Optimization

$\boxed{\text{Fast run time}}$ $\boxed{\text{More efficient}}$ — run faster
more smoothly
more FPS

$O(\dots)$

lower the complexity

---

arguments ?

function → value ⤳ cost
return
time
$\mathbb{R}$ efficiency ....

↓
math ...

return max / min

1) ↙ modelling

problem → function
args → $\mathbb{R}$

2) ↘ algorithmics
|
find minima
such

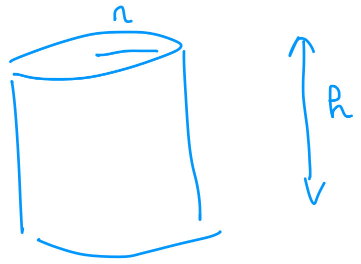
I. Example(s) → formalizing optimization

Problem of coca-cola

Manufacture cans (cylinders) → describe | the object
the model

1) → as cheap as possible
2) → volume given V_0

→ equations



model → cylinder

↕

$(r,h) \in \mathbb{R}^2$

$\boxed{\varphi(r,h) = V_0 - \pi r^2 h}$

2) $\boxed{V_0 = \pi r^2 h}$ ← satisfied

1) as cheap as possible (to manufacture ...)

↕

smallest surface

$\boxed{f(r,h) = 2 \cdot \pi r^2 + 2\pi r \times h}$ ← $\min\limits_{(r,h) \in \mathbb{R}^2}$

Pb:

function to minimize / maximize

$$\min_{(r,h) \in \cancel{\mathbb{R}^2}} f(r,h)$$

$C \subseteq \mathbb{R}^2$

$\{(r,h)$ such that $\varphi(r,h) = 0\}$

volume $= V_0$

$\equiv$

1) param.

↓

dimension n $(=2)$

2)

3) domain → $\mathbb{R}^n$

→ $C \subseteq \mathbb{R}^n$

Optimization pb

Optimization pb:

$f : \mathbb{R}^n \longrightarrow \mathbb{R} \longrightarrow \min_{\vec{x} \in C} f(\vec{x})$, $\boxed{C \subseteq \mathbb{R}^n}$ → constrained optimization

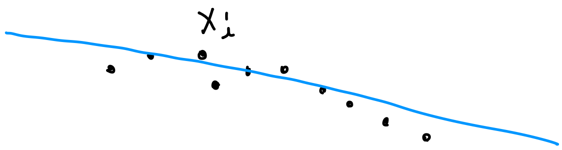
$\downarrow$

$\min_{\vec{x} \in \mathbb{R}^n} f(\vec{x})$ → non constrained optimization

II - Other examples

→ point cloud processing

① Model fitting



↓ plane ↓ as close as possible from the points

$X_i \in \mathbb{R}^3$

$\; = (x_i, y_i, z_i)$

1) → parameters ?

|

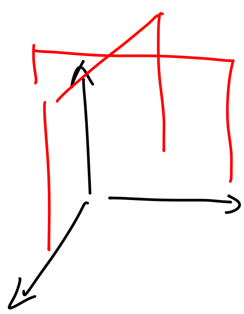
params. of the plane

$\vec{n} \rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} + d$

general eq. $\quad 2(ax + by + cz + d) = 0$

$c \neq 0$

$c \neq 0$ — non vertical planes



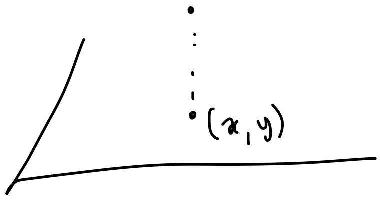
Specific eq (non vertical planes) $\quad z = a_1 x + a_2 y + a_3$

→ cartesian / elevation equation

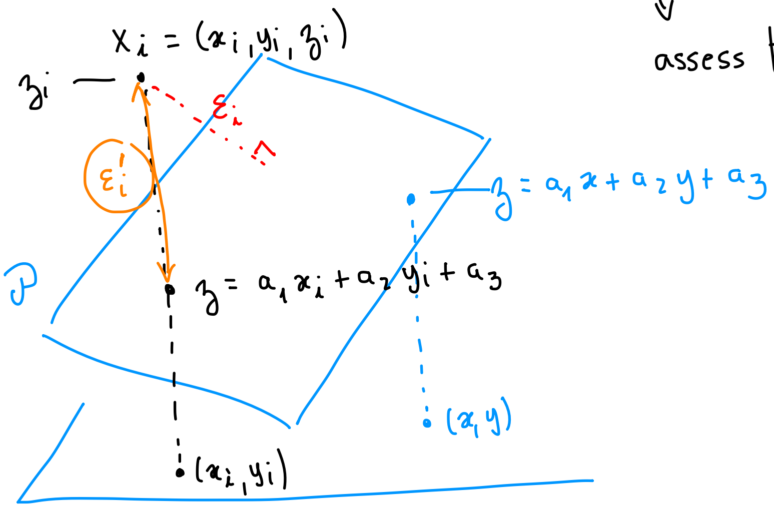
plane
↓
$z = a_1 x + a_2 y + a_3$

parameters : a_1, a_2, a_3

$n = 3$



1) → $f(a_1, a_2, a_2)$ — plane "fits" the points
↓
assess the error points / plane



error between x_i / $\mathcal{P}$

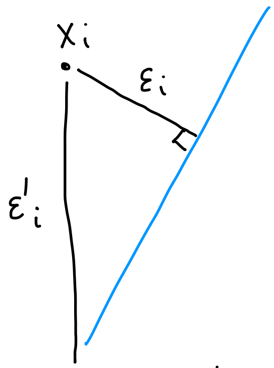
ε_i — $d(x_i, \mathcal{P})$?

not easy to compute ...

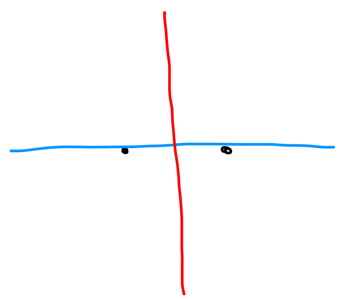
write an error using the eq. of the plane

$$\boxed{\varepsilon_i' = a_1 x_i + a_2 y_i + a_3 - z_i}$$

err. at X_i



$\varepsilon_i \leqslant \overset{\min}{\varepsilon_i'}$



global error ?

$$\vec{\varepsilon} = \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_N \end{pmatrix}$$

error vector

global error $\longrightarrow$ size of $\vec{\varepsilon}$?

$$\vec{\varepsilon} \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_N \end{pmatrix}$$

$\downarrow$

norm ...

$$f(a_1, a_2, a_3) = \|\vec{\varepsilon}\| \quad \cancel{\|\cdot\|_1} \quad \cancel{\|\cdot\|_\infty} \quad \|\cdot\|_2$$

min ?

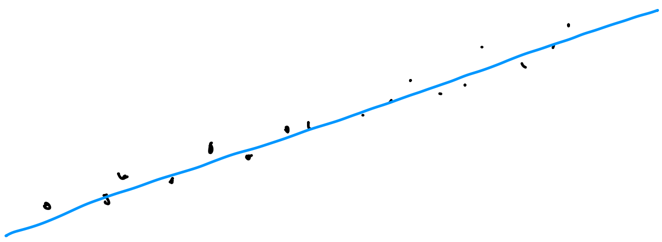
$\boxed{\text{mean square optimization}}$

- $f(x) = |x|$
  f' not defined in 0
- $\|\cdot\|_\infty$ .... not derivable

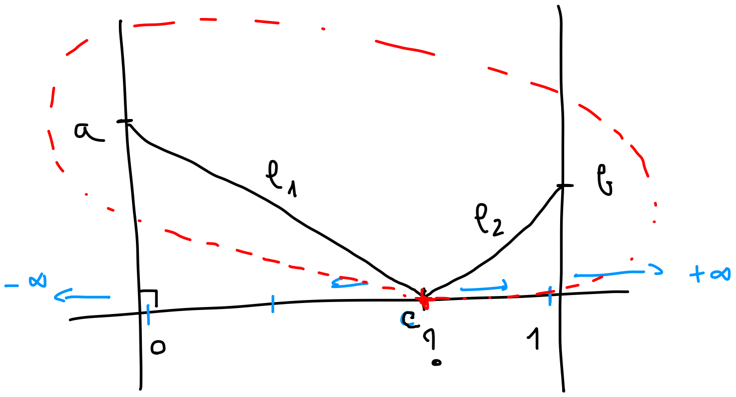
$$f(a_1, a_2, a_3) = \sum_{i=1}^{N} \left(z_i - (a_1 x_i + a_2 y_i + a_3)\right)^2$$

min ?
$(a_1, a_2, a_3) \in \mathbb{R}^3$

non constrained optimization



(2) String example



$\longrightarrow$ length minimal of the string

| a, b given

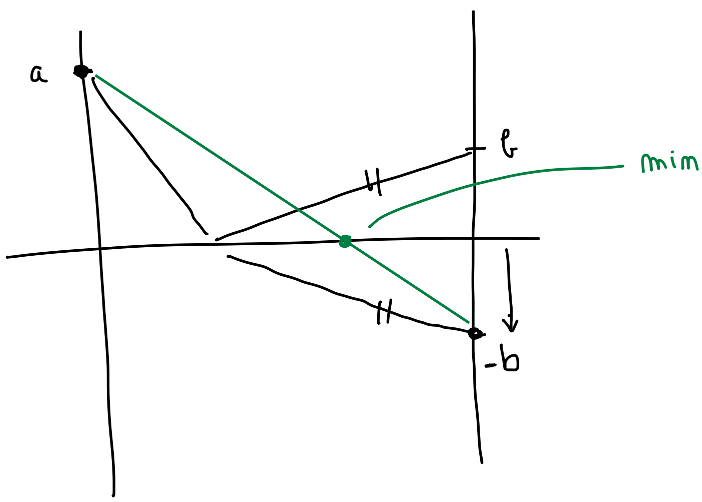
$\longrightarrow$ <u>parameters</u> $\left(\begin{array}{l} c \\ n = 1 \end{array} \right.$

$f : \mathbb{R} \longrightarrow \mathbb{R}$

$\longrightarrow$ $f(c)$: length

$$f(c) = \underbrace{\sqrt{a^2 + c^2}}_{\ell_1} + \sqrt{b^2 + (1-c)^2} \quad \text{— min ?}$$

$f'(c) = 0$



III. Roadmap of the course | optimization

Optimization

$f : \mathbb{R}^n \longrightarrow \mathbb{R}$

n parameters

meta-heuristics

stochastic alg.

→ genetic alg.

→ simulated annealing (recuit simulé)

deterministic approaches

complexity !!!

$\min f(x)$
$x \in \mathbb{R}^n$

non constrained optimization

$\min f(x)$
$x \in C$
$C \subset \mathbb{R}^n$

constrained optimization → general case

Lagrange multipliers

1-variable functions

$n = 1$

$f : \mathbb{R} \longrightarrow \mathbb{R}$

1D optimization

n D functions

nD-optimization

descent algorithms

least square optimization

$$f(x) = \sum_{i=1}^{N} f_i(x)^2$$

f linear
C → linear eq

linear programming

simplex method (polynomial)

f quadratic
C → linear eq

quadratic prog

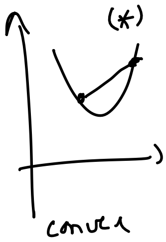
↓ linear system

(**)

f convex (*)
C → linear eq
convex

convex prog

programming ⟷ constrained optimization



convex

This course