Identification of Unknown Functions in Dynamic Systems

Stéphane Binczak, Eric Busvelle, Jean-Paul Gauthier and Sabir Jacquir

***Abstract*— We consider the problem of representing a complex process by a simple model, in order to perform advanced control for instance. In many cases, the main dynamic of the process is well known and some knowledge-based equations can be written, but some parts of the process are unknown. In this paper, we will present one such application, but we have already encountered many other situations of this kind.**

Depending on measurements and control variables, it is sometimes possible to identify the unknown part of the model and the unmeasured state variables. We will briefly recall some theoretical results concerning this problem, and we will also present a general methodology to perform this identification. Then we will explain more deeply how we apply this approach to an electronic circuit representing a neuron. We will estimate effectively the unknown function from actual measurements.

I. THEORY

A. Identifiability

In this paper, we will consider processes that can be represented by smooth continuous time systems of the form

$$\Sigma\left\{\begin{array}{rcl}\dfrac{dx}{dt} & = & f\left(x,\varphi\left(x\right)\right)\\ y & = & h\left(x,\varphi\left(x\right)\right)\end{array}\right. \tag{1}$$

where the state $x=x\left(t\right)$ lies in a n–dimensional analytic manifold X, the observation y is $\mathbb{R}^{d_y}$–valued, and f, h are respectively a smooth (parametrized) vector field and a smooth function. The function φ is an unknown function of the state. $x_0=x\left(0\right)$ is also supposed to be an unknown initial state.

Our goal is to estimate (on-line) both state variable x and unknown function $\varphi : X \longrightarrow I\subset\mathbb{R}$. More precisely, we want to reconstruct the piece of the graph of φ visited during any experiment. Usually, the function φ represents an unknown part of the process which is very difficult to model *a priori* (in the applied part of the paper – see section II – we will present an electronic system, Σ_{FHN}, where φ is denoted by g).

Let us recall some definitions and results from our previous papers ([4], [5]).

Let $\Omega = X\times L^{\infty}\left[I\right]$, where $L^{\infty}\left[I\right] = \left\{\hat{\varphi} : [0,T_{\hat{\varphi}}]\mapsto I,\ \hat{\varphi}\text{ measurable}\right\}$. We can define the input/output mapping

$$P_{\Sigma}:\quad\begin{array}{rcl}\Omega & \longrightarrow & L^{\infty}\left[\mathbf{R}^{d_y}\right]\\ \left(x_0,\hat{\varphi}\left(\cdot\right)\right) & \longrightarrow & y\left(\cdot\right)\end{array}$$

LE2I, UMR CNRS 5158, Université de Bourgogne, Aile des Sciences de l'Ingénieur, BP 47870, 21078 Dijon Cedex, France busvelle@u-bourgogne.fr

Definition 1: Σ is said to be identifiable if P_{Σ} is injective.

This definition is the natural definition of identifiability: it says that the system is identifiable if one can retrieve from measurements the graph of φ along the trajectories.

As for observability, we define an infinitesimal version of identifiability. Let us consider the first variation of Σ (where $\hat{\varphi}\left(t\right)=\varphi\circ x\left(t\right)$):

$$T\Sigma\left\{\begin{array}{rcl}\dfrac{dx}{dt} & = & f\left(x,\hat{\varphi}\right)\\ \dfrac{d\xi}{dt} & = & T_x f\left(x,\hat{\varphi}\right)\xi+d_{\varphi}f\left(x,\hat{\varphi}\right)\eta\\ \hat{y} & = & d_x h\left(x,\hat{\varphi}\right)\xi+d_{\varphi}h\left(x,\hat{\varphi}\right)\eta\end{array}\right.$$

and the input/output mapping of $T\Sigma$

$$P_{T\Sigma}:\ \begin{array}{rcl}T_{x_0}X\times L^{\infty}\left[\mathbb{R}\right] & \longrightarrow & L^{\infty}\left[\mathbb{R}^{d_y}\right]\\ \left(\xi_0,\eta\left(\cdot\right)\right) & \longrightarrow & \hat{y}\left(\cdot\right)\end{array}$$

Definition 2: Σ is said to be infinitesimally identifiable if $P_{T\Sigma}$ is injective for any $\left(x_0,\hat{\varphi}\left(\cdot\right)\right)\in\Omega$ i.e. $\ker\left(P_{T\Sigma}\right)=\left\{0\right\}$ for any $\left(x_0,\hat{\varphi}\left(\cdot\right)\right)$.

In [4], [5], we have shown the very important following result: identifiability is a generic property if and only if the number of observation d_y is greater or equal to 3. On the contrary, if d_y is equal to 1 or 2, identifiability is a very restrictive hypothesis (infinite codimension). Moreover, in the case $d_y=1$ or 2, we have completely classified infinitesimally identifiable systems by certain geometric properties that are equivalent to the normal forms presented in Theorems [4], [5] below. We will apply these two theorems to study identifiability and to build an identification algorithm for an electronic neuron. Therefore, we recall these results. We recall them in coordinates (exhibiting normal forms). One can find intrinsic statements in the paper [5].

Theorem 1: $(d_y=1)$ If Σ is uniformly infinitesimally identifiable, then, there is a subanalytic closed subset Z of X, of codimension 1 at least, such that for any $x_0\in X\backslash Z$, there is a coordinate neighborhood $(x_1,\ldots,x_n,\ V_{x_0})$, $V_{x_0}\subset X\backslash Z$ in which Σ (restricted to V_{x_0}) can be written:

$$\Sigma_1\left\{\begin{array}{rcl}\dot{x}_1 & = & x_2\\ & \vdots & \\ \dot{x}_{n-1} & = & x_n\\ \dot{x}_n & = & \psi(x,\varphi)\\ y & = & x_1\end{array}\right.\quad\text{and }\frac{\partial}{\partial\varphi}\psi(x,\varphi)\neq 0 \tag{2}$$

Theorem 2: ($d_y = 2$) If Σ is uniformly infinitesimally identifiable, then, there is an open-dense semi-analytic subset $\tilde{U}$ of $X \times I$, such that each point (x_0, φ_0) of $\tilde{U}$, has a neighborhood $V_{x_0} \times I_{\varphi_0}$, and coordinates x on V_{x_0} such that the system Σ restricted to $V_{x_0} \times I_{\varphi_0}$, denoted by $\Sigma_{|V_{x_0} \times I_{\varphi_0}}$, has one of the three following normal forms:

-type 1 normal form:

$$\Sigma_{2,1}\left\{\begin{array}{lllllll} y_1 & = & x_1 & & y_2 & = & x_2 \\ \dot{x}_1 & = & x_3 & & \dot{x}_2 & = & x_4 \\ & \vdots & & & & \vdots & \\ \dot{x}_{2k-3} & = & x_{2k-1} & & \dot{x}_{2k-2} & = & x_{2k} \\ \dot{x}_{2k-1} & = & f_{2k-1}(x_1, ..., x_{2k+1}) & & & & \\ \dot{x}_{2k} & = & x_{2k+1} & & & & \\ & \vdots & & & & & \\ \dot{x}_{n-1} & = & x_n & & & & \\ \dot{x}_n & = & f_n(x, \varphi) & & & & \end{array}\right.$$

with $\frac{\partial f_n}{\partial \varphi} \neq 0$.

-type 2 normal form:

$$\Sigma_{2,2}\left\{\begin{array}{llllll} y_1 & = & x_1 & y_2 & = & x_2 \\ \dot{x}_1 & = & x_3 & \dot{x}_2 & = & x_4 \\ & \vdots & & & \vdots & \\ \dot{x}_{2r-3} & = & x_{2r-1} & \dot{x}_{2r-2} & = & x_{2r} \\ \dot{x}_{2r-1} & = & \psi(x, \varphi) & \dot{x}_{2r} & = & F_{2r}(x_1, \ldots, \\ & & & & & x_{2r+1}, \psi(x, \varphi)) \\ & & & \dot{x}_{2r+1} & = & F_{2r+1}(x_1, \ldots, \\ & & & & & x_{2r+2}, \psi(x, \varphi)) \\ & & & & \vdots & \\ & & & \dot{x}_{n-1} & = & F_{n-1}(x, \psi(x, \varphi)) \\ & & & \dot{x}_n & = & F_n(x, \varphi) \end{array}\right.$$

with $\frac{\partial \psi}{\partial \varphi} \neq 0$, $\frac{\partial F_{2r}}{\partial x_{2r+1}} \neq 0, ..., \frac{\partial F_{n-1}}{\partial x_n} \neq 0$

-type 3 normal form:

$$\Sigma_{2,3}\left\{\begin{array}{llllll} y_1 & = & x_1 & y_2 & = & x_2 \\ \dot{x}_1 & = & x_3 & \dot{x}_2 & = & x_4 \\ & \vdots & & & \vdots & \\ \dot{x}_{n-3} & = & x_{n-1} & \dot{x}_{n-2} & = & x_n \\ \dot{x}_{n-1} & = & f_{n-1}(x, \varphi) & \dot{x}_n & = & f_n(x, \varphi) \end{array}\right.$$

with $\frac{\partial}{\partial \varphi}(f_{n-1}, f_n) \neq 0$

One can easily read on these normal forms that the converses of Theorems 1 and 2 are "almost true".

For the case $d_y \geq 3$, where identifiability is a generic property, we provide a normal form in [3].

B. Identification

For a given system, it is very important to study identifiability in a general context before developping an algorithm in order to perform identification. Indeed, in some cases, it is possible to develop an identifier for non-identifiable systems (see [6]). Moreover, studying the identifiability allows us to put the system into a suitable form for developping a reasonably general identification algorithm.

Indeed, let us assume that we have a system Σ which is uniformly infinitesimally identifiable. Then this system can be written in some identifiability canonical form as explained above. Let us assume that the system can be put globally under this identifiability canonical form (even if this strong hypothesis can be sometimes relaxed). In [6], [8], we developped a general methodology for the synthesis of nonlinear observers of observable systems. Comparable algorithms will be developped for identification of the function φ, when the system is in identifiability canonical form.

More precisely, we will assume a **local polynomial model** for the unknown function φ: at a given time, assume that φ can be written as a function of t of the form $\varphi = a_n t^n + \cdots + a_1 t + a_0$. Hence, we will use the following **local** dynamic model corresponding to $\frac{d^{n+1}\varphi}{dt^{n+1}} = 0$:

$$\left\{\begin{array}{lll} \dot{\varphi} & = & \varphi_1 \\ \dot{\varphi}_1 & = & \varphi_2 \\ & \vdots & \\ \dot{\varphi}_n & = & 0 \end{array}\right.$$

With this approach, any of the previous canonical form of identifiability yields the following canonical form of observability:

$$\left\{\begin{array}{lll} \dot{x} & = & Ax + b(x) \\ y & = & Cx \end{array}\right.$$

where

$$A = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ 0 & 0 & 0 & \ddots & 0 \\ \vdots & & & \ddots & 1 \\ 0 & & \cdots & & 0 \end{pmatrix} \text{ and } C = (1, 0, ..., 0)$$

and b is a smooth vector field, depending triangularly on x and compactly supported:

$$b = b_1(x_1)\frac{\partial}{\partial x_1} + b_2(x_1, x_2)\frac{\partial}{\partial x_2} + ... + b_n(x_1, ..., x_n)\frac{\partial}{\partial x_n}$$

Then, we could apply any kind of high-gain observer in order to reconstruct both state variables and φ as a function of t, that is $\varphi(x(t))$. Combining these informations, we obtain the graph of φ in restriction to the experimental trajectory.

In this study, we will use a "high-gain/non-high-gain" observer which is asymptotically an extended Kalman filter. This observer has been described in [6] and has the following form

$$\left\{\begin{array}{lll} \dot{z} & = & Az + b(z) - S(t)^{-1}C'r^{-1}(Cz - y(t)) \\ \dot{S} & = & -(A + b^*(z))'S - S(A + b^*(z)) \\ & & \qquad + C'r^{-1}C - SQ_\theta S \\ \dot{\theta} & = & \lambda(1 - \theta) \end{array}\right.$$

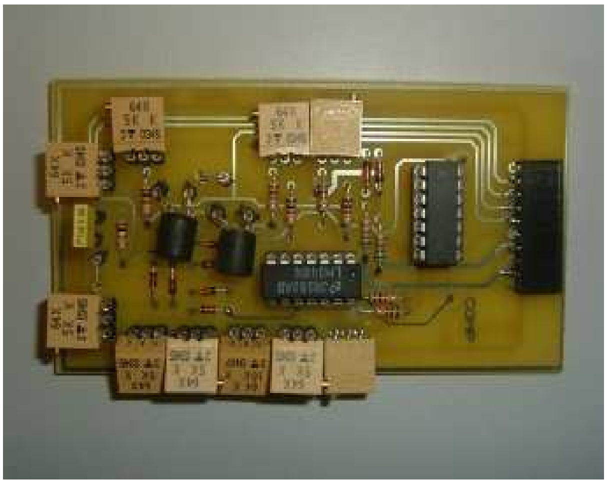

Fig. 1. Electronic neuron

We have the following result ([6])

Theorem 3: There exist λ_0 such that for all $0 \leq \lambda \leq \lambda_0$, for all θ_0 large enough, depending on λ, for all $S_0 \geq c\,Id$, for all $K \subset \mathbb{R}^n$, K a compact subset, for all $\varepsilon_0 = z_0 - x_0$, $\varepsilon_0 \in K$, the following estimation holds, for all $\tau \geq 0$:

$$||\varepsilon(\tau)||^2 \leq R(\lambda, c)e^{-a\,\tau}||\varepsilon_0||^2\Lambda(\theta_0, \tau, \lambda),$$
$$\Lambda(\theta_0, \tau, \lambda), = {\theta_0}^{2(n-1)+\frac{a}{\lambda}}e^{-\frac{a}{\lambda}\theta_0(1-e^{-\lambda\tau})},$$

where $a > 0$. $R(\lambda, c)$ is a decreasing function of c.

Moreover for all $T > 0$, $\tau \leq T$, for all $\theta_0 \geq \bar{\theta}_0$, $\bar{\theta}_0 = e^{\lambda T}(\frac{L'}{Q_m\alpha} - 1) + 1$, where L' is the sup of the partial derivatives of b w.r.t. x:

$$||\varepsilon(\tau)||^2 \leq \theta(\tau)^{2(n-1)}H(c)e^{-(a_1\theta(T)-a_2)\tau}||\varepsilon(0)||^2$$

Using this theorem, we have explained in [4]–[6] how to construct an observer which is both globally convergent and robust w.r.t. noise. In the next section, we will explain how to use this theorem in order to develop practically an observer for a neuron system.

II. APPLICATION

In this section, we will describe an application of the previous theory to a simple but practical system.

A. The electronic neuron

The modelisation of neurons is of a strong interest in neuroscience research. There exists a large number of papers describing more or less accurate models of one isolated neuron or interaction between neurons.

An accurate model of a single isolated biological neuron has been proposed by Hodgkin and Huley in 1952 ([10]) This model has been extensively studied and is considered as a reasonably good quantitative model of neuron. But in order to perform some theoretical studies, simplified models have been proposed. One of these models is the Fitztugh-Nagumo model introduced by Fitzhugh, Nagumo & al. in early 1960's ([7], [11]). In [2], one can find a short historical story of this very simple model.

The Fitzhugh–Nagumo (FHN) model has been studied by mathematicians, physiologists and computer scientists for several purposes. Recently, in our laboratory (LE2I, Université de Bourgogne), an analogue circuit which implement a modified version of FHN model has been developped (see Fig. 1) . The main objective of these realization was to quantitatively study this modified Fitzhugh–Nagumo (MFHN) model in experimental conditions.

In order to retrieve the nonlinear complexity of an actual biological neuron, a part of the electronic circuit corresponds to a nonlinear function (in fact, a piecewise linear function). We will call this function g and we will describe g and the dynamical MFHN model in the next subsection.

The electronic circuit description and results have been published in [1], [2]. The conclusion of this study was that this electronic neuron is able to reflect the main qualitative behavior of an actual neuron, especially excitability, oscillatory dynamics and bistability. These properties have been obtained using a particular choice of the function g. However, in a biological neuron, and assuming the MFHN model may be used, the function g is unknown. Hence, in order to model the neuron by a MFHN model, it will be necessary to identify this function g.

Before using our approach with actual data, we will validate the method with our electronic neuron. We will identify the function g without using any *a priori* information about it. Since the actual function is known, we will be able to compare our estimation of g with the actual function and hence to validate the method. Since we use a real process and electric measurements, we will also study the effect of the noise on the identification algorithm.

In the next subsection, we will present the model. Then, we will present our algorithm and finally the results.

B. The FitzHugh-Nagumo model

In [1], [2], the realization of an electronic circuit which practically realizes the FHN model has been described. More precisely, the electronic system is supposed to implement the following equations:

$$\Sigma_{FHN}\left\{\begin{array}{rcl}\dfrac{dV}{dt} & = & V - V^3 - W\\ \dfrac{dW}{dt} & = & \varepsilon\left(g\left(V\right) - W - \eta\right)\end{array}\right. \tag{3}$$

For details concerning the electronic realization of this system, see [2]. Let us just point out that, in our electronic circuit, V corresponds to a voltage (Fig. 2) and W corresponds to a current (Fig. 3). In a biological model, V represents the membrane voltage and W is the recovery variable. ε and η are some constant parameters.

In the electronic circuit, the function g corresponds to the modification of the original model and is a known piecewise linear function:

$$g\left(V\right) = \left\{\begin{array}{ll}\beta\,V & \text{if } V > 0\\ \alpha\,V & \text{if } V \leq 0\end{array}\right. \tag{4}$$

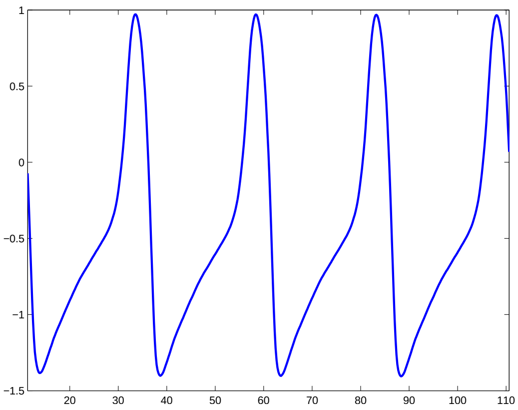

Fig. 2. V versus time

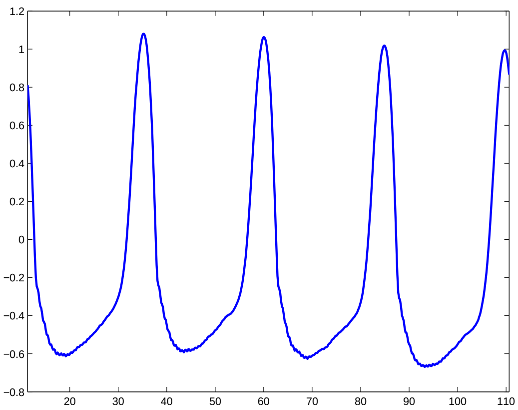

Fig. 3. W versus time

The choice of this function is more or less arbitrary: this kind of function can be easily implemented in a circuit and is sufficient to reproduce the main dynamical properties of a neuron. The values of the shapes α and β of the linear parts of the function were tuned in order to observe these well known properties (excitability, oscillatory dynamics and bistability). From a mathematical point of view, a piecewise linear function is not so simple, since it is not differentiable at some points. Since many identification techniques require more regularity of the function, these methods will fail in this case. However, our method does not require such a preliminary assumption.

C. *Identification of the g–function*

Let us consider the system (3). Both state variables are measured but only V will be used as a measured variable, in order to be in the same operating conditions as for a biological neuron experiment. Hence, we are in the case of a system with only one output. Althrough the identifiability is not a generic property in this case, it is clear that our system can be written globally in the generic identifiability canonical form (2) (and hence is identifiable and uniformly infinitesimally identifiable). Using the same notations as in the theoretical part, let us denote $y = x_1 = V$. Then, in order to have $\dot{x}_1 = x_2$, let us denote $x_2 = V - V^3 - W$. The system can be written

$$\tilde{\Sigma}_{FHN} \left\{ \begin{array}{lcl} \frac{dx_1}{dt} & = & x_2 \\ \frac{dx_2}{dt} & = & \psi\left(x,\varphi\right) \end{array} \right.$$

where $x = (x_1, x_2)$, $\varphi\left(x\right) = \varphi\left(x_1\right) = g\left(V\right)$ and

$$\begin{aligned} \psi\left(x,\varphi\right) = & \left(1 - 3x_1^2\right) x_2 \\ & - \varepsilon\left(\varphi\left(x_1\right) - \eta - x_1 + x_1^3 + x_2\right) \end{aligned}$$

Since $\varepsilon \neq 0$, the system is clearly identifiable. Moreover, we can see that if η is an unknown parameter, the whole function $\varphi\left(x_1\right) - \eta$ could also be identified. However, in the following, we will simply identify g.

We used the general algorithm as described in section I-B, that is to say we apply a high–gain/non high gain extended Kalman filter to the system

$$\left\{ \begin{array}{lclclcl} \dfrac{dx_1}{dt} & = & x_2 & \quad & \dfrac{dx_2}{dt} & = & \psi\left(x,\tilde{\varphi}\right) \\ \dfrac{d\varphi}{dt} & = & \varphi_1 & & \dfrac{d\varphi_1}{dt} & = & \varphi_2 \\ \dfrac{d\varphi_2}{dt} & = & \varphi_3 & & \dfrac{d\varphi_3}{dt} & = & 0 \end{array} \right. \tag{5}$$

From this algorithm (using a local polynomial of order 3) we should obtain an estimation $\left(\hat{V}\left(t\right), \hat{g}\left(t\right)\right)$ along the time. $\hat{V}\left(t\right)$ is just an estimation of the measured output $V\left(t\right)$ (hence a Kalman filtering version of $V\left(t\right)$). $\hat{g}\left(t\right)$ is an estimation of $g\left(V\left(t\right)\right)$. Therefore, the parametric curve $\left(\hat{V}\left(t\right), \hat{g}\left(t\right)\right)$ represents the graph of the function g versus its variable V.

Our first results were very bad. It was impossible to estimate an unknown function $\varphi\left(x_2\right)$. Practically, instead of a graph, we obtained a chaotic curve which was not at all a graph. In fact, this kind of result proves that there is no function φ in the structure Σ_{FHN} which can explain the observed dynamic. Indeed, thanks to this study, we conclude that the electronic circuit didn't simulate a FHN model. Watching more deeply the circuit, we observed that one electronic component (an operational amplifier) was in fact used outside its operating range. After changing this device, results became correct. This hardware problem was in fact very interesting because we were able to detect the problem only after our identification study. So we hope to obtain the same kind of results with biological measurements (even if we hope that the conclusion will be that some φ does exist).

Because of the non differentiability of V, the graph was not estimated very efficiently arround $V = 0$. Instead of a graph, we obtained a closed curve arround the graph. Indeed, the local approximation of $g\left(V\right)$ by a polynomial of degree 3 was not justified each time V crossed the 0-axis. The consequence was a delay for the estimation of $g\left(V\right)$, and then a trigger effect which explains that we obtained a closed loop instead of a graph.

In order to improve the estimation, we just separate the estimation of the shape (*i.e.* of α and β) depending on the

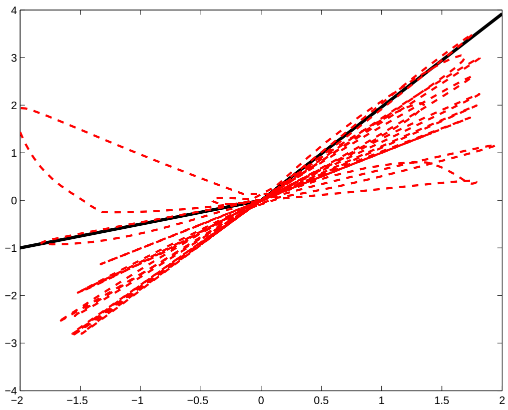

Fig. 4. Estimation of g after 8 cycles

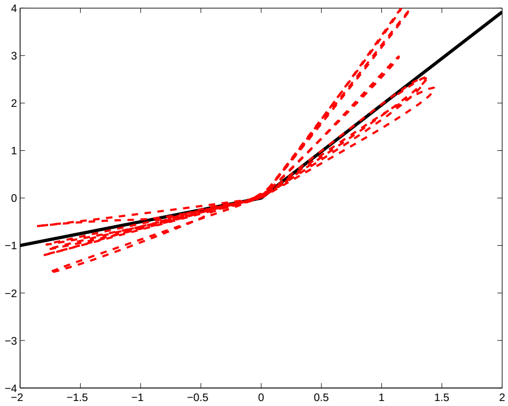

Fig. 5. Estimation of g after 13 cycles

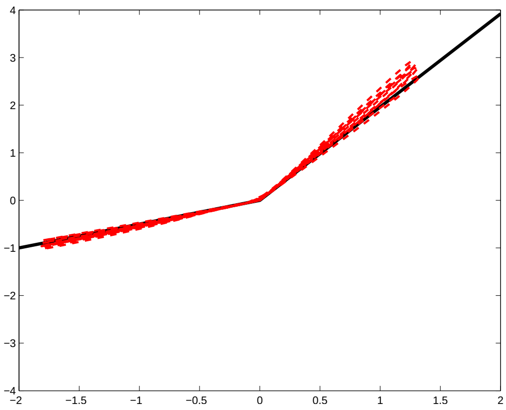

Fig. 6. Estimation of g after 19 cycles

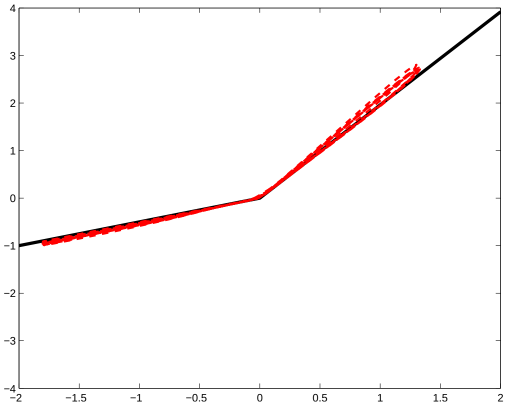

Fig. 7. Estimation of g after 25 cycles

sign of V, that is to say we replace the system (5) by the following system

$$\left\{\begin{array}{lllll} \frac{dx_1}{dt} & = x_2 & \frac{dx_2}{dt} & = \bar{\psi}\left(x,\hat{\alpha},\hat{\beta}\right) \\ \frac{d\hat{\alpha}}{dt} & = \alpha_1 & \frac{d\hat{\beta}}{dt} & = \beta_1 \\ \frac{d\alpha_1}{dt} & = \alpha_2 & \frac{d\beta_1}{dt} & = \beta_2 \\ \frac{d\alpha_2}{dt} & = \alpha_3 & \frac{d\beta_2}{dt} & = \beta_3 \\ \frac{d\alpha_3}{dt} & = 0 & \frac{d\beta_3}{dt} & = 0 \end{array}\right. \tag{6}$$

where, as for the definition of g in (4)

$$\bar{\psi}\left(x,\hat{\alpha},\hat{\beta}\right) = \begin{cases} \psi\left(x,\hat{\beta}\right) & \text{if } x_1 > 0 \\ \psi\left(x,\hat{\alpha}\right) & \text{if } x_1 \leq 0 \end{cases} \tag{7}$$

D. Results

Figures 4 to 7 show the results obtained using the previous algorithm on the electronic neuron, based upon the measurements of Fig. 2. The piecewise linear black curve is the implemented function g. The red dotted line represents the trajectory of $\left(\hat{V}(t), \hat{g}(t)\right)$. On the first figure (Fig. 4), the trajectory is plotted from time $t = 0$ to time $t \simeq 200\,\mu$s, corresponding to 8 cycles of the almost periodic trajectory of $(V(t), W(t))$. The second curve (Fig. 5) represents the same parametric curve, for the 5 next cycles, and so on for Figures 6 and 7. On Figure 4, the algorithm had not enough time to converge. On Figure 7, the observer has correctly estimated the graph of φ.

One can see a very small error on the shape both for the positive values and negative values of V. These small errors are more clear on figure 8, where we have plotted $\hat{\alpha}$ and $\hat{\beta}$ *versus* time. In our case, since the function is a piecewise linear function, $\hat{\alpha}$ and $\hat{\beta}$ should be constant parameters, after the transient part. It is important to keep in mind that, even if we plotted $\hat{\alpha}$ and $\hat{\beta}$ versus time, we did not assume that $\hat{\alpha}$ and $\hat{\beta}$ are constant parameters *i.e.* we performed identification and not only parametric estimation (see [4], [5]).

If we simulate a FHN-model rather than using actual measurements of an electronic device, our identification procedure will exhibit $\hat{\alpha}$ and $\hat{\beta}$ as perfect constants. Here, we can see some periodic errors: these errors reflect small modelling errors, due to the fact that the circuit does not implement exactly the FHN model, especially around $V = 0$ since the switch between shapes is just implemented with non ideal diodes. Once again, this behavior is interesting because it exhausts the ranges

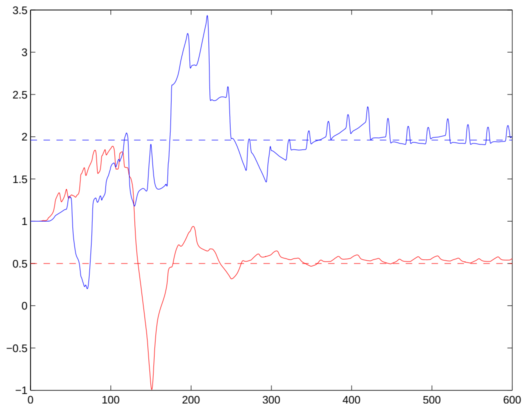

Fig. 8. $\hat{\alpha}$ and $\hat{\beta}$ estimation *vs* time

where the model does not succeed to perfectly explain the system.

III. CONCLUSIONS AND FUTURE WORKS

In this paper, we have reconstructed an unknown function from electronic experimental data. In this way, we have validated our theoretical identification method by comparing the identified function with the actual function, in a noisy context.

Results obtained on an artificial neuron were very promising. Despite the fact that the system is far less complicated than a biological neuron, the Fitzhugh–Nagumo model is good enough to represent main dynamics of a real neuron. Our method was able to estimate the unknown function for an electronic circuit which simutale this model.

Therefore, we will be able to apply the identification procedure to actual measurements of a biological neuron. If it is impossible to identify a function corresponding to measurements, then we will conclude that the Fitzhugh–Nagumo is not good enough to quantitatively model the main dynamic of a biological neuron. Hence we will use the model which has been designed for this purpose: the Hodgkin–Huley model. But if a function can be identified then we will obtain a simple model of biological neuron.

The next step will be to study several connected neurons. We have already studied the case of several electronic neurons and estimated the unknown function in this case.

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