Simple Priced Timed Games are not that simple

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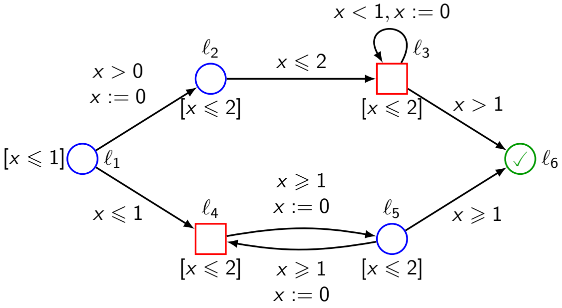
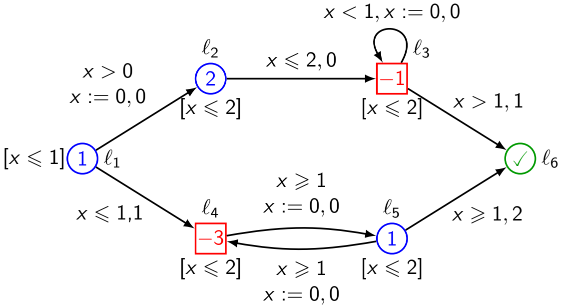
Priced Timed Games



Timed Automaton
with partition of states
between 2 players
+ reachability objective
+ rates in locations
+ costs over transitions

Semantics in terms of
infinite game with weights

Priced Timed Games



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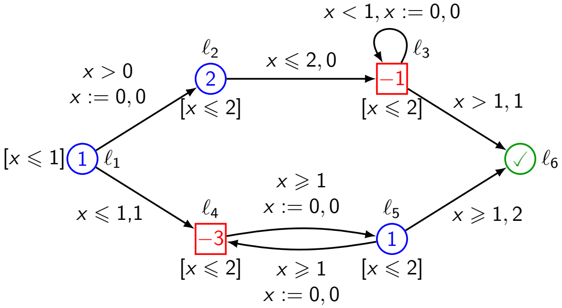
$(\ell_1, 0)$

Priced Timed Games



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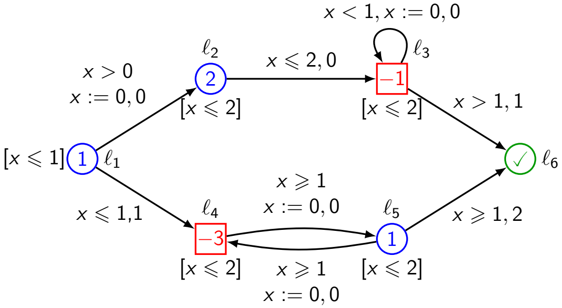
$$(\ell_1, 0) \xrightarrow{0.4, \searrow} (\ell_4, 0.4)$$

Priced Timed Games



$(\ell_1, 0) \xrightarrow{0.4, \searrow} (\ell_4, 0.4) \xrightarrow{0.6, \rightarrow} (\ell_5, 0)$

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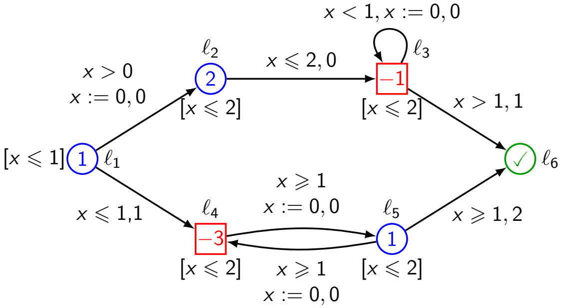

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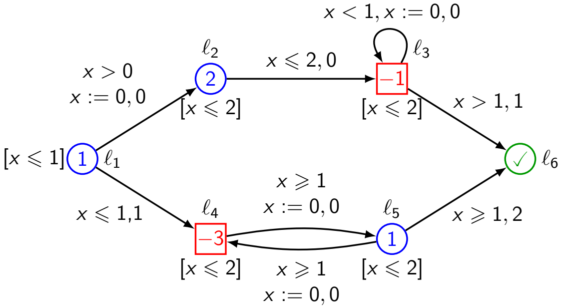
Priced Timed Games



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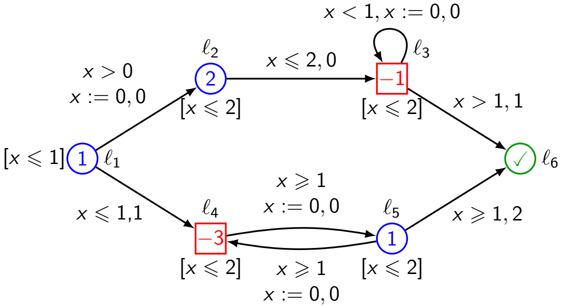
$$0.4 + 1 \quad -3 \times 0.6 \quad +1.5 \quad -3 \times 1.1 \quad +2 \times 2 + 2 \quad = 3.8$$

Priced Timed Games



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$$(\ell_1, 0) \xrightarrow[0.4+1]{0.4, \searrow} (\ell_4, 0.4) \xrightarrow[-3 \times 0.6]{0.6, \rightarrow} (\ell_5, 0) \xrightarrow[+1.5]{1.5, \leftarrow} (\ell_4, 0) \xrightarrow[-3 \times 1.1]{1.1, \rightarrow} (\ell_5, 0) \xrightarrow[+2 \times 2 + 2]{2, \nearrow} (\checkmark, 2) \quad = 3.8$$

$$(\ell_1, 0) \xrightarrow[0.2]{0.2, \nearrow} (\ell_2, 0) \xrightarrow[+0.9]{0.9, \rightarrow} (\ell_3, 0.9) \xrightarrow[-0.2]{0.2, \circlearrowleft} (\ell_3, 0) \xrightarrow[-0.9]{0.9, \circlearrowleft} (\ell_3, 0) \quad \cdots \quad = +\infty \ (\checkmark \text{ not reached})$$

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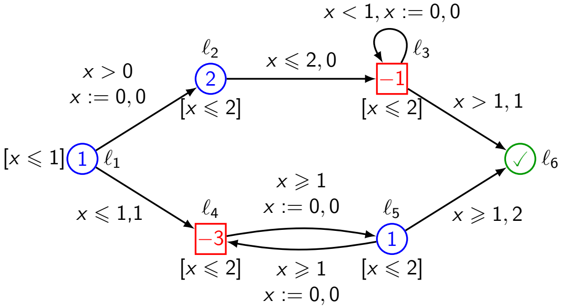

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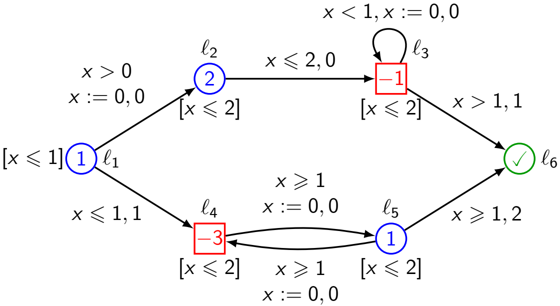
$$0.2 \qquad +0.9 \qquad -0.2 \qquad -0.9 \qquad \cdots \qquad = +\infty \ (\checkmark \text{ not reached})$$

Cost of a play: $\begin{cases} +\infty & \text{if } \checkmark \text{ not reached} \\ \text{total payoff up to } \checkmark & \text{otherwise} \end{cases}$

Strategies and objectives



Strategy for each player: mapping of finite runs to a delay and an action

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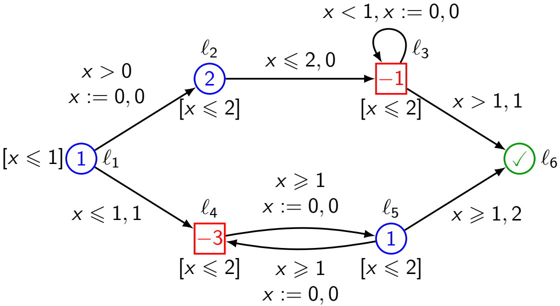

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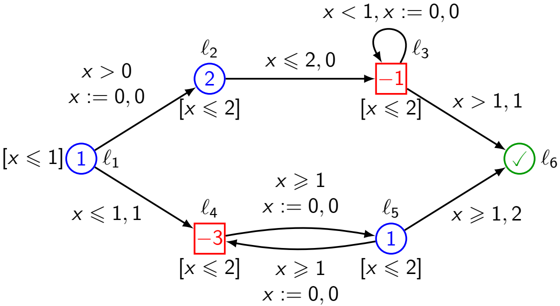

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Main object of interest:

$$\overline{\mathsf{Val}}(\ell, v) = \inf_{\sigma_\bigcirc \in \mathsf{Strat}_\bigcirc} \sup_{\sigma_\square \in \mathsf{Strat}_\square} \mathsf{Wt}(\mathsf{Play}((\ell, v), \sigma_\bigcirc, \sigma_\square)) \in \mathbf{R} \cup \{-\infty, +\infty\}$$

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What can players guarantee as a payoff? and design *good* strategies

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$\mathsf{F}_{\leqslant K}\checkmark$: $\exists$ a strategy in the PTG (priced timed game) for player $\bigcirc$ reaching $\checkmark$ with a cost $\leqslant K$?

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This talk: **PTGs with negative costs**

More complex when negative costs

- Value $-\infty$: detection is as hard as mean-payoff. No hope for complexity better than $\mathbf{NP} \cap \mathbf{co\text{-}NP}$, or pseudo-polynomial

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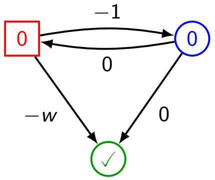
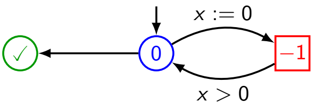
- Memory complexity
  - Player $\bigcirc$ needs memory, even in the untimed setting



  - Player $\square$ may require infinite memory



Known results with negative costs [Brihaye, Geeraerts, Krishna, Manasa, Monmege, and Trivedi, 2014]

- $\mathsf{F}_{\leqslant K}$ ✓ undecidable for 2 or more clocks

Proof by reduction of 2-counter machines.

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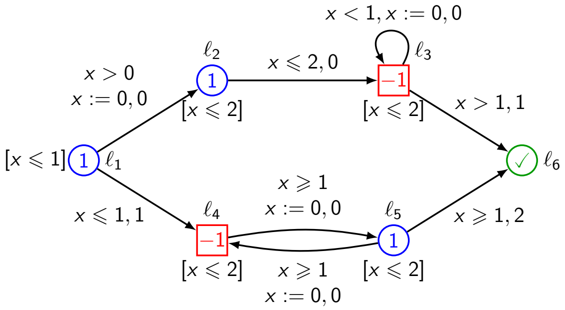
- Pseudo-polynomial algorithm for One-clock Bi-valued PTG

Assumption: rates of locations $\{p^-, p^+\}$ **included in** $\{0, +d, -d\}$ **($d \in \mathbf{N}$)** (no assumption on costs of transitions)



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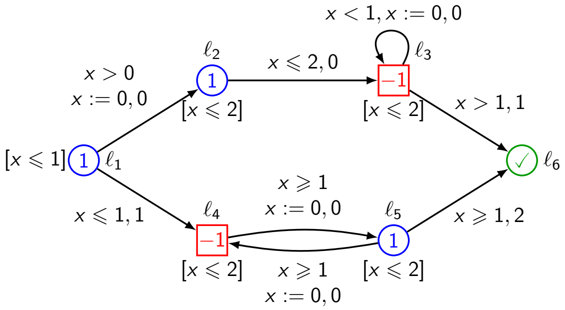
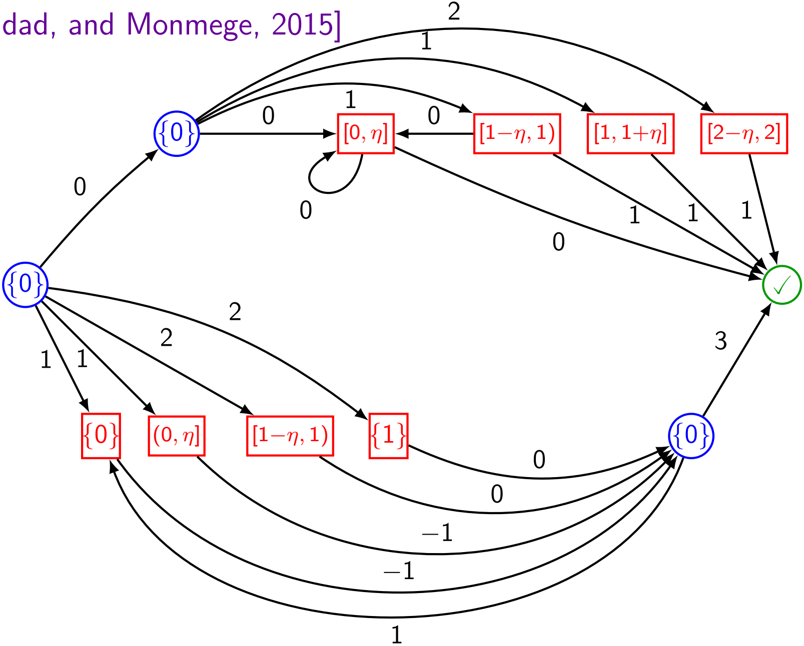


Method: Corner point abstraction.

Solving min-cost reachability games [Brihaye, Geeraerts, Haddad, and Monmege, 2015]



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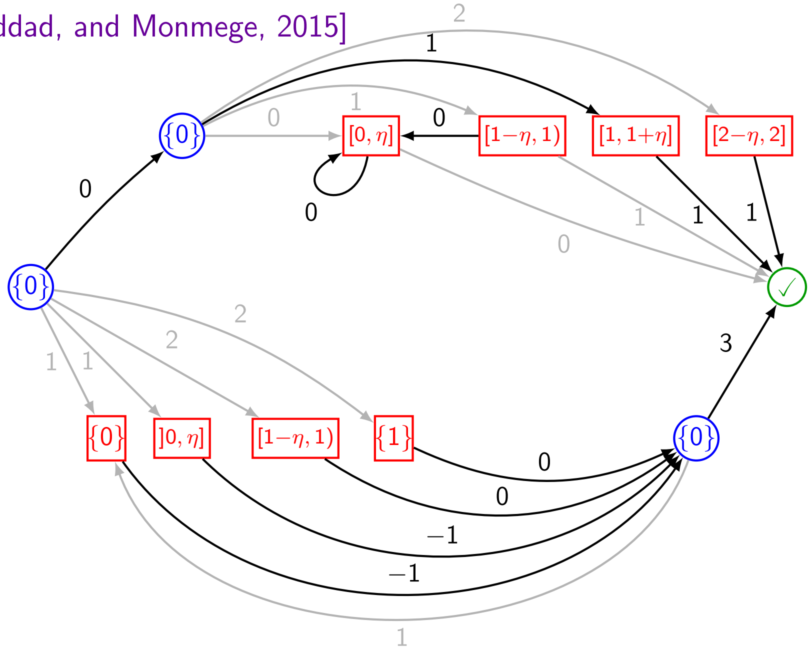
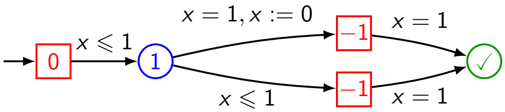


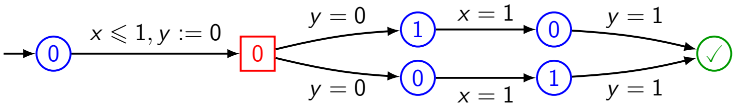
1BPTG: maximal fragment for corner-point abstraction

Players may need to play far from corners...

- With 3 weights in $\{-1, 0, +1\}$: value $1/2$...



- With 2 weights in $\{-1, 0, +1\}$ but 2 clocks: value $1/2$...



Inspired by other previous techniques for 1-clock PTGs?

[Hansen, Ibsen-Jensen, and Miltersen, 2013]: strategy improvement algorithm

[Bouyer, Larsen, Markey, and Rasmussen, 2006b, Rutkowski, 2011]: iterative elimination of locations

- precomputation: polynomial-time cascade of simplification of 1-clock PTGs into simple 1-clock PTGs (SPTGs)
  - clock bounded by 1, no guards/invariants, no resets

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- precomputation: polynomial-time cascade of simplification of 1-clock PTGs into simple 1-clock PTGs (SPTGs)
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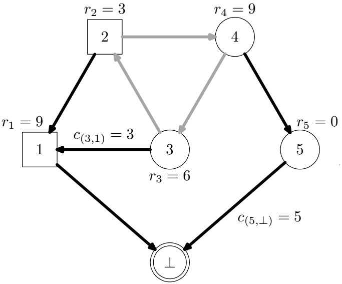
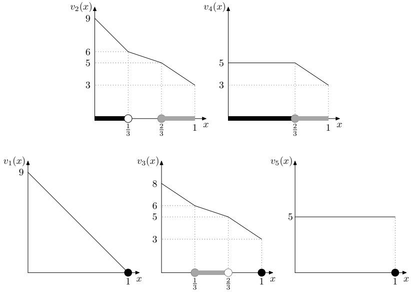
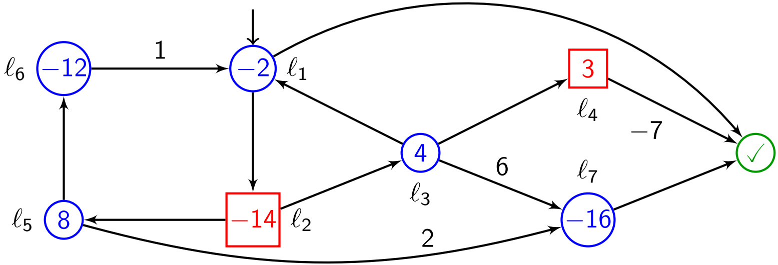
- for SPTGs: compute value functions $\overline{\mathsf{Val}}(\ell, x)$.





SPTGs with arbitrary weights



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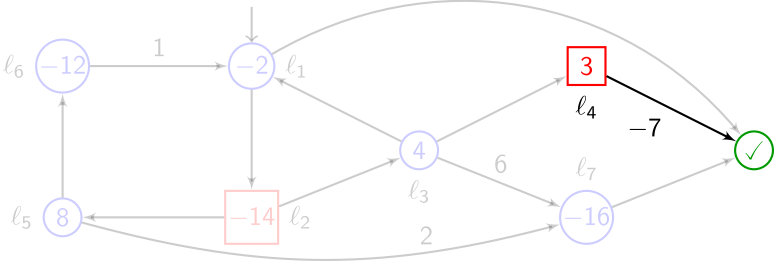

$$\mathsf{Val}(\ell_4, x) = 3(1-x) - 7 = -3x - 4$$

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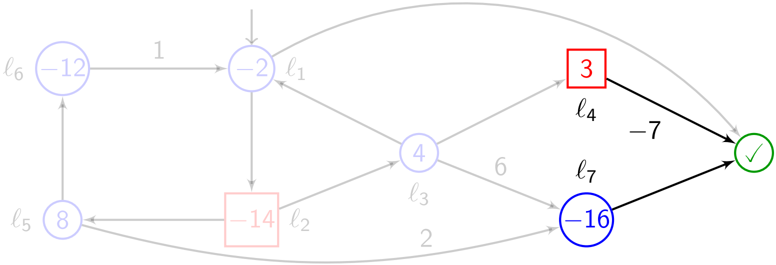

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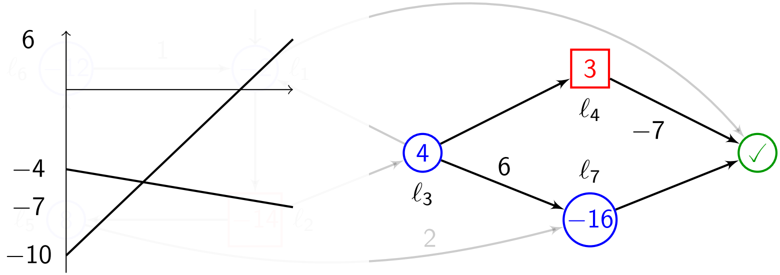

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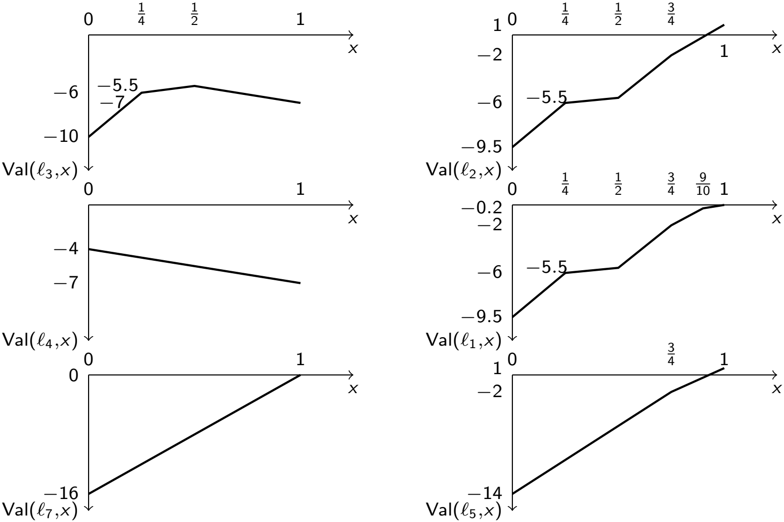
$$\mathrm{Val}(\ell_3, x) = \inf_{0 \leqslant t \leqslant 1-x}[4t + \min(-3(x+t) - 4, 6 - 16(1-(x+t)))] = \min(-3x - 4, 16x - 10)$$

SPTGs with arbitrary weights



Recursive elimination of states

Intuition from [Bouyer, Larsen, Markey, and Rasmussen, 2006b, Rutkowski, 2011]:

- Player ○ prefers to stay as long as possible in locations with **minimal price**: add a final location allowing him to stay until the end, and make the location urgent

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Problem: intuition not always true... you may have to change decision!
Recursive algorithm + construction of the value functions from right ($x = 1$) to left ($x = 0$)

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Challenges with arbitrary weights:

- Proof of correctness does not generalise: initially two distinct proofs for ○ and □
- Proof of termination does not generalise: difficult because of the double recursion...

Make a symmetric treatment of $\bigcirc$ and $\square$

Theorem

PTGs are determined ($\overline{\mathsf{Val}} = \underline{\mathsf{Val}}$), and value functions are continuous (over regions).

Determinacy follows from Gale-Stewart determinacy result...

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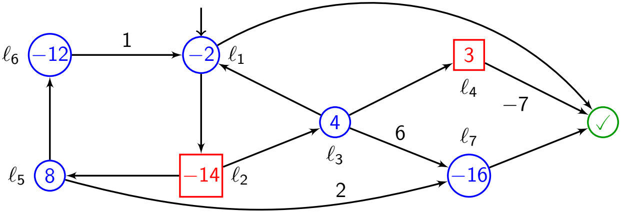
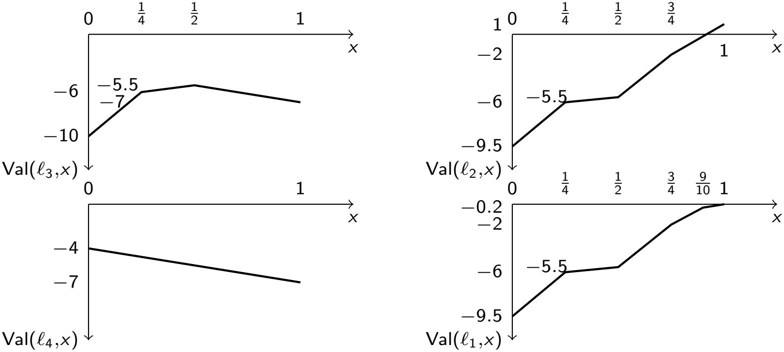

ℓ6
−12
1
−2
ℓ1
3
ℓ4
−7
4
6
ℓ7
ℓ3
ℓ5
8
−14
ℓ2
−16
2


Val(ℓ3,x)
Val(ℓ2,x)
Val(ℓ4,x)
Val(ℓ1,x)
x
−5.5
−7
−6
−10
−9.5
−2
−4
−0.2

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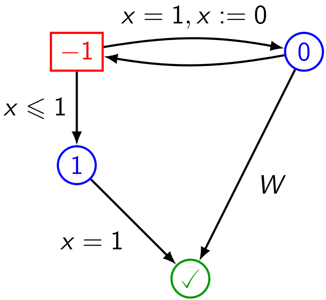
For every SPTG, all value functions are piecewise affine, with at most an exponential number of cutpoints (in number of locations).

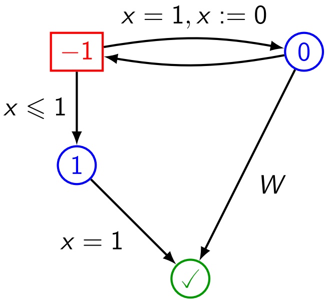
For general 1-clock PTGs?

- removing guards and invariants: previously used techniques work!
- removing resets: previously, bound the number of resets...

Bounding the number of resets needed is not possible



Bounding the number of resets needed is not possible



Player $\bigcirc$ can guarantee (i.e., ensure to be below) value ε for all $\varepsilon > 0$...

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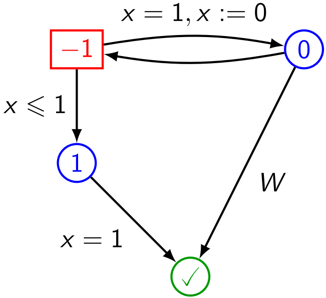

Player ○ can guarantee (i.e., ensure to be below) value ε for all $\varepsilon > 0$...

... but cannot obtain 0: hence, no optimal strategy...

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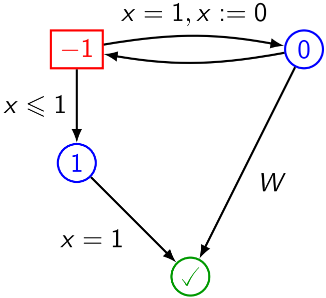

Player $\bigcirc$ can guarantee (i.e., ensure to be below) value ε for all $\varepsilon > 0$...

... but cannot obtain 0: hence, no optimal strategy...

... moreover, to obtain ε, $\bigcirc$ needs to loop at least $W + \lceil 1/\log \varepsilon \rceil$ times before reaching $\checkmark$!

Current solution: Reset-acyclic 1-clock PTGs

exponential time algorithm for reset-acyclic 1-clock PTGs with arbitrary weights

Summary and Future Work

Results

- Extension of iterative elimination for reset-acyclic 1-clock PTGs with arbitrary weights
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Thank you for your attention

References I

Rajeev Alur, Mikhail Bernadsky, and P. Madhusudan. Optimal reachability for weighted timed games. In *Proceedings of the 31st International Colloquium on Automata, Languages and Programming (ICALP'04)*, volume 3142 of *Lecture Notes in Computer Science*, pages 122–133. Springer, 2004.

Patricia Bouyer, Franck Cassez, Emmanuel Fleury, and Kim G. Larsen. Optimal strategies in priced timed game automata. In *Proceedings of the 24th Conference on Foundations of Software Technology and Theoretical Computer Science (FSTTCS'04)*, volume 3328 of *Lecture Notes in Computer Science*, pages 148–160. Springer, 2004.

Patricia Bouyer, Thomas Brihaye, and Nicolas Markey. Improved undecidability results on weighted timed automata. *Information Processing Letters*, 98(5):188–194, 2006a.

Patricia Bouyer, Kim G. Larsen, Nicolas Markey, and Jacob Illum Rasmussen. Almost optimal strategies in one-clock priced timed games. In *Proceedings of the 26th Conference on Foundations of Software Technology and Theoretical Computer Science (FSTTCS'06)*, volume 4337 of *Lecture Notes in Computer Science*, pages 345–356. Springer, 2006b.

Patricia Bouyer, Thomas Brihaye, Véronique Bruyère, and Jean-François Raskin. On the optimal reachability problem of weighted timed automata. *Formal Methods in System Design*, 31(2):135–175, 2007.

References II

Thomas Brihaye, Véronique Bruyère, and Jean-François Raskin. On optimal timed strategies. In *Proceedings of the Third international conference on Formal Modeling and Analysis of Timed Systems (FORMATS'05)*, volume 3829 of *Lecture Notes in Computer Science*, pages 49–64. Springer, 2005.

Thomas Brihaye, Gilles Geeraerts, Shankara Narayanan Krishna, Lakshmi Manasa, Benjamin Monmege, and Ashutosh Trivedi. Adding Negative Prices to Priced Timed Games. In *Proceedings of the 25th International Conference on Concurrency Theory (CONCUR'13)*, volume 8704 of *Lecture Notes in Computer Science*, pages 560–575. Springer, 2014.

Thomas Brihaye, Gilles Geeraerts, Axel Haddad, and Benjamin Monmege. To reach or not to reach? Efficient algorithms for total-payoff games. In Luca Aceto and David de Frutos Escrig, editors, *Proceedings of the 26th International Conference on Concurrency Theory (CONCUR'15)*, volume 42 of *LIPIcs*, pages 297–310. Schloss Dagstuhl–Leibniz-Zentrum für Informatik, September 2015.

John Fearnley and Marcin Jurdziński. Reachability in two-clock timed automata is PSPACE-complete. In *Proceedings of ICALP'13*, volume 7966 of *Lecture Notes in Computer Science*, pages 212–223. Springer, 2013.

Christoph Haase, Joël Ouaknine, and James Worrell. On the relationship between reachability problems in timed and counter automata. In *Proceedings of RP'12*, pages 54–65, 2012.

References III

Thomas Dueholm Hansen, Rasmus Ibsen-Jensen, and Peter Bro Miltersen. A faster algorithm for solving one-clock priced timed games. In *Proceedings of the 24th International Conference on Concurrency Theory (CONCUR'13)*, volume 8052 of *Lecture Notes in Computer Science*, pages 531–545. Springer, 2013.

Donald A. Martin. An extension of borel determinacy. *Annals of Pure and Applied Logic*, 49:279–293, 1990.

Michał Rutkowski. Two-player reachability-price games on single-clock timed automata. In *Proceedings of the Ninth Workshop on Quantitative Aspects of Programming Languages (QAPL'11)*, volume 57 of *Electronic Proceedings in Theoretical Computer Science*, pages 31–46, 2011.