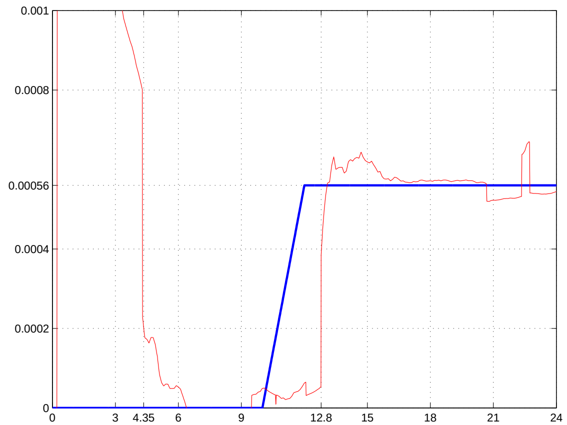




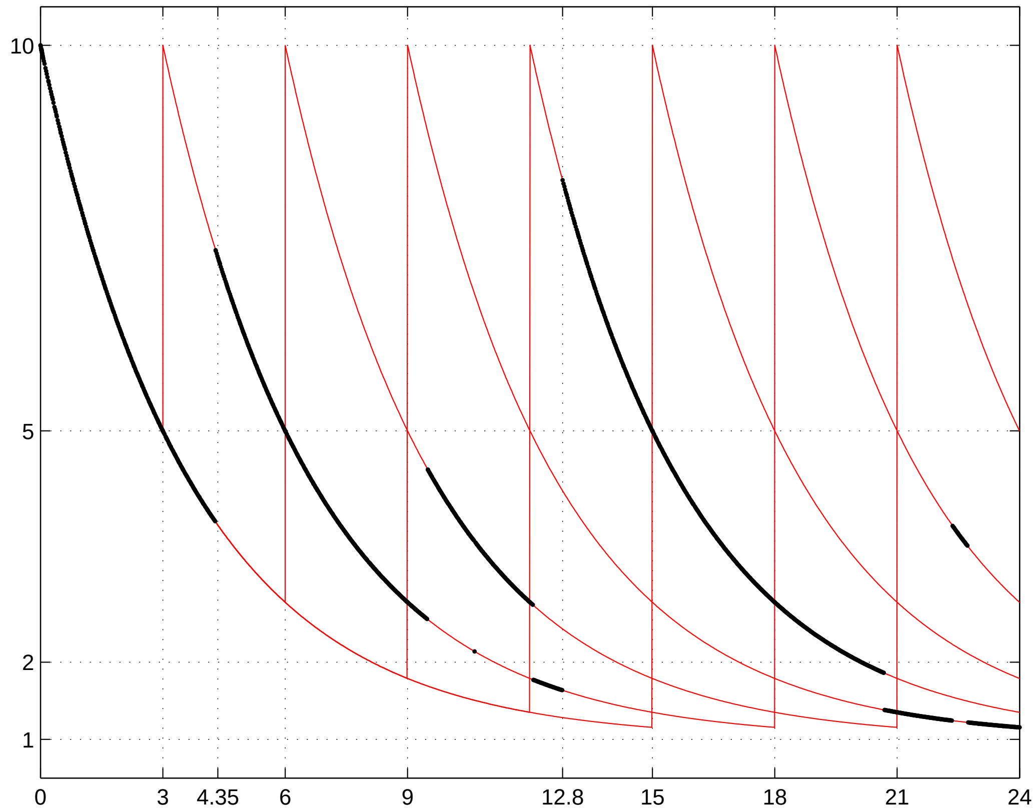
Observer: Parameter estimation

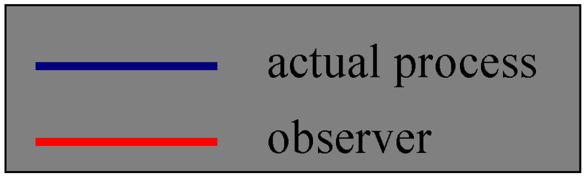


Norm of the gain

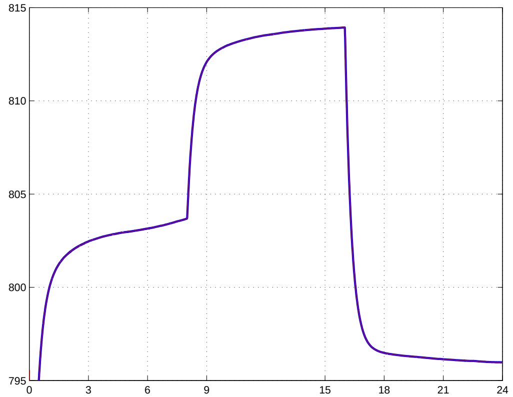


F_{cf}

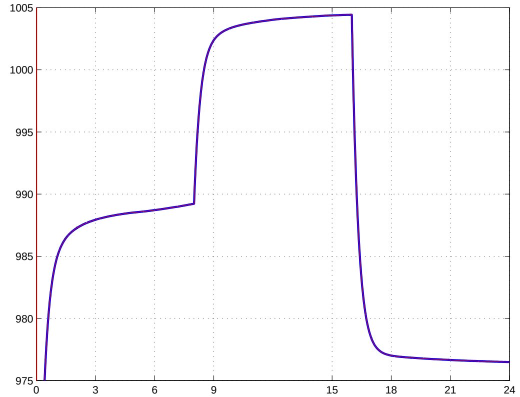




Identification: state estimation



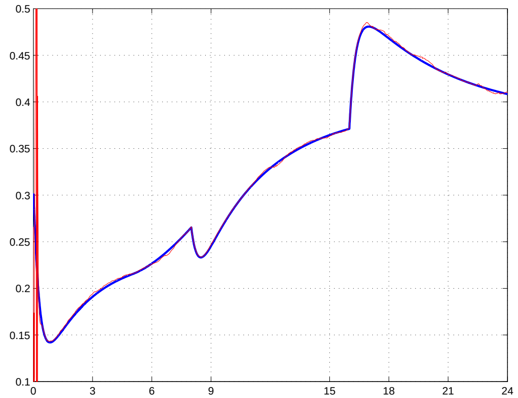
T_{ra}



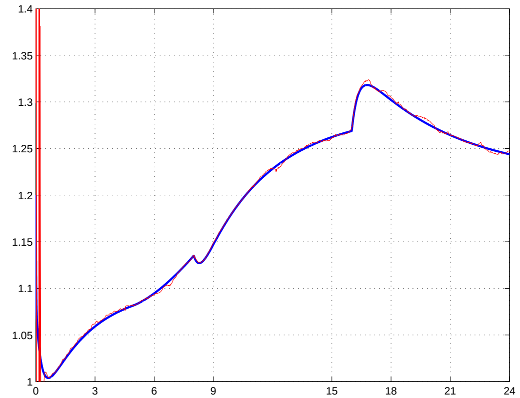
T_{rg}

Reactor

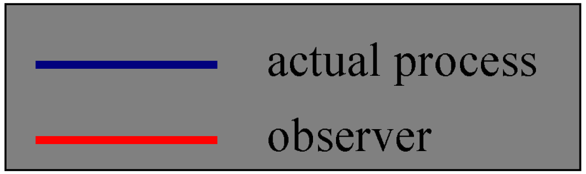
Regenerator



C_{sc}



C_{rc}



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Identification of unknown function