GIPSA-lab summer school on
Nonlinear Observers and Applications
Observer synthesis with Matlab/Simulink

September 7, 2007

1. SERIES CONNECTED DC MOTOR

In this first part we propose the implementation of both a high-gain Luenberger observer and a high gain extended Kalman filter on a rather simple SISO system, a series connected DC motor.
We recall below the equations of the model of this process derived from the equivalent circuit representation shown Fig.1 and Newton's law. We consider the following model hypothesis:
* the motor is operated below saturation (electromechanical torque is equal to $K_m L_{af} I^2$, where $K_m L_{af}$ is a constant and writen L_{af} from now)
* ideal (100%) efficiency in energy conversion (L_{af} is the same in both equations, otherwise it may differ)
* the forces acting on the the motor's shaft are a viscous torque and a load torque (e.g. a brake)

$$\left\{ \begin{array}{rcl} L.\dot{I} & = & V - R.I - L_{af}.I.w_r \\ J.\dot{w}_r & = & L_{af}.I^2 - B.w_r - T_l \end{array} \right.$$

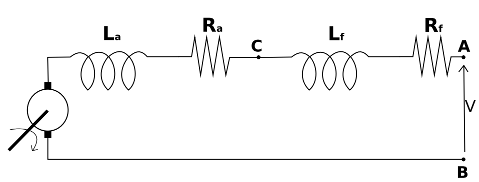

Figure 1: equivalent circuit representation

quantity	physical meaning
L, R, L_{af}	circuit total inductance, resistance and mutual inductance
J, B	system inertia, viscous friction coeficient
V, I	voltage (input), current (output)
w_r	motor's speed
T_l	load torque

The Matlab S-function ***DCseries.m*** *(S-FUNCTION)* and the Simulink diagram ***SimDC_grenoble07.mdl*** are provided to simulate this model.

-> Check differential observability when I is measured.

-> In order to estimate the torque load, it is now taken as a state variable. Propose a simple differential equation for the torque load and find the change of variables that gives the SISO observability canonical form.

-> Being given this form, implement the high-gain Luenberger observer
$\dot{z} = A(u).z + b(z,u) - K_{\Theta}.(C.z - y)$
with $K_{\Theta} = \left(diag(\Theta, \Theta^2, \Theta^3)\right).K$ and K is such that $(A(u) - K.C) < 0$
Note 1: K' may be obtained from the Matlab command lqr(A',C',Q,R) of the Control System Toolbox (Q and R are set by the user, symmetric and positive definite) or using a pole placement approach.
Note 2: you may get inspiration from the files ***DCluenberger.m*** *(S-FUNCTION) and* ***gain_Luenberger.m*** *(SCRIPT).*

-> Implement a high-gain Kalman filter for the series connected DC motor

$$\dot{z} = A(u).z + b(z,u) - P.C'.R^{-1}.(Cz - y)$$

$$\dot{P} = P.(A(u) + b^*(z,u))' + (A(u) + b^*(z,u)).P - P.C'.R^{-1}.C.P + Q_{\Theta}$$

with $Q_{\Theta} = \Theta^2 \Delta.Q.\Delta$ and $\Delta = diag(1, \Theta, \Theta^2)$
Note 1: as far as the Riccati matrix is symmetric, one only need to compute $n*(n+1)/2$ values to solve the second equation. Consequently if z(1:3) denotes the estimated state vector then z(3+1:3+6) will denote the computed values of the Riccati equation and the state vector of the Kalman filter will be of length 9. SymReshape.m is a routine that transforms a symetric square matrix $(n \times n)$ into a $n*(n+1)/2$ vector and vice versa.
Note 2: you may get inspiration from the files ***DCkalman.m*** *(S-FUNCTION).*
Note 3: the two above mentionned programs may be run using the ***Observers_grenoble07.mdl*** *Simulink diagram.*

2. DISTILLATION COLUMN

In this second part we consider a binary distillation column which is a 2 inputs / 2 outputs system. We use the *constant molar overflow* model (one of the simplest for that kind of process). We consider the following model hypothesis :
* Thermal balance is replaced by Lewis Hypothesis which implies that vapor and liquid flow rates are constant along the 'stripping' section (lower part of the column) and the 'rectification' section (upper part of the column)
* on each tray the vapor (x_i) and liquid (y_j) compositions are linked by the liquid/vapor equilibrium relation $y_i = k(x_i)$ where k is the monotonic function $k(x) = \frac{\alpha.x}{(1+(\alpha-1)x)}$, $\alpha > 1$

$H_1\dot{x_1} = (V + FV)(y_2 - x_1)$

$H_j\dot{x_j} = L(x_{j-1} - x_j) + (V + FV)(y_{j+1} - y_j)$ for $j = 2, .., f - 1$ (i.e. the rectifying section)

$H_f\dot{x_f} = FL(Z_f - x_f) + FV(k(Z_f) - y_f) + L(x_{f-1} - x_f) + V(y_{f+1} - y_f)$ the feed tray

$H_j\dot{x_j} = (L + FL)(x_{j-1} - x_j) + V(y_{j+1} - y_j)$ for $j = f + 1, .., n - 1$ (i.e. the stripping section)

$H_n\dot{x_n} = (L+FL)(x_{n-1} - x_n) + (V+FV)(x_n - y_n)$ bottom of the column

quantity	Physical meaning
n	number ot trays
f	index of the feed tray
H_j	liquid hold up on the j^{th} tray (constant)
x_j	liquid composition on the j^{th} tray
$y_j = k(x_j)$	vapor composition on the j^{th} tray
FL, FV	feed (liquid and vapor) flow
L, V	reflux and vapor flow (control variables)
Z_f	feed composition (molar fraction of light component in the feed)

Output variables are x_1 and x_n.

-> Check observability using the differential approach.

-> Find a change of variables that leads to a MIMO observability form.

The observer implementation in that case is a little bit long therefore the Simulink diagrams ***SimColonne_grenoble2007.mdl*** and ***colombine_grenoble2007.mdl*** are proposed. Only a high-gain Kalman filter has been implemented for this example.

-> Compare the different behaviours of the observer when changing the values of the Q and R matrices of the Riccati equation and notice what the changes in the Q_Θ matrix are.

-> Suppose now that Z_f, the feed concentration, is an unmeasured perturbation. Is it possible to identify this quantity ?