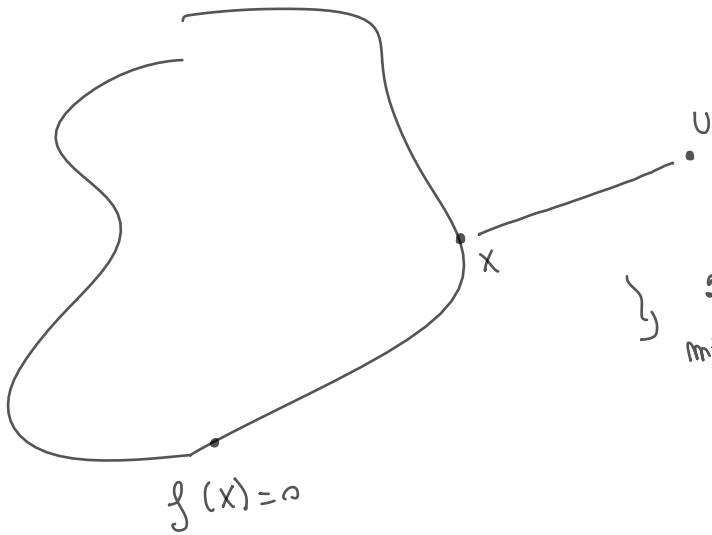


TD4

Ex. 1



1) Paramètres

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3$$

$(m=3)$

2) minimise $d(x, U)$
 ~~$\|X - U\|$~~
 $\|X - U\|^2$

min $\|X - U\|^2$ $\rightarrow J(x)$
 $X \in \mathbb{R}^3$
 surface

3) Contraintes !

ici $\leftrightarrow f(x) = 0$
 1 contrainte $(m=1)$

6m minimise $J(x) = \|X - U\|^2$ $(m=3)$

sur $X \in \text{surface}$

\rightarrow 1 eq. de contrainte : $f(x) = 0$ $(m=1)$

$L : \mathbb{R}^4 \rightarrow \mathbb{R}$
 $(X, \lambda) \mapsto J(x) + \lambda \cdot f(x) = \|X - U\|^2 + \lambda \cdot f(x)$

Plan

$ax + by + cz - d = 0$

$f = \underbrace{x^T A x}_{d^{\circ 2}} + \underbrace{b^T x}_{d^{\circ 1}} + \underbrace{c^T x - d}_{d^{\circ 0}}$

$\nabla f = (A + A^t)X + B$

ici \rightarrow

$A = 0$

$B = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$c^t x = -d$

Ellipsoids

$$ax^2 + by^2 + cz^2 - 1 = 0$$

↓

$$\underbrace{x \cdot A \cdot x} + \underbrace{b x} + \underbrace{c z} = 0$$

| |
 B=0

$$A = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$$

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$