Logical Characterization of Weighted Pebble Walking Automata

Benjamin Monmege
Université libre de Bruxelles, Belgium

Benedikt Bollig and Paul Gastin (LSV, ENS Cachan, France)
Marc Zeitoun (LaBRI, Bordeaux University, France)

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Equivalence between automata and logic

- Well-known and studied model of computation: NFA over words
- Existing extensions over trees, grids, graphs...
- Robustness of automata intrinsically linked to logical characterization

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- Engelfriet-Hoogeboom: pebble walking automata vs FOposTC
- Droste-Gastin: weighted automata vs restricted weighted MSO

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- Droste-Gastin: weighted automata vs restricted weighted MSO

- Aim: **extend Engelfriet-Hoogeboom result to the quantitative setting, relating weighted pebble walking automata with weighted FOposTC**

Graphs as a general model

Words: $D = \{\rightarrow\}$
computations of sequential programs

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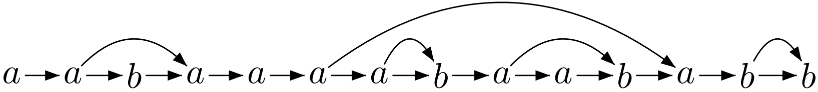
Nested words: $D = \{\rightarrow, \curvearrowright\}$
computations of recursive programs, XML documents



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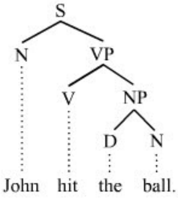
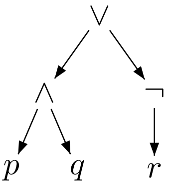
Ranked trees: $D = \{\downarrow_1, \downarrow_2\}$

expressions, formulae, parse trees





Graphs as a general model

Definition: directed graphs

$G = (V, (E_d)_{d\in D}, \lambda, \iota)$ where

- V is a nonempty and finite set of vertices;
- for all edge label $d \in D$, $E_d \subseteq V \times V$ is an *irreflexive relation*, describing the d-edges of the graph, which is *deterministic and codeterministic*;
- $\lambda\colon V \to A$ is a vertex-labeling function;
- $\iota \in V$ is an initial vertex.

For all edge label d, we consider its reverse d^{-1} letting $E_{d^{-1}} = (E_d)^{-1}$

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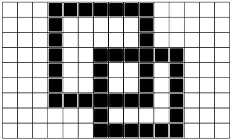
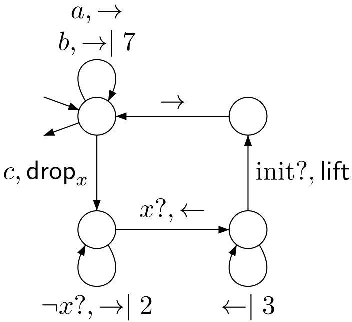
Grids: $D = \{\rightarrow, \uparrow\}$
pictures



Weighted pebble walking automata

Definition:

- Finite number of states, initial and final states
- Ability to navigate in the graph (using the deterministic edge labels)
- Bounded supply of pebbles able to mark temporarily a position
- Pebbles are treated with a stack policy: first pebble to lift is the last dropped pebble
- Transitions equipped with weights in a complete semiring $(\mathbf{S}, +, \times, 0, 1)$



Examples of complete semirings:
$(\{0,1\}, \vee, \wedge, 0, 1)$
$(\mathbb{R}^+ \cup \{+\infty\}, +, \times, 0, 1)$
$(\mathbb{Z} \cup \{+\infty, -\infty\}, \min, +, +\infty, 0)$
$(\mathbb{Z} \cup \{+\infty, -\infty\}, \max, +, -\infty, 0)$
$([0,1], \min, \max, 1, 0)$
$(2^{\Sigma^\star}, \cup, \cdot, \emptyset, \{\varepsilon\})$

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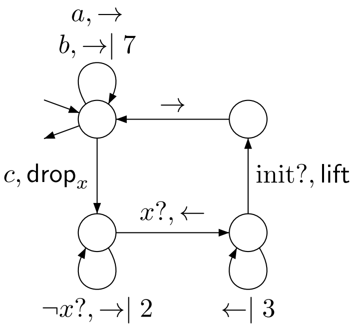
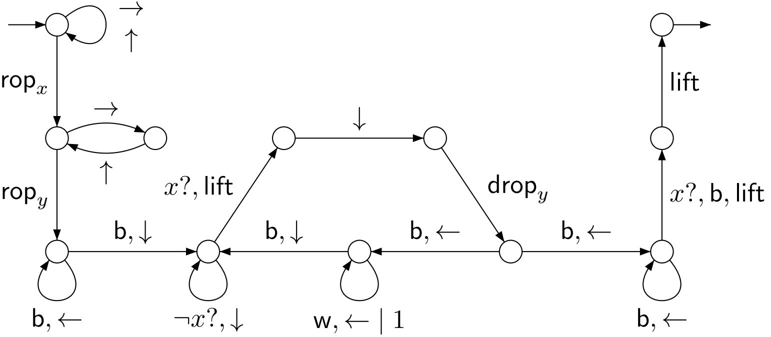
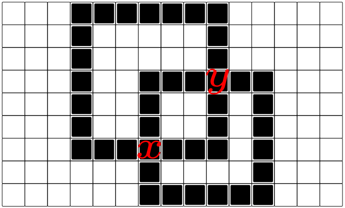


Definition: Semantics over $(\mathbf{S}, +, \times, 0, 1)$

- Configurations over a graph G: (G, q, π, v) with state q, stack π of pebble positions and current vertex v
- Weight of a run: multiplication of the weights of transitions
- Semantics $[\![\mathcal{A}]\!](G)$: sum of weights of accepting runs over G

Example of weighted pebble walking automaton





computes the biggest size of frames (empty black square)

$(\mathbb{N} \cup \{-\infty\}, \max, +, -\infty, 0)$

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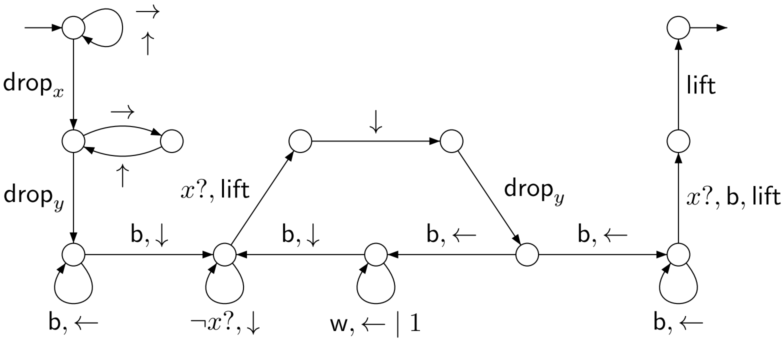

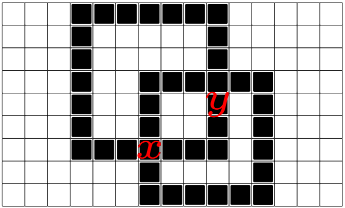

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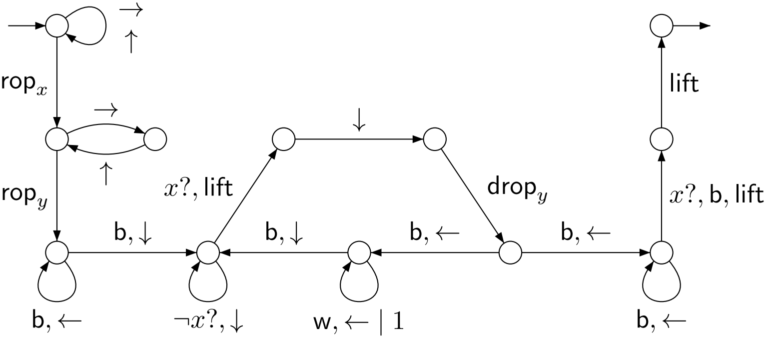

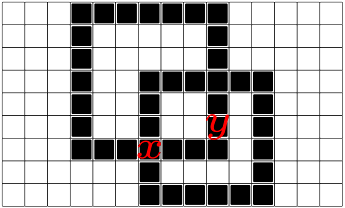

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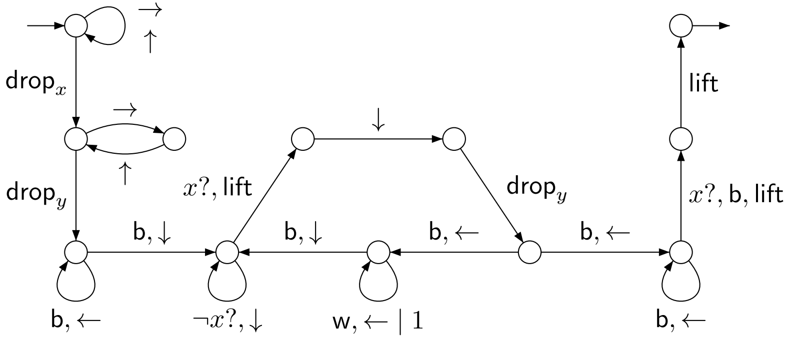

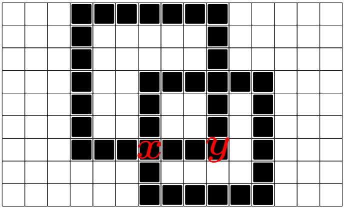

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Logical characterization

Classical weighted automata are **one-way** (sometimes branching) and **without pebbles**

Logical characterization for them in terms of a restricted weighted MSO logic:

- over words [Droste and Gastin, 2009]
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Restricted weighted MSO does not even contain full weighted FO a priori

Theorem: Our contribution

Weighted pebble walking automata over graphs $(\mathrm{wPWA}) = \mathrm{wFOTC}$

Weighted first-order logic

Definition:

Classical first-order logic

$$\varphi ::= \top \mid (x = y) \mid \mathrm{init}(x) \mid P_a(x) \mid R_d(x,y) \mid R_d^+(x,y) \mid \neg\varphi \mid \varphi \vee \varphi \mid \exists x\, \varphi$$

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Weighted first-order logic over graphs with weights in a semiring $(\mathbf{S}, +, \times, 0, 1)$

$$\Phi ::= s \mid \varphi \,?\, \Phi : \Phi \mid \Phi \oplus \Phi \mid \Phi \otimes \Phi \mid \bigoplus_x \Phi \mid \bigotimes_x \Phi$$

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Semantics over a graph G and a valuation σ of free variables

$$[\![\varphi \,?\, \Phi_1 : \Phi_2]\!](G,\sigma) = \begin{cases} [\![\Phi_1]\!](G,\sigma) & \text{if } G,\sigma \models \varphi \\ [\![\Phi_2]\!](G,\sigma) & \text{otherwise} \end{cases}$$

$$[\![\Phi_1 \oplus \Phi_2]\!](G,\sigma) = [\![\Phi_1]\!](G,\sigma) + [\![\Phi_2]\!](G,\sigma) \qquad [\![\bigoplus_x \Phi]\!](G,\sigma) = \sum_{v \in V} [\![\Phi]\!](G, \sigma[x \mapsto v])$$

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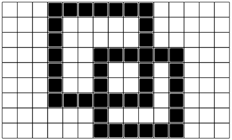
Examples in $(\mathbb{N}\cup\{-\infty\}, \max, +, -\infty, 0)$

$\Phi_{\mathrm{b}} = \bigotimes_x P_{\mathrm{b}}(x) \,?\, 1 : 0$ $\qquad$ $\Phi_{\mathrm{w}} = \bigotimes_x P_{\mathrm{w}}(x) \,?\, 1 : 0$ $\qquad$ $\Phi_{\mathrm{b}} \oplus \Phi_{\mathrm{w}}$

Transitive closure in graphs



Binary predicate $R_{\nearrow}(x,y) = \exists z[R_{\rightarrow}(x,z) \wedge R_{\uparrow}(z,y)]$
Transitive Closure $\mathrm{TC}_{x,y} R_{\nearrow}(x,y)$
test if square (not doable in FO)

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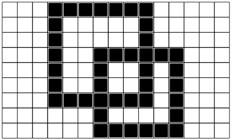

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verifies that it is a square **and** computes the length of its diagonal

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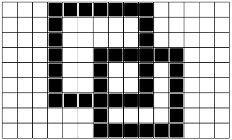

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Semantics given in a complete semiring $(\mathbf{S}, +, \times, 0, 1)$

$$[\![[\mathrm{TC}_{x,y}\Phi](x',y')]\!](G,\sigma) = \sum_{\substack{v_0,v_1,\dots,v_m\ (m>0)\\ \sigma(x')=v_0,\sigma(y')=v_m}} \prod_{0\leqslant k\leqslant m-1} [\![\Phi]\!](G,\sigma[x\mapsto v_k, y\mapsto v_{k+1}])$$

sum along
sequences of stop-vertices

multiplication along
the sequence

Bounding the Transitive Closure

- A necessary restriction to obtain a fragment of logic expressively equivalent to wPWA
- But not so restrictive in most of the cases!

$$\mathrm{TC}^{N}_{x,y}\Phi(x,y) = \mathrm{TC}_{x,y}[\mathsf{dist}(x,y) \leqslant N \,?\, \Phi(x,y) : 0]$$

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Definition: Logic wFOTC

$$\Phi ::= s \mid \varphi \, ? \, \Phi : \Phi \mid \Phi \oplus \Phi \mid \Phi \otimes \Phi \mid \bigoplus_x \Phi \mid \bigotimes_x \Phi \mid \mathrm{TC}^{N}_{x,y}\Phi$$

with $s \in \mathbf{S}$, $\varphi \in \mathsf{FO}$, $x, y \in \mathrm{Var}$ and $N \in \mathbb{N} \setminus \{0\}$.

Comparison with restricted wMSO:

- unrestricted use of $\bigoplus_x$ and $\otimes$, presence of $\mathrm{TC}_{x,y}$, absence of $\bigoplus_X$

Contribution

Theorem:

over searchable graphs: $\mathrm{wFOTC} \longrightarrow$ weighted pebble walking automata

over zonable graphs: weighted pebble walking automata $\longrightarrow \mathrm{wFOTC}$

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$\Longrightarrow$ (un)decidability and complexity results over automata transfer to wFOTC

Translation of wFOTC in wPWA

Definition: Hypothesis: searchable graphs

- linear order $\leqslant$ on vertices with ι (initial vertex) as minimal element
- existence of a guide: walking automaton $\mathcal{A}_{\mathcal{G}}$ computing $\leqslant$

All previously classes of graphs are searchable

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Inductive translation:

$\Phi \oplus \Psi$ disjoint union of automata

$\Phi \otimes \Psi$ $\longrightarrow \boxed{\mathcal{A}_\Phi} \xrightarrow{\text{reset to } \iota} \boxed{\mathcal{A}_\Psi} \longrightarrow$

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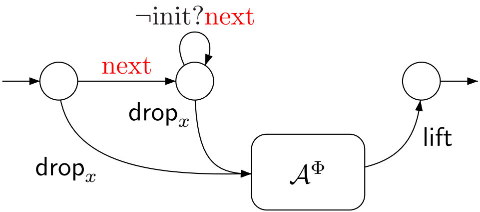

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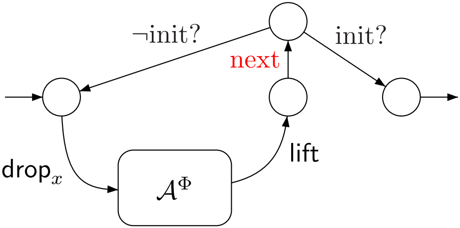

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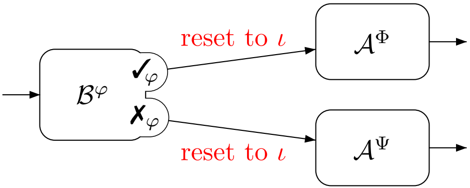

Boolean fragment: **linear size automata** (pebble and navigation)

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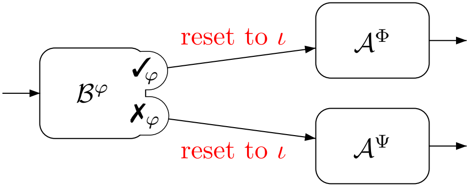

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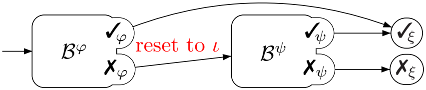
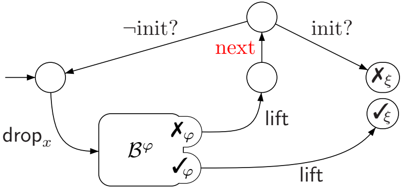
disjunction $\xi = \varphi \vee \psi$



existential quantification $\xi = \exists x\ \varphi$



Translation of wFOTC in wPWA

Case of a formula $[\mathrm{TC}^{N}_{x,y}\Phi(x,y)]\underbrace{(x',y')}_{\text{fresh free variables}}$ with $\mathcal{A}$ a wPWA for Φ: construction of a wPWA $\mathcal{A}'$ with two more layers of pebbles that does the following

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1. search free variable x', and drop pebble x
2. guess a sequence of moves of length $\leqslant N$, follow it, and drop pebble y (*then flush the sequence to save memory*)
3. reset to ι and simulate $\mathcal{A}$
4. search pebble y
5. guess sequence π of moves of length $\leqslant N$, follow it, check that it holds x
6. lift pebbles y and x (hence returning to the vertex of x)
7. follow π^R to reach back the vertex that held y, and drop pebble x
8. if y' is held by the current vertex, enter a final state
9. in every case, go back to step 2

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2. guess a sequence π of moves of length $\leqslant N$, follow it, and drop pebble y (*then flush the sequence to save memory*)
    - test that π is minimal amongst all sequences going from x to y
3. reset to ι and simulate $\mathcal{A}$
4. search pebble y
5. guess sequence π of moves of length $\leqslant N$, follow it, check that it holds x
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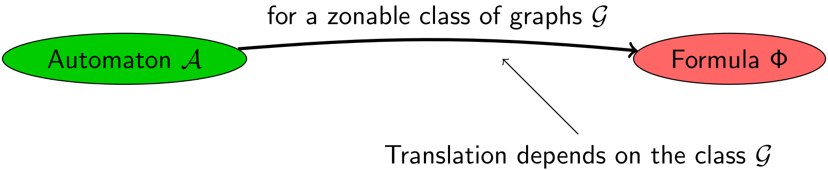
Translation of wPWA in wFOTC

Theorem:

Let $\mathcal{G}$ be a **zonable** class of graphs. Then, for every wPWA $\mathcal{A}$, we can construct a formula Φ of wFOTC such that for every graph $G \in \mathcal{G}$, and valuation σ of free variables, $[\![\mathcal{A}]\!](G, \sigma) = [\![\Phi]\!](G, \sigma)$.



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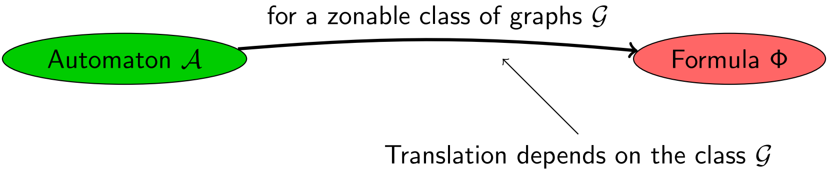


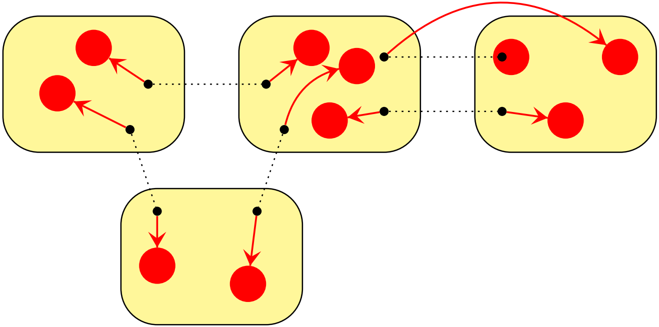
Proof in two steps:

- For the considered class of graphs, prove the **zonability**;
- **Generic** translation of automata into formulae for zonable class of graphs

Zonable classes of graphs

A zoning of a graph G with parameter N:

- an equivalence relation $\sim$, decomposing a graph into *zones* of diameter bounded by a constant M;
- set $\mathcal{W}$ of wires = (directed) edges relating different zones;
- an injective encoding function $enc\colon \mathcal{W} \times \{0, \dots, N-1\} \to V$



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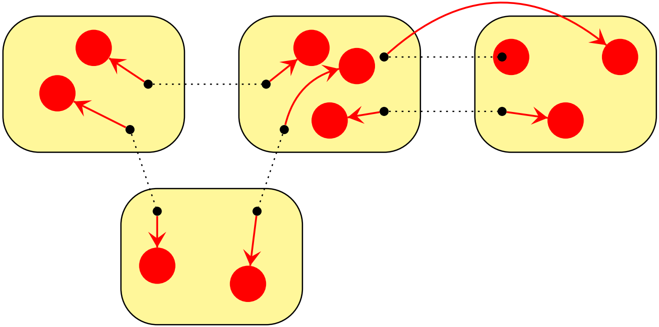
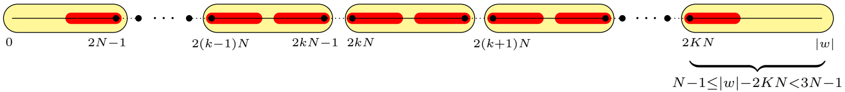


and $\sim$ and *enc* must be expressible by some formulae $\mathsf{zone}(z, z')$ and $\mathsf{enc}_n(z, z', x)$ (for $n \in \{0, \dots, N-1\}$) in wFOTC

Examples



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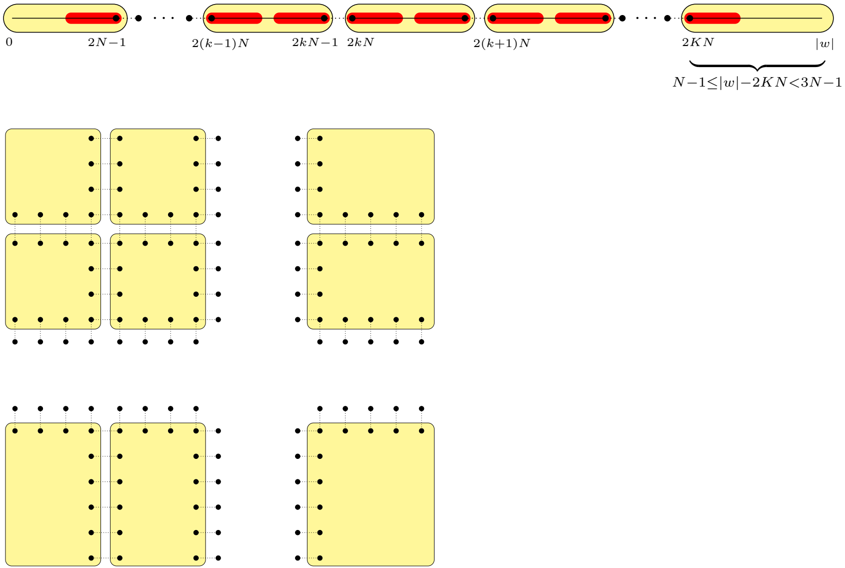

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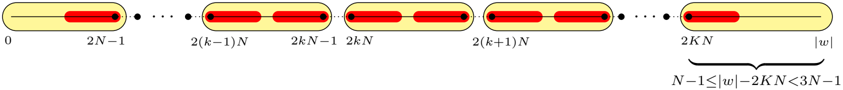
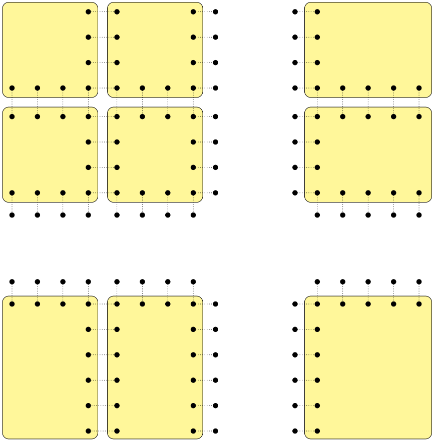
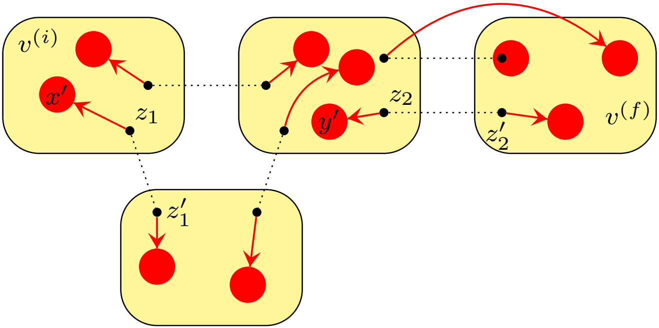




but also trees, nested words, Mazurkiewicz traces, rings...

Translation in a zonable class of graphs

- External (bounded) transitive closure jumping from zone to zone: state at the wires encoded using *enc*;
- Internal (bounded) transitive closures to compute the weights of the infinite set of runs restricted to a zone: computation by McNaughton-Yamada algorithm, state directly encoded in the formulae.



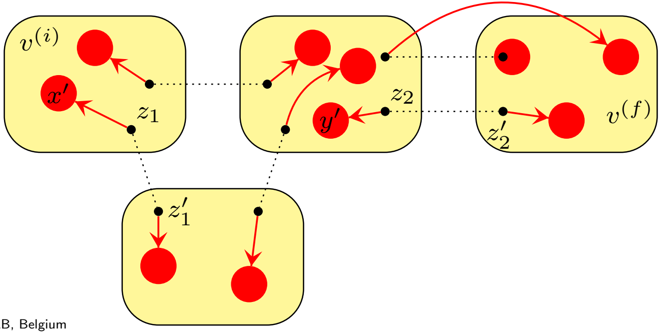
Translation in a zonable class of graphs

Weight of the runs from z_i in state q_i to z_f in state q_f:

$$\bigoplus_{x',y'}\Big[\bigoplus_{z_1,z_1'}\bigoplus_{q_1\in Q}\mathsf{enc}_{q_1}(z_1,z_1',x')\otimes\Phi_{q_i,q_1}(z_i,z_1)\Big]\otimes[\mathrm{TC}^{3M}_{y_1,y_2}\Psi](x',y')$$
$$\otimes\bigoplus_{z_2,z_2'}\bigoplus_{q_2,q_2'\in Q}\Big[\mathsf{enc}_{q_2}(z_2,z_2',y')\otimes\mathsf{tr}_{q_2,q_2'}(z_2,z_2')\otimes\Phi_{q_2',q_f}(z_2',z_f)\Big]$$

with $\Psi(y_1,y_2)$ the formula

$$\bigoplus_{\substack{z_1,z_1',\\ z_2,z_2'}}\bigoplus_{\substack{q_1,q_1',\\ q_2\in Q}}\Big[\mathsf{enc}_{q_1}(z_1,z_1',y_1)\otimes\mathsf{tr}_{q_1,q_1'}(z_1,z_1')\otimes\mathsf{enc}_{q_2}(z_2,z_2',y_2)\otimes\Phi_{q_1',q_2}(z_1',z_2)\Big]$$



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with $\Psi(y_1,y_2)$ the formula

$$\bigoplus_{\substack{z_1,z_1',\\ z_2,z_2'}} \bigoplus_{\substack{q_1,q_1',\\ q_2\in Q}} \Big[\mathsf{enc}_{q_1}(z_1,z_1',y_1) \otimes \mathsf{tr}_{q_1,q_1'}(z_1,z_1') \otimes \mathsf{enc}_{q_2}(z_2,z_2',y_2) \otimes \Phi_{q_1',q_2}(z_1',z_2)\Big]$$

$\Phi_{q,q'}(x,x')$ formula computing the weight of the runs from x in q to x' in q', staying in the zone containing both x and x'

- built by McNaughton-Yamada algorithm, with cascade of **bounded** transitive closures (**since zones have bounded diameter**)

Conclusion and Perspectives

- Expressive equivalence between weighted pebble walking automata and weighted first-order logic with bounded transitive closure, over arbitrary complete semirings
- Additional reasonable requirements on the classes of graphs (searchable and zonable), met by usual examples of graphs (words, nested words, trees, grids, Mazurkiewicz traces, rings...)
- Interesting special case: a logic for **graph-to-word transducers** (non-commutative semiring of languages over an alphabet Σ)

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Thank you!

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