Laboratoire d'Analyse Appliquée et Optimisation

Observation and Identification for Nonlinear Systems

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Introduction

$$\Sigma : \begin{cases} \frac{dx}{dt} &= f(x, u(t), \varphi \circ \pi(x(t))) \\ y &= h(x, u(t), \varphi \circ \pi(x(t))) \end{cases}$$

where $\varphi \circ \pi : \begin{array}{ccccc} X & \longrightarrow & Z & \longrightarrow & I \subset \mathbf{R} \\ x & \longrightarrow & z = \pi(x) & \longrightarrow & \varphi(\pi(x)) \end{array}$

and $P_\Sigma : \begin{array}{ccccccc} X & \times & L^\infty[U] & \times & L^\infty[I] & \longrightarrow & L^\infty[\mathbf{R}^{d_y}] \\ (x_0 & , & u(.) & , & \widehat{\varphi}(.)) & \longrightarrow & y(.) \end{array}$

φ is an unknown function of $\pi(x)$

$\widehat{\varphi}$ is a function of time

P_Σ is the input/output function of Σ

Identifiability

Definition 1: Σ is identifiable at

$$(u(.), y(.)) \in L^\infty[U] \times L^\infty[\mathbf{R}^{d_y}]$$

if there is at most a single couple

$$(x_0, \widehat{\varphi}) \in X \times L^\infty(I)$$

such that for almost all t

$$P_\Sigma(x_0, u, \widehat{\varphi})(t) = y(t)$$

and $\widehat{\varphi}(t) = \varphi \circ \pi(x(t))$ for some smooth function $\varphi : Z \to I$.

Σ is identifiable if it is identifiable at any admissible $(u(.), y(.))$.

Infinitesimal Identifiability

Definition 2:

$$T\Sigma : \begin{cases} \frac{d\xi}{dt} &= T_{x,\varphi} f(x, u, \varphi; \xi, \eta) \\ \widehat{y} &= d_{x,\varphi} h(x, u, \varphi; \xi, \eta) \end{cases}$$

where $(\xi, \eta) \in T_x X \times T_\varphi I$, we set

$$\begin{aligned} P^t_{T\Sigma}(\xi_0, \eta) &= d_{x,\varphi} h(x, u, \widehat{\varphi}; T_{x,\varphi}\phi_t(x, u, \widehat{\varphi}; \xi_0, \eta), \eta) \\ &= T_{x,\varphi} P^t_\Sigma(\xi_0, \eta) \end{aligned}$$

Σ is infinitesimally identifiable at $(x_{0,}\ u,\ \widehat{\varphi}) \in X \times L^\infty[U] \times L^\infty[I]$ if $P^t_{T\Sigma}$ is injective $\forall t > 0$

Σ is uniformly infinitesimally identifiable if this is true at all $(x_{0,}\ u,\ \widehat{\varphi})$

Differential Identifiability

Let $D_k\Phi = X \times (U \times \mathbf{R}^{(k-1)d_u}) \times (I \times \mathbf{R}^{k-1})$ be the space of k-jets of the system Σ $\left(j^k(u) = \left(u(0), u'(0), \ldots, u^{(k-1)}(0)\right)\right)$, we set

$$
\begin{array}{cccc}
\Phi_k^\Sigma: & D_k\Phi & \rightarrow & \mathbf{R}^{kd_y} \\
 & (x_0, j^k(u), j^k(\widehat{\varphi})) & \rightarrow & j^k(y)
\end{array}
$$

$$
\begin{array}{cccc}
\Phi_{k,2}^{\Sigma,*}: & D_k\Phi \times D_k\Phi & \rightarrow & \mathbf{R}^{kd_y} \times \mathbf{R}^{kd_y} \\
 & (z_1, z_2) & \rightarrow & (\Phi_k^\Sigma(z_1), \Phi_k^\Sigma(z_2))
\end{array}
$$

Definition 3: Σ is differentially identifiable of order k if
$\Phi_{k,2}^{\Sigma,*}(z_1, z_2) \in \Delta_k \Rightarrow (x_1, \widehat{\varphi}_1(0)) = (x_2, \widehat{\varphi}_2(0))$

Genericity (without control)

Proposition. Differential Identifiability $\Rightarrow$ Identifiability

Theorem 1.

- If $d_y \geq 3$, differential identifiability of order $2n+1$ is a **generic property** in the class of C^∞ systems.
- If $d_y < 3$, differential identifiability **is not** a generic property.

Passer la preuve

Proof of genericity 1/2

$$Z_i = \left(x_i, \varphi_i, \varphi_i', \ldots, \varphi_i^k, j_\Sigma^k(x_i, \varphi_i)\right),\ i = 1, 2$$
$$Z = (Z_1, Z_2)$$
$$\Phi(Z) = \Phi_k^\Sigma(Z_1) - \Phi_k^\Sigma(Z_2) \in R^{k\,d_y},$$
$$k = 2n + 1,\ d_y \geq 3$$

Let us suppose that Φ is a submersion

$\mathrm{codim}\Phi^{-1}(0) = k\,d_y$

Let $\Pi\Phi^{-1}(0) = \left(x_i, \varphi_i, j_\Sigma^k(x_i, \varphi_i)\right)_{i=1,2}$

$$\begin{aligned} \mathrm{codim}\Pi\Phi^{-1}(0) &\geq k\,d_y - 2\,(k-1) = k\,(d_y - 2) + 2 \\ &\geq k + 2 \geq 2n + 3 \end{aligned}$$

Proof of genericity 2/2

$$\begin{array}{rccc} \rho_\Sigma : & (X \times I)^2 \setminus \Delta & \to & \left(J^k_\Sigma\right)^2 \\ & (x_1, \varphi_1, x_2, \varphi_2) & \to & \left(x_i, \varphi_i, j^k_\Sigma(x_i, \varphi_i)\right)_{i=1,2} \end{array}$$

Multijet transversality theorem: the set of Σ such that ρ_Σ is transversal to $\Pi\Phi^{-1}(0)$ is residual.

$$\text{dim}\,(X \times I)^2 \setminus \Delta = 2n + 2$$
$$\Downarrow$$
$$\text{generically, } \rho_\Sigma \text{ avoids } \Pi\Phi^{-1}(0)$$

Single-output case

Theorem 2. If Σ is uniformly infinitesimally identifiable then

i) $\frac{\partial}{\partial\varphi}\left\{h, L_{f_\varphi}h, \ldots, (L_{f_\varphi})^{n-1}h\right\} \equiv 0$

ii) $\frac{\partial}{\partial\varphi}L_{f_\varphi}^{n}h \neq 0$

iii) $d_x h \wedge \ldots \wedge d_x L_{f_\varphi}^{n-1}h \neq 0$,

Therefore, locally, the system can be written

$$\begin{cases} \dot{x}_1 &=& x_2 \\ & \vdots & \\ \dot{x}_{n-1} &=& x_n \\ \dot{x}_n &=& \psi(x,\varphi) \\ y &=& x_1 \end{cases} \quad \text{and } \frac{\partial}{\partial\varphi}\psi(x,\varphi) \neq 0$$

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Single-output case, pseudo-converse

Theorem 3. If Σ meets the following conditions,

i) $\frac{\partial}{\partial \varphi}\left\{h, L_{f_\varphi}h, \ldots, (L_{f_\varphi})^{n-1}h\right\} \equiv 0$

ii) $\frac{\partial}{\partial \varphi} L^n_{f_\varphi} h \neq 0$

iii) $d_x h \wedge \ldots \wedge d_x L^{n-1}_{f_\varphi} h \neq 0$,

then Σ is

1) locally identifiable,

2) loc. unif. infinitesimally identifiable,

3) loc. diff. identifiable of order $n+1$.

Passer la preuve

Proof of the single-output case 1/2

Let $k < n$ be the first k such that $d_\varphi L_f^k h \not\equiv 0$:

$$\Sigma \quad \begin{cases} y & = & x_1 \\ \dot{x}_1 & = & x_2 \cdots \\ \dot{x}_{k-1} & = & x_k \\ \dot{x}_k & = & L_f^k (x, \varphi) = f_k(x, \varphi) \cdots \\ \dot{x}_n & = & f_n(x, \varphi) \end{cases}$$

$$T\Sigma \quad \begin{cases} \dot{x} & = & f\,(x, \varphi) \\ \widehat{y} & = & \xi_1 \\ \dot{\xi}_1 & = & \xi_2 \cdots \\ \dot{\xi}_{k-1} & = & \xi_k \\ \dot{\xi}_k & = & d_x f_k\,(x, \varphi)\,\xi + d_\varphi f_k\,(x, \varphi)\,\eta \end{cases}$$

Proof of the single-output case 2/2

A feedback $\eta = -\dfrac{d_x f_k(x, \varphi_0)\,\xi}{d_\varphi f_k(x, \varphi_0)}$ in φ_0 s.t. $d_\varphi f_k(x, \varphi_0) \neq 0$ gives $\frac{d\xi_k}{dt} = 0$ which contradict observability.

If $\frac{\partial}{\partial \varphi} L^n_{f_\varphi} h = 0$ at (x, φ)

$$\begin{array}{ccc} X \times I & \supset & E = \left\{(x,\varphi)\,;\; d_\varphi L^n_f h = 0\right\} \\ & & \downarrow \Pi \\ X & \supset & \Pi E \end{array}$$

Hardt's theorem $\Rightarrow \exists \widehat{\varphi}$

$$\left\{ \begin{array}{ccccccccc} y & = & x_1, & \dot{x}_1 & = & x_2, & \ldots & \dot{x}_n & = & \psi(x, \widehat{\varphi}(x)) \\ \widehat{y} & = & \xi_1, & \dot{\xi}_1 & = & \xi_2, & \ldots & \dot{\xi}_n & = & d_x\psi(x, \widehat{\varphi}(x)) + 0 \end{array} \right.$$

Two output case: definitions of k and r

Define $E_l = \left\{ d_x h_i, d_x L_{f_\varphi} h_i, \ldots, d_x L_{f_\varphi}^{l-1} h_i \,,\; i = 1, 2 \right\}$ and $N(l) = \text{rank}\,(E_l)$ at a generic point:

k is defined by

$N(0)$	$N(1)$	$\cdots$	$N(k-1)$	$N(k)$	$N(k+1)$	$\cdots$	$N(k+m)$
0	2		$2k-2$	$2k$	$2k+1$		$2k+m$

$(2k + m \leq n)$

The **order** of the system is the first integer r such that $d_\varphi L_{f_\varphi}^r (h_1, h_2) \neq 0$.

Classification

Lemma: If Σ is uniformly infinitesimally identifiable then (1) $2k + m = n$

(2) $r \leq k + m$

Proof:

(1) $\varphi = \varphi_0$ =cte $\begin{cases} \dot{x} = f(x, \varphi_0) \\ \dot{\xi} = g(x, \xi, \varphi_0) \\ y = h(x, \varphi_0) \end{cases}$ contradict observability

(2) $\begin{cases} \dot{x} = f(x) \\ y = h(x) \end{cases}$ contradict identifiability

Définition 5. A system Σ is regular if (1) and (2) holds.

Type 3: $r = k$ and $n = 2k$

$$\begin{cases} y_1 = x_1 & y_2 = x_2 \\ \dot{x}_1 = x_3 & \dot{x}_2 = x_4 \\ \vdots & \vdots \\ \dot{x}_{n-3} = x_{n-1} & \dot{x}_{n-2} = x_n \\ \dot{x}_{n-1} = f_{n-1}(x, \varphi) & \dot{x}_n = f_n(x, \varphi) \end{cases}$$

with $\frac{\partial}{\partial \varphi}(f_{n-1}, f_n) \neq 0$

$N(l)$ increases by steps of 2 until the last derivative and apparition of φ.

Type 1: *r>k*

$$\begin{cases} y_1 = x_1 & y_2 = x_2 \\ \dot{x}_1 = x_3 & \dot{x}_2 = x_4 \\ \quad\vdots & \quad\vdots \\ \dot{x}_{2k-3} = x_{2k-1} & \dot{x}_{2k-2} = x_{2k} \\ \dot{x}_{2k-1} = f_{2k-1}(x_1, ..., x_{2k+1}) \\ \dot{x}_{2k} = x_{2k+1} \\ \quad\vdots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = f_n(x, \varphi) \end{cases}$$

with $\frac{\partial f_n}{\partial \varphi} \neq 0$.

$N(l)$ increases by steps of 1 when φ appears for the first time, $\simeq$ single-output case.

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Type 2: $r<k$

$$
\begin{array}{lllllll}
y_1 & = & x_1 & y_2 & = & x_2 \\
\dot{x}_1 & = & x_3 & \dot{x}_2 & = & x_4 \\
 & \vdots & & & \vdots & \\
\dot{x}_{2r-3} & = & x_{2r-1} & \dot{x}_{2r-2} & = & x_{2r} \\
\dot{x}_{2r-1} & = & \psi(x,\varphi) & \dot{x}_{2r} & = & F_{2r}(x_1,\ldots,x_{2r+1},\psi(x,\varphi)) \\
 & & & \dot{x}_{2r+1} & = & F_{2r+1}(x_1,\ldots,x_{2r+2},\psi(x,\varphi)) \\
 & & & & \vdots & \\
 & & & \dot{x}_{n-1} & = & F_{n-1}(x,\psi(x,\varphi)) \\
 & & & \dot{x}_n & = & F_n(x,\varphi)
\end{array}
$$

with $\frac{\partial\psi}{\partial\varphi} \neq 0, \frac{\partial F_{2r}}{\partial x_{2r+1}} \neq 0, \ldots., \frac{\partial F_{n-1}}{\partial x_n} \neq 0$

φ appears when N (l) increases by steps of 2.

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$r=k$ and $2k<n$

If $r = k$ before the last derivative:

$$d_x h_1 \wedge \cdots \wedge d_x L_{f_\varphi}^{k-1} h_1 \wedge d_x L_{f_\varphi}^{k-1} h_2 \wedge d_x L_{f_\varphi}^{k} h_2 \not\equiv 0$$

If $d_\varphi L_{f_\varphi}^{k} h_1 \neq 0$, we obtain φ using y_1 and $x_{2k}, \ldots, x_n$ using y_2

$\longrightarrow$ **Type 2**

If $d_\varphi L_{f_\varphi}^{k} h_1 \equiv 0$, we obtain φ using y_2

$\longrightarrow$ **Type 1**

Canonical form for observer construction

$$\begin{cases} \dot{x}_1 &= F_1(x_1, x_2, u) \\ \dot{x}_2 &= F_2(x_1, x_2, x_3, u) \\ &\vdots \\ \dot{x}_n &= F_n(x, u) \end{cases} \qquad \frac{\partial F_1}{\partial x_2} \neq 0 \quad \frac{\partial F_2}{\partial x_3} \neq 0$$

$$\xi_1 = y = x_1, \ \xi_2 = F_1(x_1, x_2, u)$$

$$\xi_3 = \frac{\partial F_1}{\partial x_2} F_2(x_1, x_2, u), \cdots$$

$$\xi_{i+1} = \frac{\partial F_1}{\partial x_2} \cdots \frac{\partial F_{i-1}}{\partial x_i} F_i\left(x_1, \ldots, x_{i+1}, u\right)$$

$$\downarrow$$

$$\begin{cases} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= \xi_3 + \frac{\partial F_1}{\partial x_1}\dot{x}_1 + \frac{\partial F_1}{\partial u}\dot{u} \\ &\vdots \\ \dot{\xi}_n &= G(x, u, \dot{u}) \end{cases}$$

Ref. H. **Hammouri, M. Farza,** *Nonlinear observers for local uniform observable systems,* to appear

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Canonical form of observability

$$\begin{cases} \dfrac{dx}{dt} & = A(t)x + b(x,u) \\ y & = C(t)x \end{cases}$$

$$A(t) = \begin{pmatrix} 0 & a_2(t) & 0 & \cdots & 0 \\ & & a_3(t) & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ & & & & a_n(t) \\ 0 & & \cdots & & 0 \end{pmatrix}$$

$$C(t) = \begin{pmatrix} a_1(t) & 0 & \cdots & 0 \end{pmatrix}$$

$$0 < a_m \leq a_i(t) \leq a_M$$

$$b(x,u) = b_1(x_1,u)\frac{\partial}{\partial x_1} + b_2(x_1,x_2,u)\frac{\partial}{\partial x_2} + b_n(x_1,\ldots,x_n,u)\frac{\partial}{\partial x_n}$$

Modified Extended Kalman filter

$$
\begin{aligned}
\frac{dz}{dt} &= A(t)z + b(z,u) - S(t)^{-1}C(t)' r^{-1}(C(t)z - y(t)) \\
\frac{dS}{dt} &= -(A(t) + b^*(z,u))'S - S(A(t) + b^*(z,u)) \\
&\quad + C(t)' r^{-1} C(t) - S Q_\theta S
\end{aligned}
$$

$$
\Delta = \begin{pmatrix} 1 & & & \\ & \frac{1}{\theta} & & \\ & & \ddots & \\ & & & (\frac{1}{\theta})^{n-1} \end{pmatrix}
\qquad Q_\theta = \theta^2 \Delta^{-1} Q \Delta^{-1}
$$

If θ is large, high-gain observer (HGEKF)

If $\theta \approx 1$, Classical Extended Kalman filter (EKF)

Modified Extended Kalman filter

$$\begin{aligned}\frac{dz}{dt} &= A(t)z + b(z,u) - S(t)^{-1}C(t)'r^{-1}(C(t)z - y(t))\\ \frac{dS}{dt} &= -(A(t) + b^*(z,u))'S - S(A(t) + b^*(z,u))\\ &\quad +C(t)'r^{-1}C(t) - SQ_\theta S\end{aligned}$$

$$\frac{d\theta}{dt} = \lambda(1-\theta)$$

$$\Delta = \begin{pmatrix} 1 & & & \\ & \frac{1}{\theta} & & \\ & & \ddots & \\ & & & (\frac{1}{\theta})^{n-1} \end{pmatrix} \qquad Q_\theta = \theta^2\Delta^{-1}Q\Delta^{-1}$$

If θ is large, high-gain observer (HGEKF)

If $\theta \approx 1$, Classical Extended Kalman filter (EKF)

Theorem

There exist $\lambda_0 > 0$ such that for any $0 \leq \lambda \leq \lambda_0$, there exist θ_0 such that for any $\theta(0) > \theta_0$, for any $S(0) \geq c\ Id$, for any compact $K \subset \mathbf{R}^n$, for any $z(0) \in K$ then if we set $\varepsilon(t) = z(t) - x(t)$ for any $t \geq 0$

$$||\varepsilon(t)||^2 \leq R(\lambda, c) e^{-at} \Lambda(\theta(0), t, \lambda) ||\varepsilon(0)||^2 \quad (1)$$

where

$$\Lambda(\theta(0), t, \lambda) = \theta(0)^{2(n-1)+\frac{a}{\lambda}} e^{-\frac{a}{\lambda}\theta(0)(1-e^{-\lambda t})}$$

and a is a positive constant and $R(\lambda, c)$ is a decreasing function of c.

Passer la preuve

Proof

Change of variables $\begin{cases} \tilde{x} &= \Delta x \\ \tilde{P} &= \frac{1}{\theta}\Delta P \Delta \end{cases}$ $\left(P = S^{-1}\right)$

$+$ time change $d\tau = \theta(t)\, dt$

We set $\varepsilon = z - x = error$ then we calculate ${}^{T}\varepsilon(\tau)\, S(\tau)\, \varepsilon(\tau)$.

Observability give us $\alpha I \leq S(\tau) \leq \beta I$ then

$$ {}^{T}\varepsilon(\tau)\, S(\tau)\, \varepsilon(\tau) \longrightarrow 0 \Longleftrightarrow \varepsilon(\tau) \longrightarrow 0 $$

When $\tau \leq T$

$$ \|\varepsilon(\tau)\|^2 \leq \theta(\tau)^{2(n-1)}\, H(c)\, e^{-(a_1\theta(T)-a_2)\tau}\, \|\varepsilon(0)\|^2 $$

Parallel high-gain and non-high-gain EKF

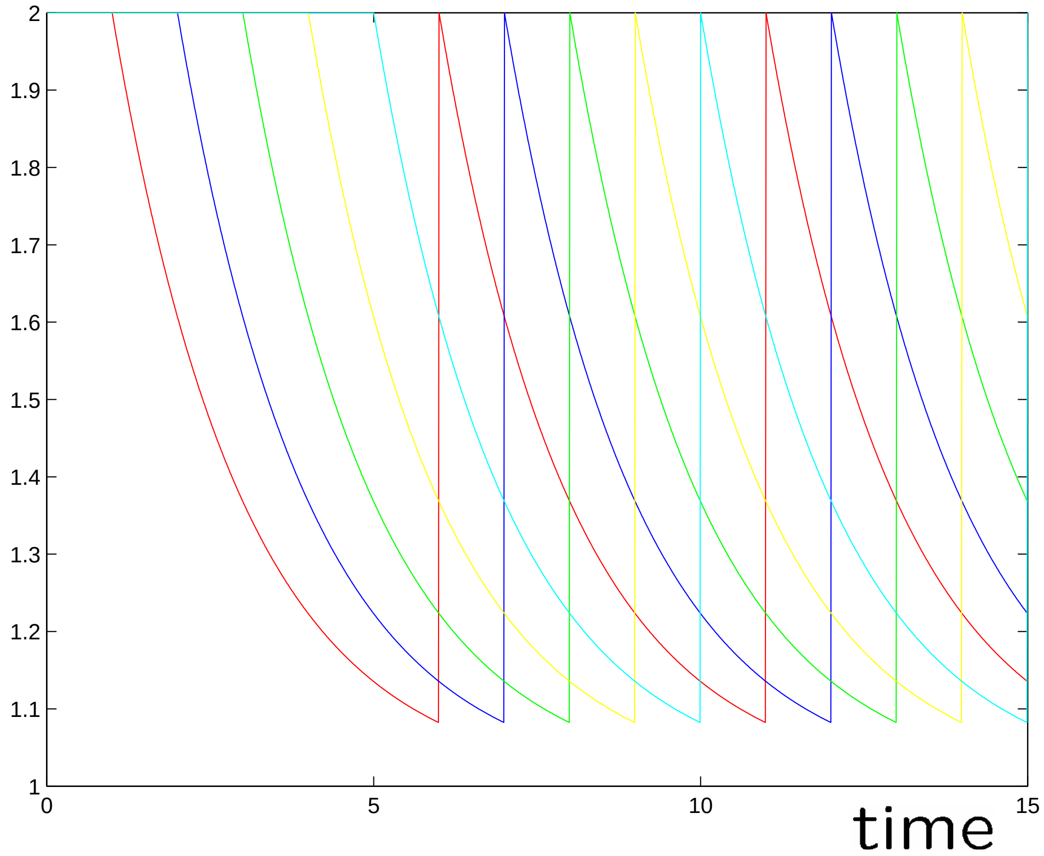
We use N observers in parallel. At times kT :

- a new observer is initialized with $\theta(kT) = \theta_0$,
- the older observer is killed.

Therefore, at any time t, we have N observers initialized at times kT, $(k-1)T \ldots (k-N+1)T$ where $k = \left\lfloor \frac{t}{T} \right\rfloor$.

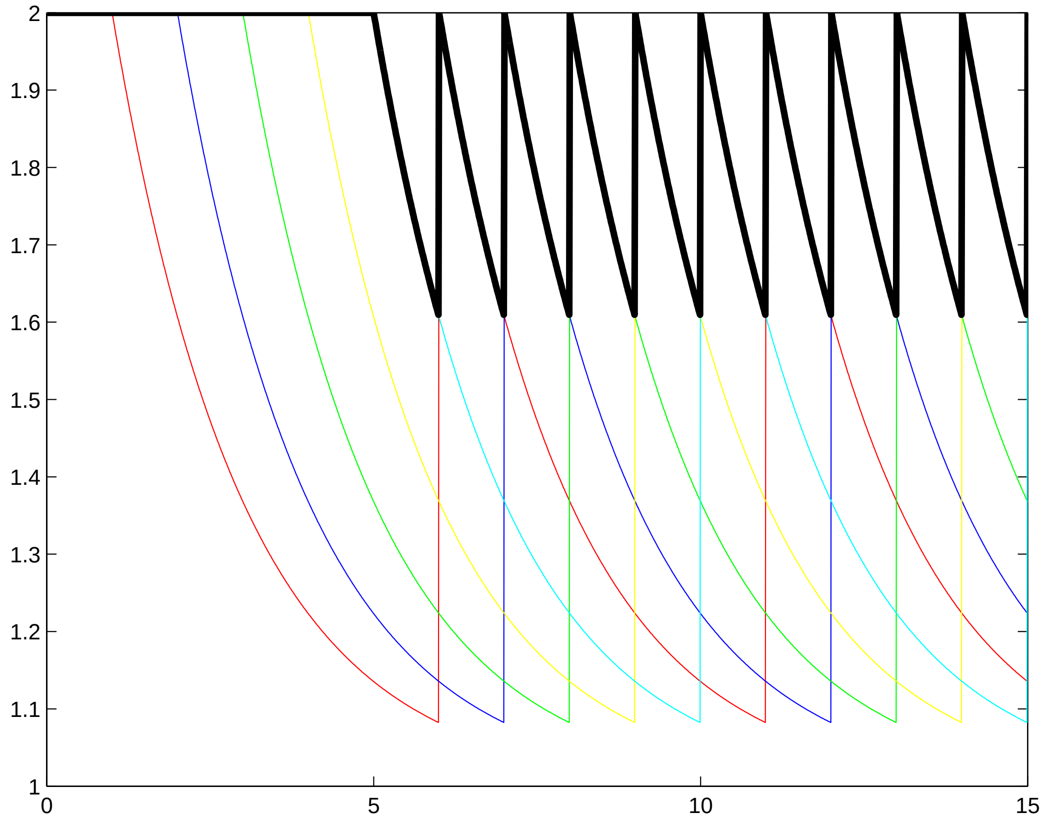
State estimation: the estimation given by the observer with smallest innovation $\|y - C\widehat{x}\|$.

Parallel Extended Kalman Filter



θ for 5 observers

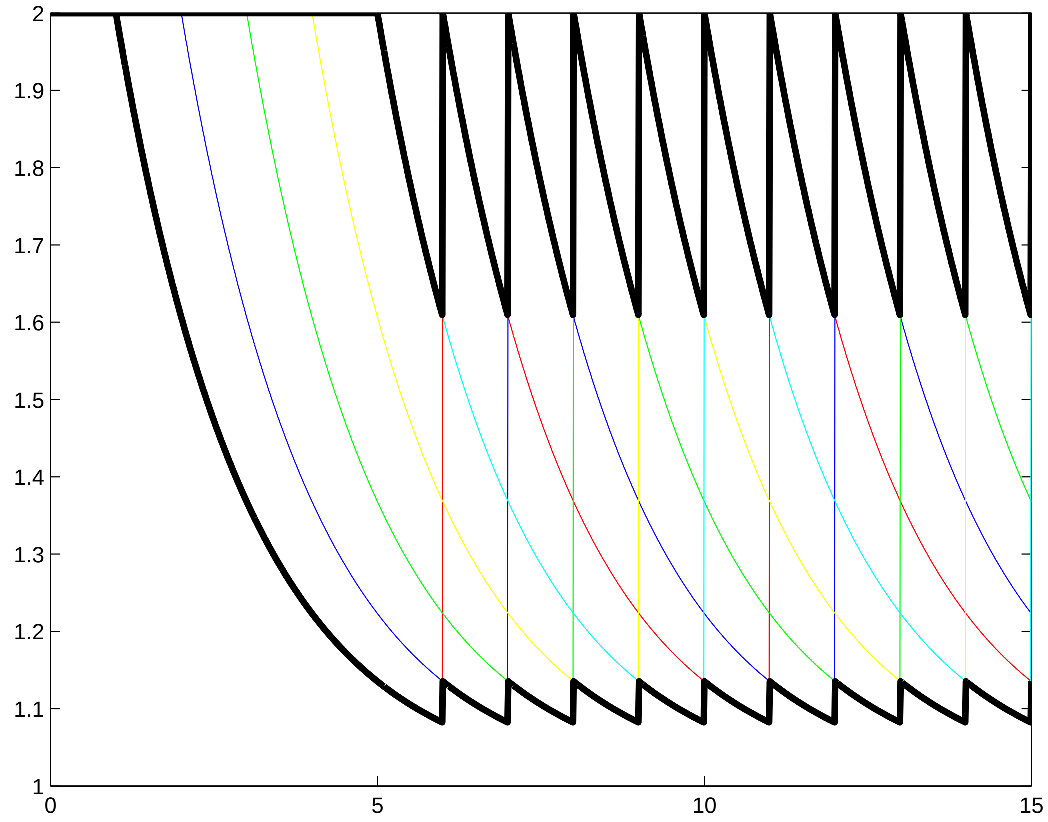
High-gain Extended Kalman Filter



θ for the youngest observer is

$$1 + e^{-\lambda(t-kT)}(\theta_0 - 1) \geq 1 + e^{-\lambda T}(\theta_0 - 1)$$

Standard Extended Kalman Filter



θ for the oldest observer is

$$1 + e^{-\lambda(t-kT+(N+1)T)}(\theta_0 - 1) \approx 1$$

Extended Kalman filtering equations

$$\frac{dx}{dt} = F(x, u)$$

Our *diffeomorphism* $\xi = \varphi(x, u)$ depend on u supposed to be smooth, hence:

$$\begin{aligned}\frac{d\xi}{dt} &= D_\varphi\left(\varphi_u^{-1}(\xi)\right) f\left(\varphi_u^{-1}(\xi), u\right) + \frac{\partial\varphi\left(\varphi_u^{-1}(\xi), u\right)}{\partial u}\dot{u} \\ &= F(\xi, u, \dot{u})\end{aligned}$$

$$\begin{cases} \frac{d\hat{\xi}}{dt} &= F\left(\hat{\xi}, u, \dot{u}\right) + PC^T R^{-1}\left(y - C\hat{\xi}\right) \\ \frac{dP}{dt} &= F^*\left(\hat{\xi}, u, \dot{u}\right) P + PF^*\left(\hat{\xi}, u, \dot{u}\right) + Q_\theta - PC^T R^{-1} CP \end{cases}$$

In the original coordinates

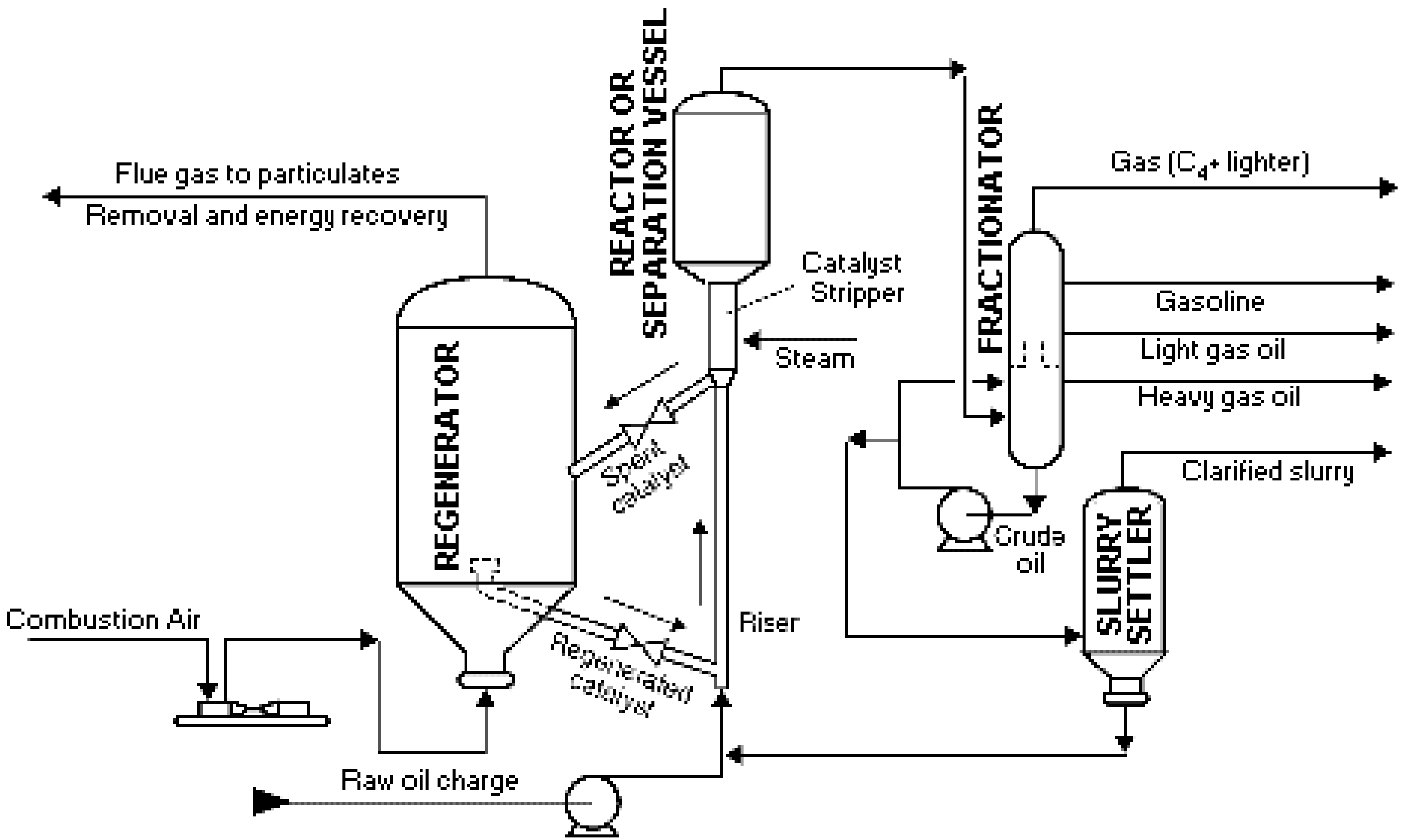
Since $C\varphi_u(x) = Cx$, equations are those of a modified extended Kalman filter

$$\left\{\begin{array}{l} \frac{d\hat{x}}{dt} = f(\hat{x},u) + pC^T R^{-1}(y - C\hat{x}) \\ \frac{dp}{dt} = f^*(\hat{x},u)\,p + pf^*(\hat{x},u)^T + q_\theta(\hat{x}) \\ \qquad -ph^*(\hat{x},u)^T R^{-1} h^*(\hat{x},u)\,p \\ \qquad +D_{\psi_u}^{-1}(\hat{x})\,D_{\psi_u}^2 \cdot \left(ph^*(\hat{x},u)^T R^{-1}(h(\hat{x},u) - y)\right) p \\ \qquad +pD_{\psi_u}^2 \cdot \left(ph^*(\hat{x},u)^T R^{-1}(h(\hat{x},u) - y)\right) D_{\psi_u}^{-1}(\hat{x})^T \end{array}\right.$$

where $q_\theta(\hat{x}) = D_{\varphi_u}(\hat{x})^{-1} Q_\theta \left(D_{\varphi_u}(\hat{x})^{-1}\right)^T$

The two last lines (transposed) correspond to the change of coordinate.

Application: Fluid Catalytic Cracker (FCC)



Reactor model

$$S_c H_{ra} \dot{T}_{ra} = S_c R_c \left(T_{rg} - T_{ra}\right) + S_{tf} R_{tf} \left(T_{tf} - T_{ra}\right) - \Delta H_{fv} R_{tf} - \Delta H_{cr} R_{tf} C_{tf}$$

$$C_{tf} = \frac{1}{1 + \frac{R_{tf}}{R_{cr}}} \qquad R_{cr} = K_{cr} P_{ra} H_{ra}$$

$$C_{cat}^2 = \frac{100 P_{ra} H_{ra}}{R_c C_{rc}^{0.06}} k_{cc} \exp\left(-\frac{A_{cc}}{R T_{ra}}\right) \qquad K_{cr} = \frac{k_{cr}}{C_{cat} C_{rc}^{0.15}} \exp\left(-\frac{A_{cr}}{R T_{ra}}\right)$$

$$H_{ra} \dot{C}_{sc} = R_c \left(C_{rc} - C_{sc}\right) + R_{cf}$$

$$R_{cf} = R_{cc} + R_{ad} \qquad R_{ad} = F_{cf} R_{tf} \qquad R_{cc} = K_{cc} P_{ra} H_{ra}$$

$$K_{cc} = \frac{k_{cc}}{C_{cat} C_{rc}^{0.06}} \exp\left(-\frac{A_{cc}}{R T_{ra}}\right)$$

Regenerator model

$$S_c H_{rg} \dot{T}_{rg} = S_c R_c (T_{ra} - T_{rg}) + S_a R_{ai} (T_{ai} - T_{rg}) + \Delta H_{rg} R_{cb}$$

$$R_{cb} = \frac{R_{ai}}{242} \left(21 - O_{fg}\right) \qquad O_{fg} = 21 \exp\left(\frac{-\frac{P_{rg} H_{rg}}{R_{ai}}}{\frac{1}{K_{od}} + \frac{1}{K_{or} C_{rc}}}\right)$$

K_{or} =unknown function of T_{rg}.

$$K_{od} = 6.34\ 10^{-9} R_{ai}^2$$

$$H_{rg} \dot{C}_{rc} = R_c (C_{sc} - C_{rc}) - R_{cb}$$

New form of the system

$$\begin{aligned}\varphi(x;u) &= \varphi\left(T_{rg}, T_{ra}, C_{rc}, C_{sc}, F_{cf}; R_{ai}\right)\\ &= \left(T_{rg}, T_{ra}, C_{tf}\left(C_{rc}, T_{ra}\right), \frac{C_{sc}}{C_{rc}}, \frac{F_{cf}}{C_{rc}}\right) = \xi\end{aligned}$$

$$\begin{cases}\dot{x}_1 = \dot{T}_{rg} &= \psi\left(x, \varphi\left(x_1\right), u\right)\\ \dot{x}_2 = \dot{T}_{ra} &= a_3(t)x_3 + f_2\left(x_1, x_2\right)\\ \dot{x}_3 \simeq \dot{C}_{rc} &= a_4(t)x_4\\ & \quad + f_3\left(x_1, x_2, x_3, \psi\left(x, \varphi\left(x_1\right), u\right), u, \dot{u}\right)\\ \dot{x}_4 \simeq \dot{C}_{sc} &= a_5(t)x_5 + f_4\left(x_1, x_2, x_3, x_4\right)\\ \dot{x}_5 \simeq \dot{F}_{cf} &= F(x)\end{cases}$$

Type 2

Here, $\psi = R_{cb}$, $\varphi = K_{or}$ and $\pi(x) = T_{rg} = x_1$.

$u = (R_{ai}, P_{ra})$

Tuning

We use a second order system to estimate K_{or} i.e. $\dfrac{d^3 K_{or}}{dt^3} = 0$

We use three parallel extended Kalman filters such that

- $\theta_0 = 3$ (starting value for each observers)
- $\theta_{HG} = 2$ (minimal value of θ ensuring high-gain)
- Time between two consecutive initializations: 2 hours

EKF with two outputs

At last,

$$\xi = \left(T_{rg}, R_{cb}, \dot{K}_{or}, \ddot{K}_{or}, T_{ra}, C_{tf}, \frac{C_{sc}}{C_{rc}}, \frac{F_{cf}}{C_{rc}} \right)$$

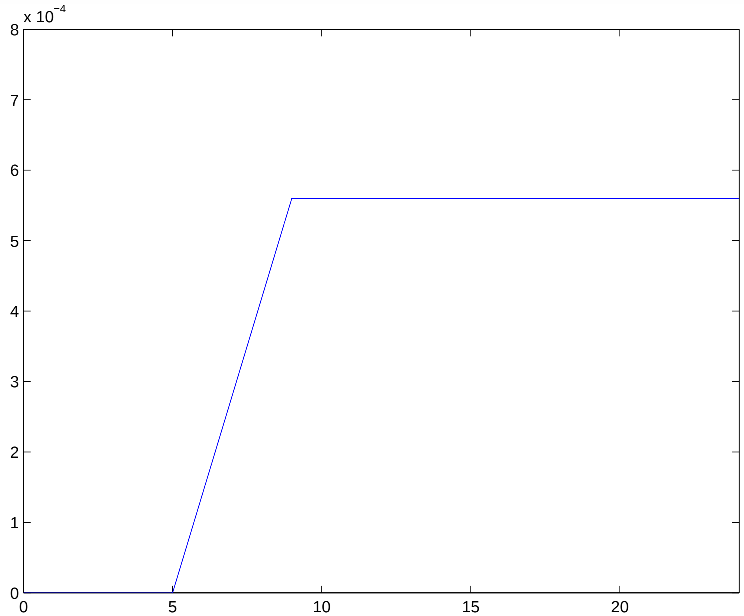
and

$$\Delta^{-1} = \mathrm{diag}\left(1, \theta, \theta^2, \theta^3, 1, \theta, \theta^2, \theta^3\right)$$

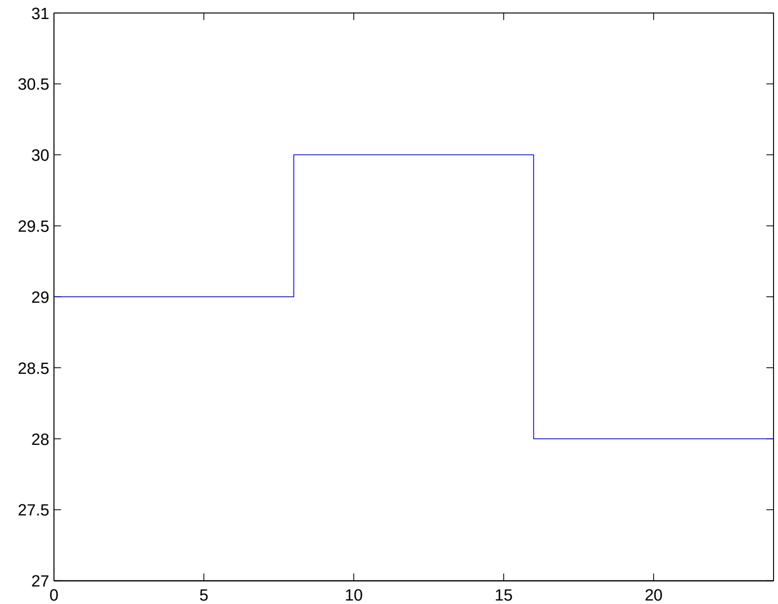
with $Q_\theta = \theta^2 \Delta^{-1} Q \Delta^{-1}$
and $R_\theta = \left(C\Delta^{-1}C'\right) R \left(C\Delta^{-1}C'\right)$

Simulations

Colored noise (Ornstein–Uhlenbeck process) on both measured variables T_{rg} and T_{ra}.



Unknown parameter F_{cf}



Control variable R_{ai}

$$K_{or} = 1.16\ 10^{-5} \exp\left(\frac{A_{or}}{R\left(\frac{1}{866.7} - \frac{1}{T_{rg}}\right)} \right)$$

Constants of the model

Reactor operating conditions

$H_{ra} = 1.85\ 10^{-4}$, $P_{ra} = 211.7$,

Feed properties

$R_{tf} = 41$, $T_{tf} = 492.8$, $S_{tf} = 3140$,

Cat.recirculation

$R_c = 290$, $S_c = 1047$,

Heat constants

$\Delta H_{cr} = 4.65\ 10^5$, $\Delta H_{fv} = 1.7410^5$,

$\Delta H_{rg} = 3.02\ 10^7$, $R = 8.314$

$k_{cr} = 8.31\ 10^{-2}$, $A_{cr} = 6.28\ 10^4$,

$k_{cc} = 2.66\ 10^{-4}$, $A_{cc} = 4.18\ 10^4$

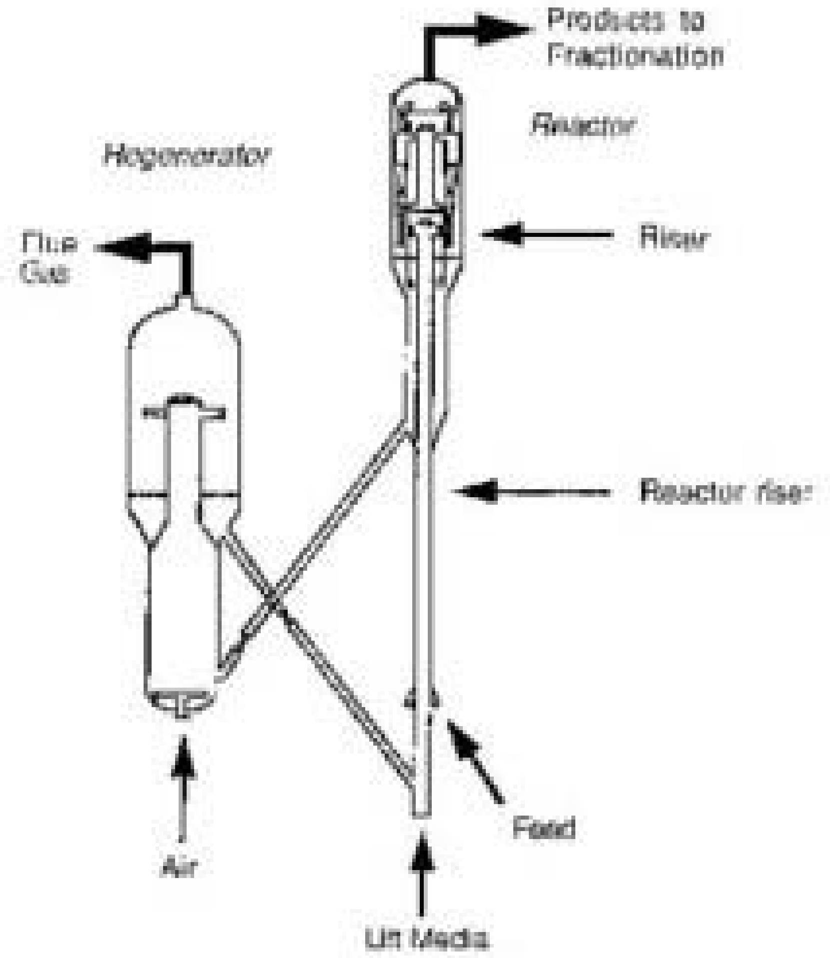
Regenerator operating conditions

$H_{rg} = 1.53\ 10^5$, $P_{rg} = 254.4$,

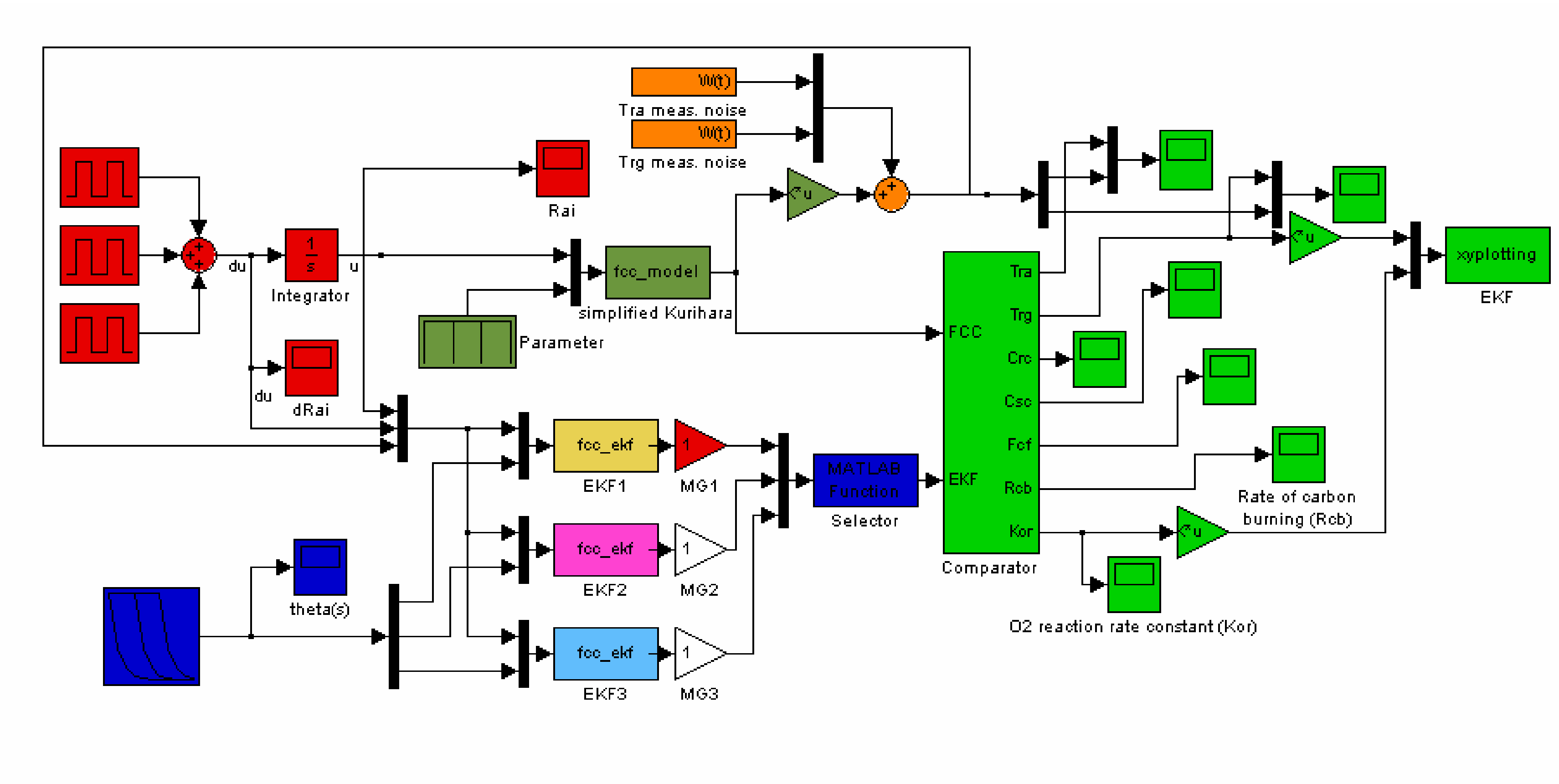
Air properties

$R_{ai} = 26$, $T_{ai} = 394$, $S_{ai} = 1130$

$A_{or} = 1.47\ 10^5$



Simulation using Matlab/Simulink



to be continued…