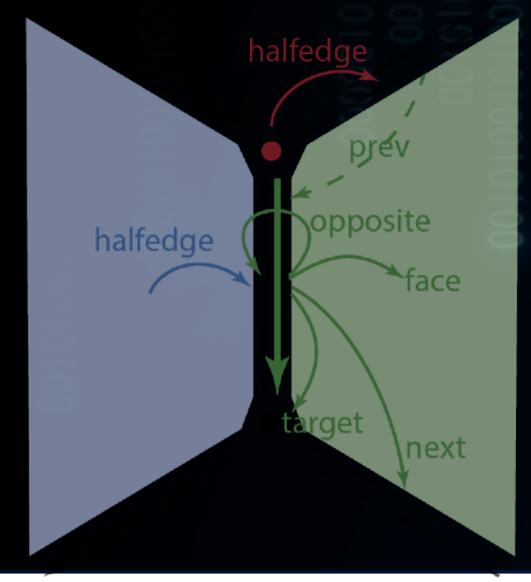
MODELISATION GEOMETRIQUE



Alexandra Bac

POLYTECH 4A INFORMATIQUE REVA

3 – COURBES ET SURFACES PARAMÉTRIQUES

Certaines illustrations sont issues du livre « polygon mesh processing »

« MODÉLISATION GÉOMÉTRIQUE » - MODÈLES PARAMÉTRIQUES

Chapitre 2

MAILLAGES

Chapitre 4

GÉOMÉTRIE DES SURFACES

Chapitre 1

MODÉLISATION DES SURFACES

Chapitre 3 (+ 5A)

SURFACES PARAMÉTRIQUES

Chapitre 5

SURFACES IMPLICITES

COURBES ET SURFACES PARAMÉTRIQUES

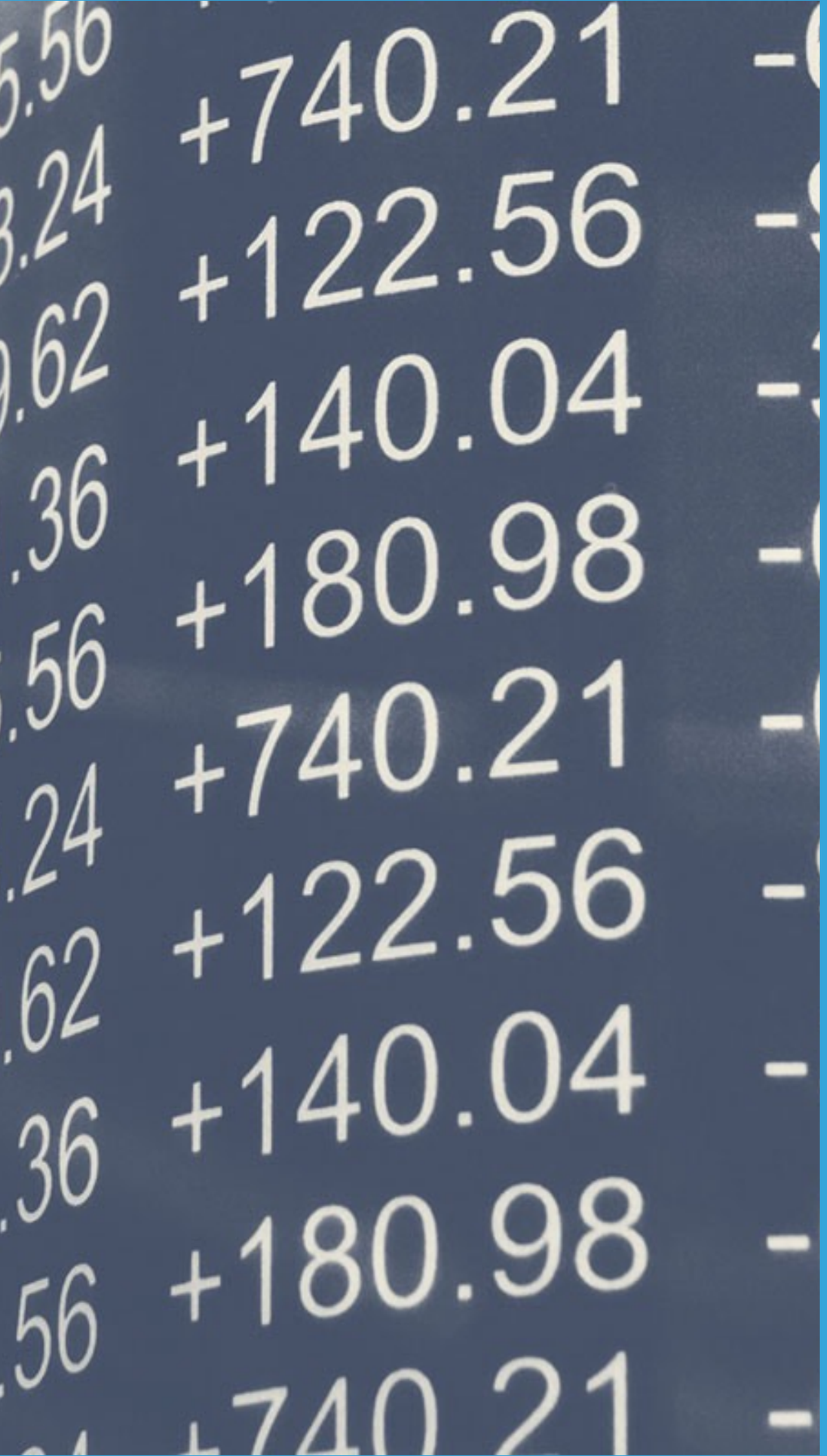
Très incomplet (nécessiterait un module entier)

A suivre en 5A …

Commençons par les courbes

- Plus simple
- Base des modèles de surfaces (produit tensoriel)

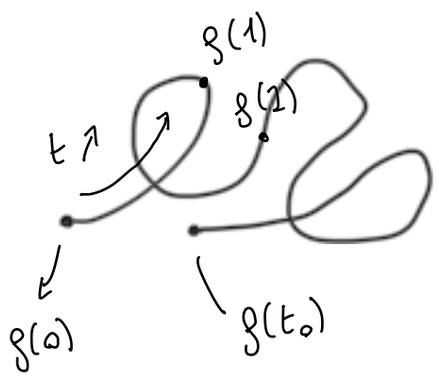
COURBES PARAMÉTRIQUES



GÉNÉRALITÉS

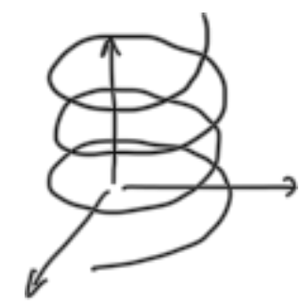
COURBES PLANES

Courbes dans $\mathbb{R}^2$



COURBES GAUCHES

Courbes dans $\mathbb{R}^3$



REPRÉSENTATION PARAMÉTRIQUE

$$f : \mathbb{R} \to \mathbb{R}^2$$
$$t \mapsto g(t) \in \mathbb{R}^2$$

1 parameter

$$f : \mathbb{R} \to \mathbb{R}^3$$

dim 1

curves in $\mathbb{R}^n$

$$g : \mathbb{R} \to \mathbb{R}^n$$

COURBES

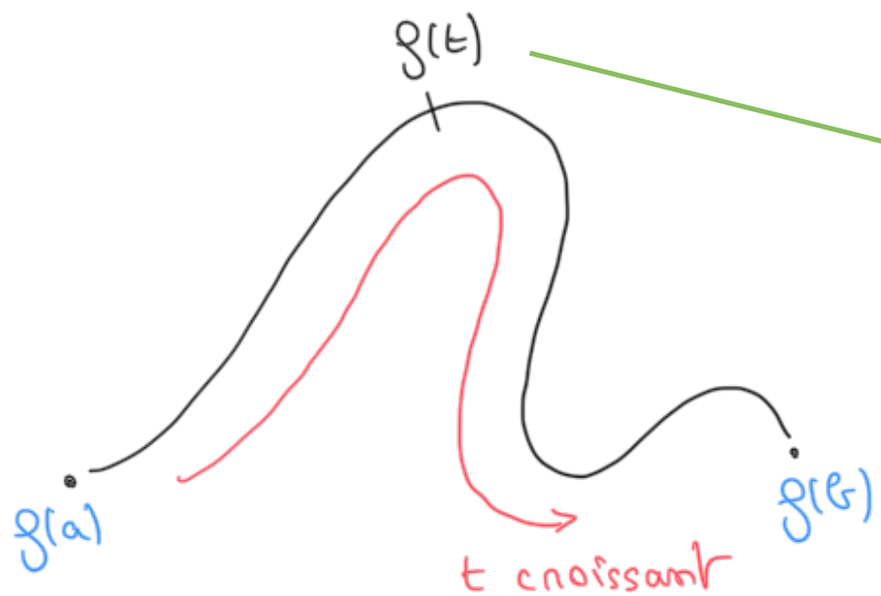
Courbe <u>finie</u> :

$f : [a, b] \subseteq \mathbb{R} \to \mathbb{R}^2$ (ou $\mathbb{R}^3$)

COURBES

Courbe <u>finie</u> :

$f\ :\ [a,b] \subseteq \mathbb{R} \to \mathbb{R}^2$ (ou $\mathbb{R}^3$)



$f(t) \in \mathbb{R}^2$ donc :

$$f(t) = \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix}$$

coord. functions :

$f_i : \mathbb{R} \to \mathbb{R}$

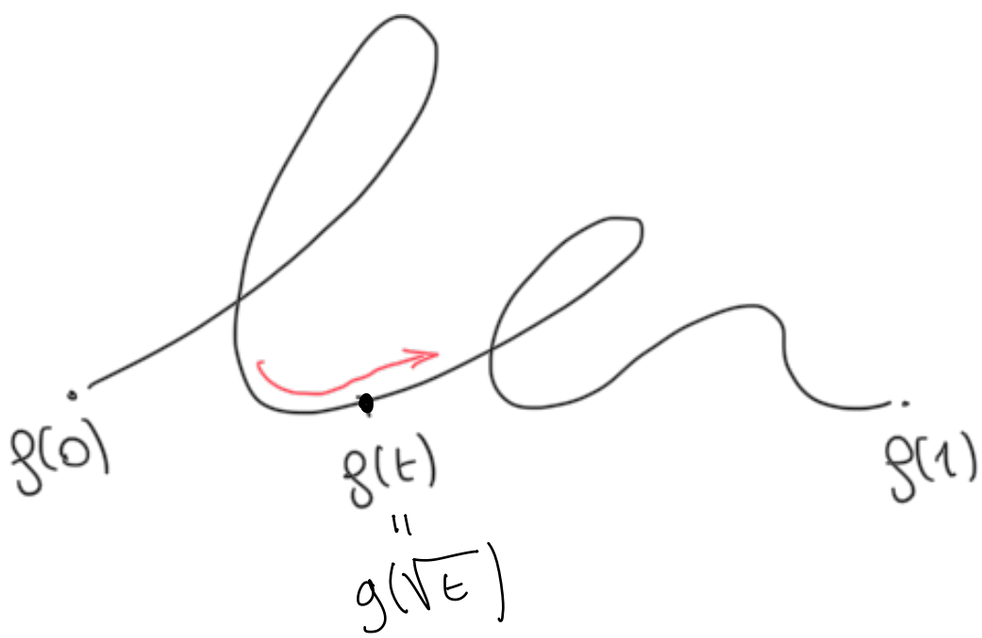
$$\Updownarrow$$

$f_1, f_2\ :\ \mathbb{R} \to \mathbb{R}$

QUESTION DE LA PARAMÉTRISATION

Courbe $\leftrightarrow$ modèle paramétrique $f : \mathbb{R} \to \mathbb{R}^k$

unique ?



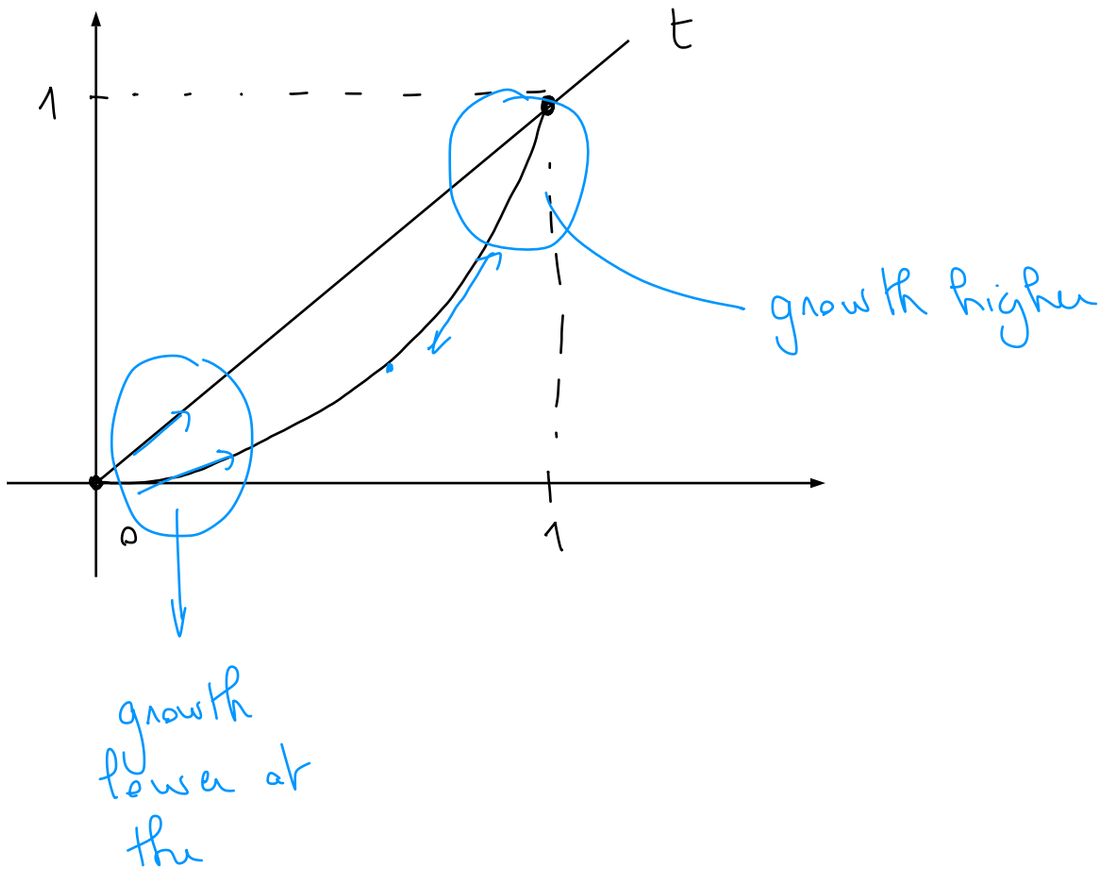
$f : [0,1] \to \mathbb{R}^2$

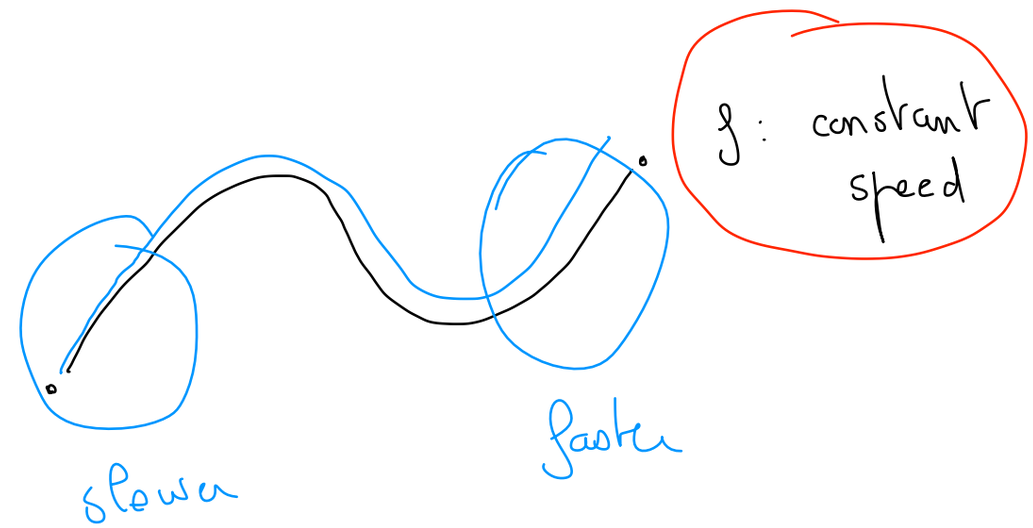
On pose $g(t) = f(t^2)$

- Quelle est la courbe décrite ?

same curve ⚠

different speeds





Parametrization → not unique

→ ≠ object we want to model

independent

PARAMÉTRISATION NORMALE

Pour toute courbe, il existe une paramétrisation selon laquelle la courbe est parcourue à vitesse constante :

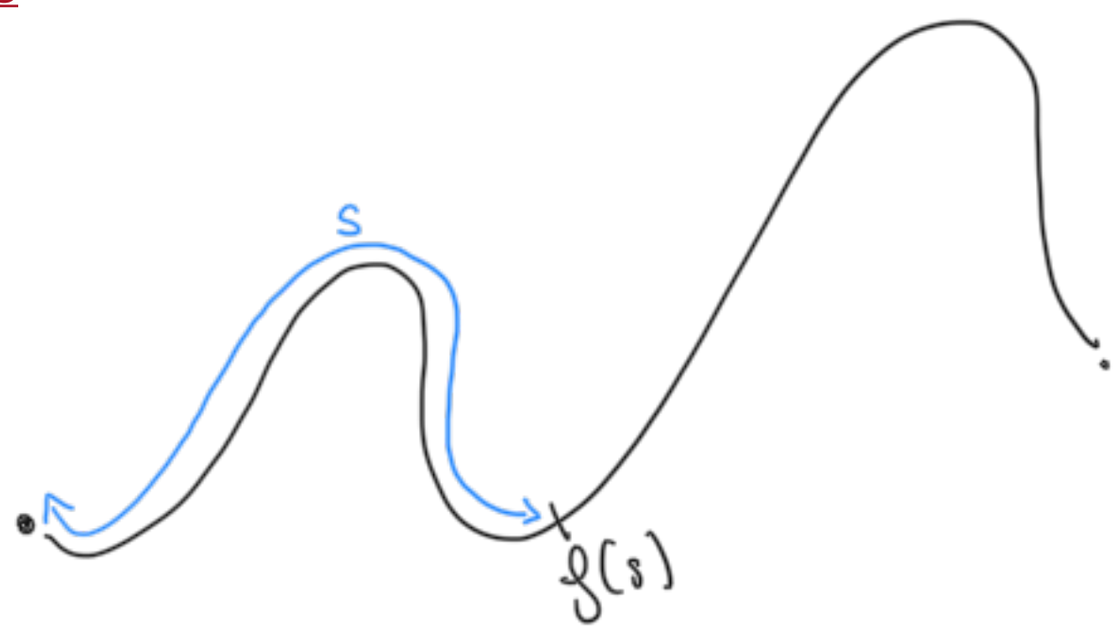
- Unique
- Paramètre noté s (**abscisse curviligne**) : longueur le long de la courbe
- Appelée **paramétrisation normale**

$\int \dots$

existe / unique

tangent → $f'(s)$

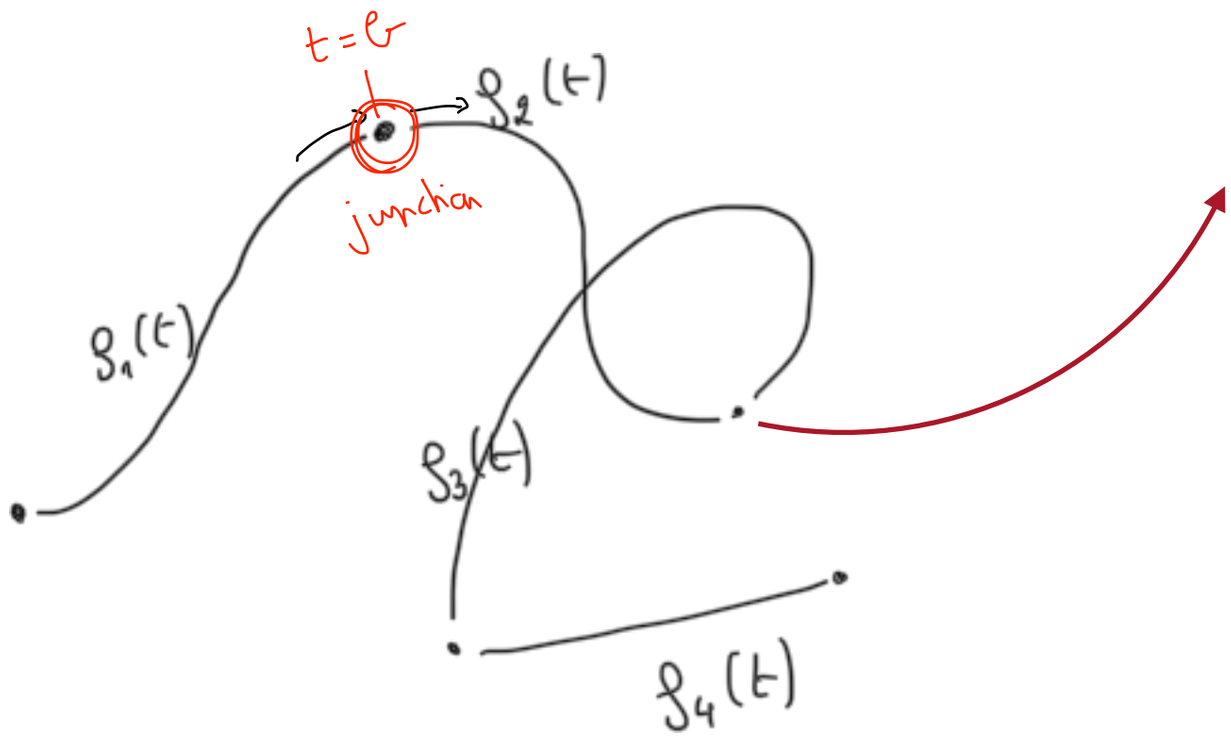
courbure ~ $f''(s)$



CONTINUITÉ GÉOMÉTRIQUE ($\mathscr{G}^k$)

$\mathscr{C}^k$

Courbes modélisée par morceaux :



$f_1 : [a, b] \to \mathbb{R}^k$

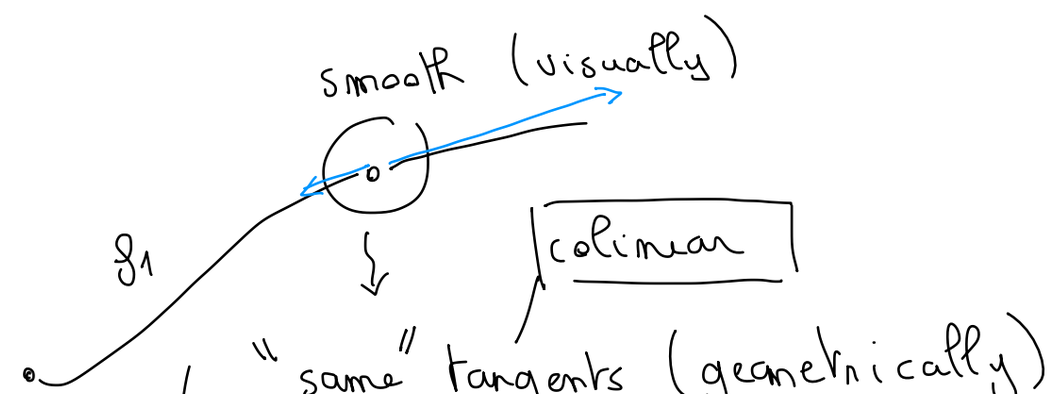
$f_2 : [b, c] \to \mathbb{R}^k$

$f_3 : [c, d] \to \mathbb{R}^k$

$f_4 : [d, e] \to \mathbb{R}^k$

Courbe continue $\longrightarrow$ $f_1(c) = f_2(c)$

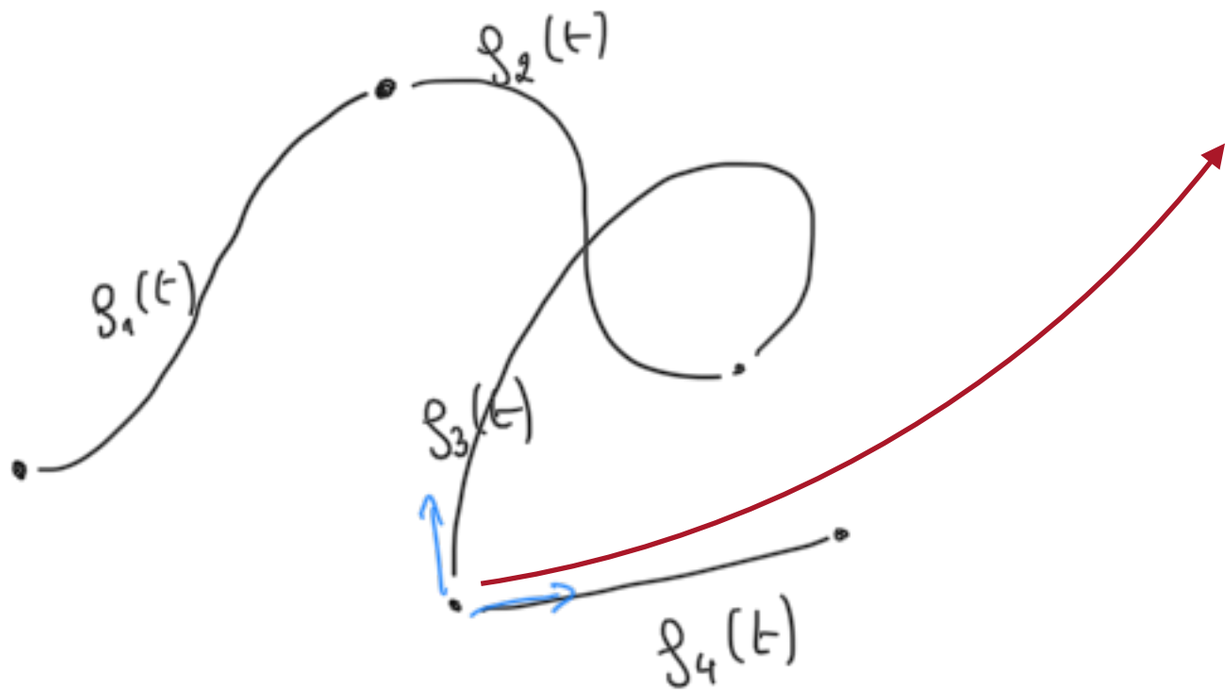
Continuité de la dérivée / tangente ?



geometric continuity $\to$ $\boxed{\mathscr{G}^k} \neq \mathscr{C}^k$

CONTINUITÉ GÉOMÉTRIQUE ($\mathscr{G}^k$)

Courbes modélisée par morceaux :



Courbe continue

Continuité de la dérivée / tangente ?

- Discontinue

$$f_1 : [a, b] \to \mathbb{R}^k$$

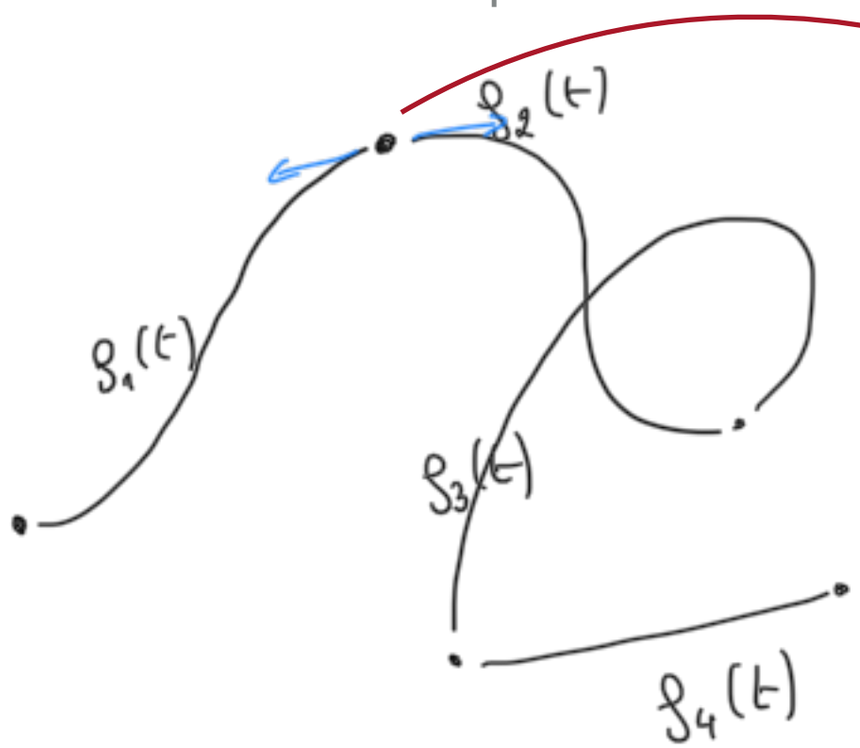
$$f_2 : [b, c] \to \mathbb{R}^k$$

$$f_3 : [c, d] \to \mathbb{R}^k$$

$$f_4 : [d, e] \to \mathbb{R}^k$$

CONTINUITÉ GÉOMÉTRIQUE ($\mathscr{G}^k$)

Courbes modélisée par morceaux :



Courbe continue

Continuité de la dérivée / tangente ?

$$f_1'(b^-) = f_2'(b^+)$$

$$f_1 : [a, b] \to \mathbb{R}^k$$

$$f_2 : [b, c] \to \mathbb{R}^k$$

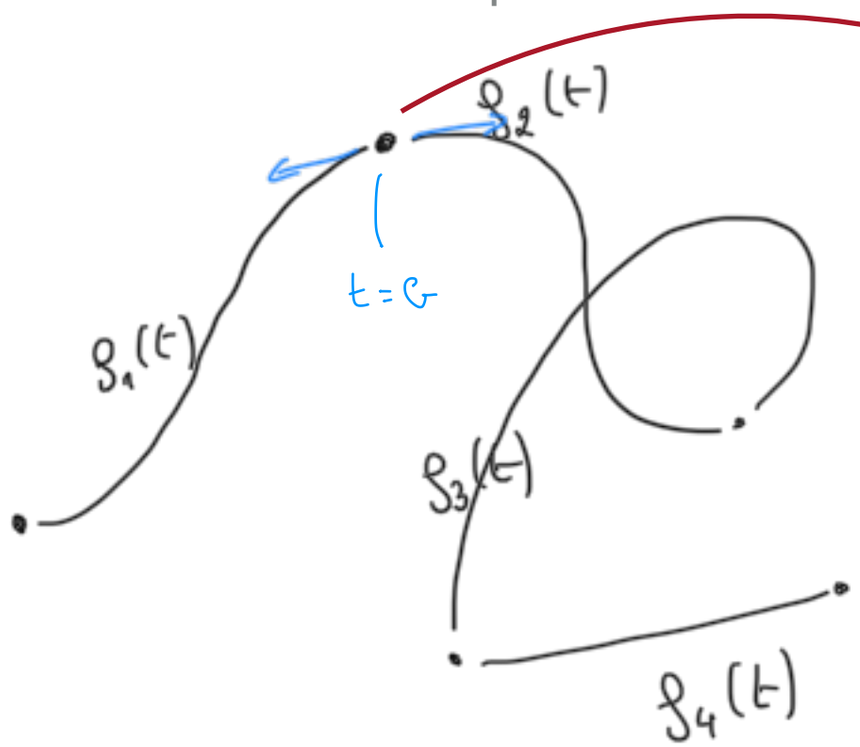
$$f_3 : [c, d] \to \mathbb{R}^k$$

$$f_4 : [d, e] \to \mathbb{R}^k$$

CONTINUITÉ GÉOMÉTRIQUE ($\mathscr{G}^k$)

$$\begin{cases} b^- : t \underset{t<b}{\longmapsto} b \\ b^+ : t \underset{t>b}{\longmapsto} b \end{cases}$$

Courbes modélisée par morceaux :



$f_1 : [a,b] \to \mathbb{R}^k$

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$f_3 : [c,d] \to \mathbb{R}^k$

$f_4 : [d,e] \to \mathbb{R}^k$

Courbe continue

Continuité de la dérivée / tangente ?

$f_1'(b^-) = f_2'(b^+)$

Continuité paramétrique $\mathscr{C}^0$
$\shortparallel$
$\mathscr{G}^0$

Tangentes colinéaires en b

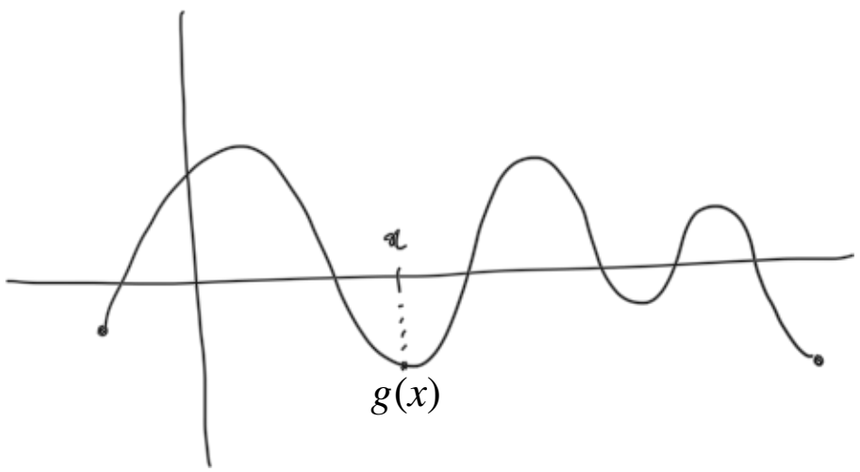
$f_1'(b^-) = \lambda \cdot f_2'(b^+)$ $\quad \lambda \geqslant 0$

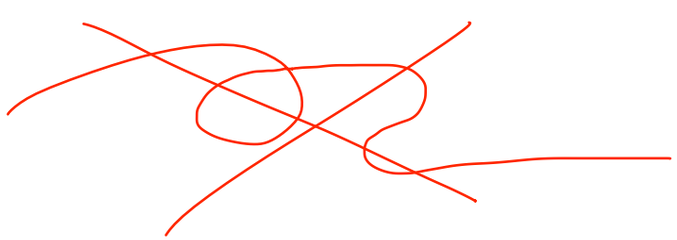
Continuité géométrique $\mathscr{G}^1$

CAS PARTICULIER : MODÈLES CARTÉSIENS

Cas particulier où le paramètre est x :

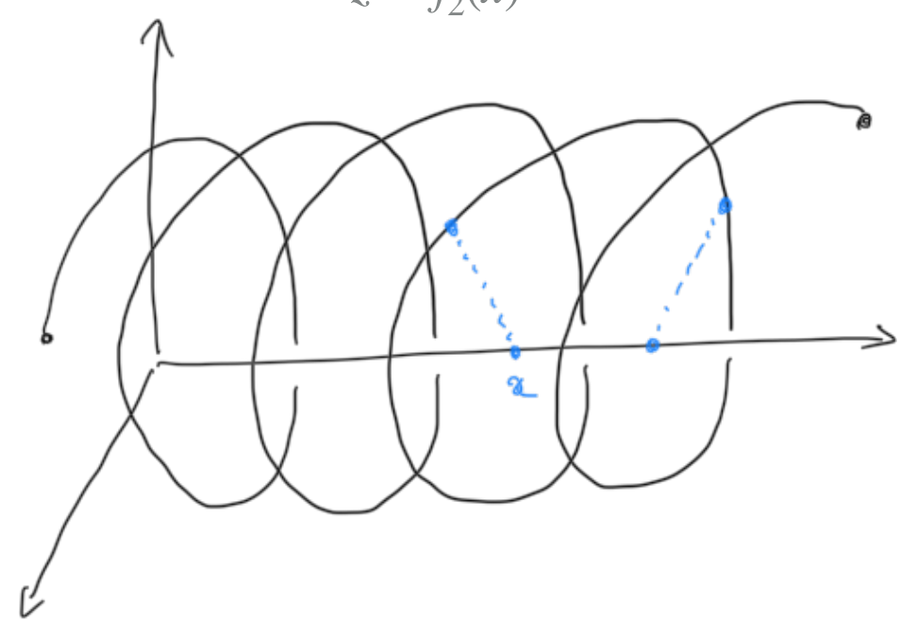
$$y = g(x)$$





$$y = f_1(x)$$

$$z = f_2(x)$$

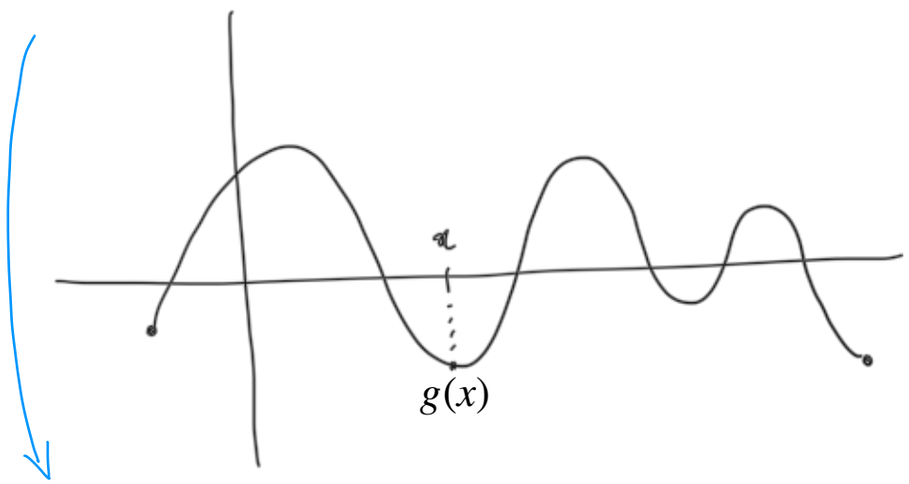


CAS PARTICULIER : MODÈLES CARTÉSIENS

Cas particulier où le paramètre est x :

$$y = g(x)$$

cartesian



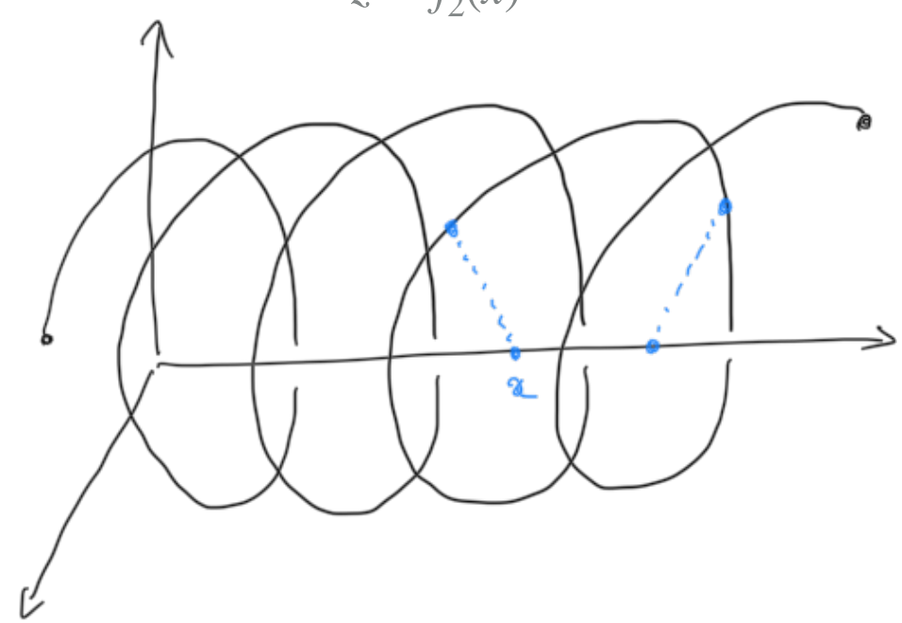
Représentation paramétrique associée :

$$f : \mathbb{R} \to \mathbb{R}^2$$

$$f(x) = \begin{pmatrix} x \\ g(x) \end{pmatrix}$$

$$y = f_1(x)$$

$$z = f_2(x)$$



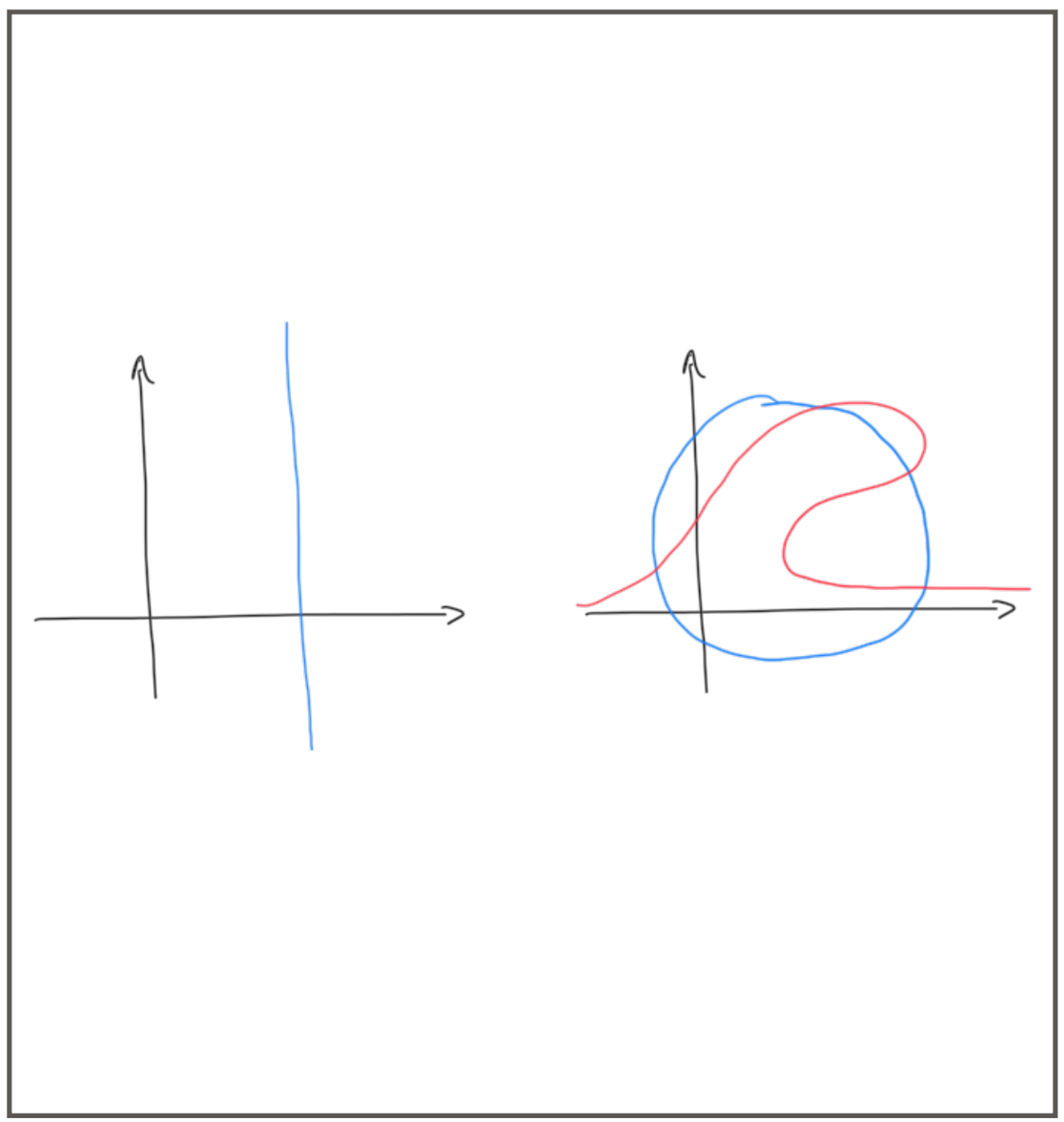
Représentation paramétrique associée :

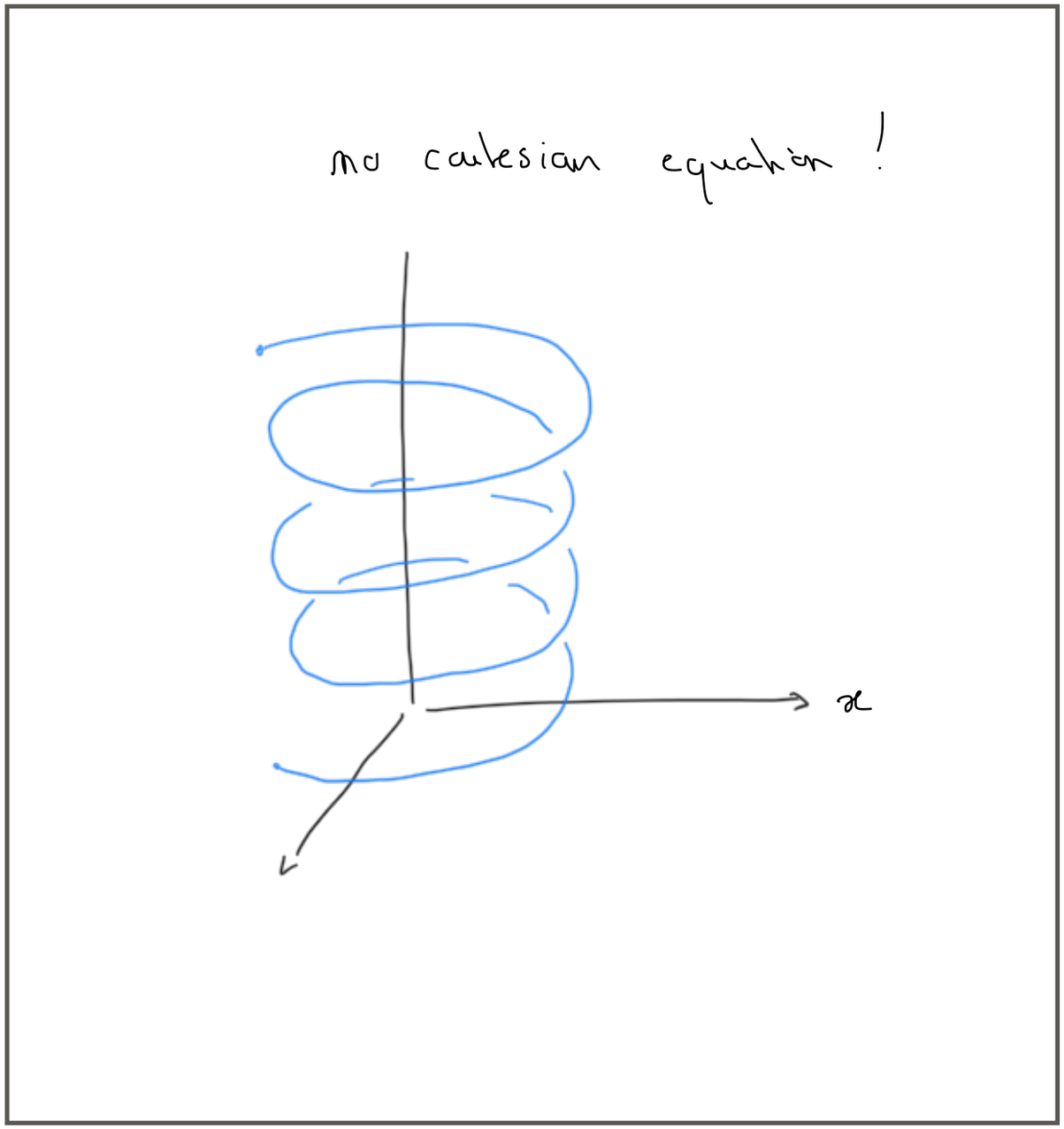
$$f : \mathbb{R} \to \mathbb{R}^3$$

$$f(x) = \begin{pmatrix} x \\ f_1(x) \\ f_2(x) \end{pmatrix}$$

CAS PARTICULIER : MODÈLES CARTÉSIENS

Attention : modèles restrictifs !





CAS PARTICULIER : MODÈLES CARTÉSIENS

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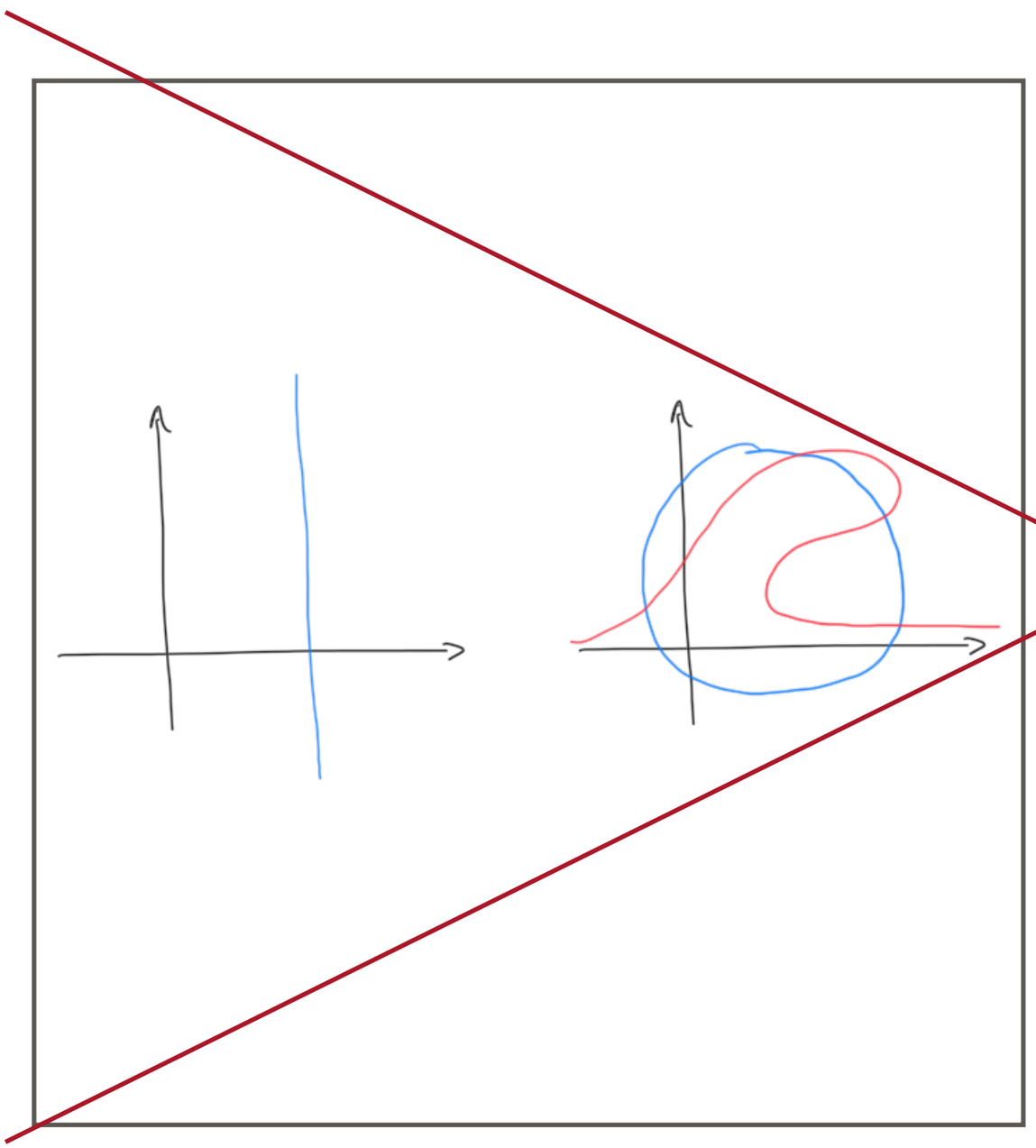
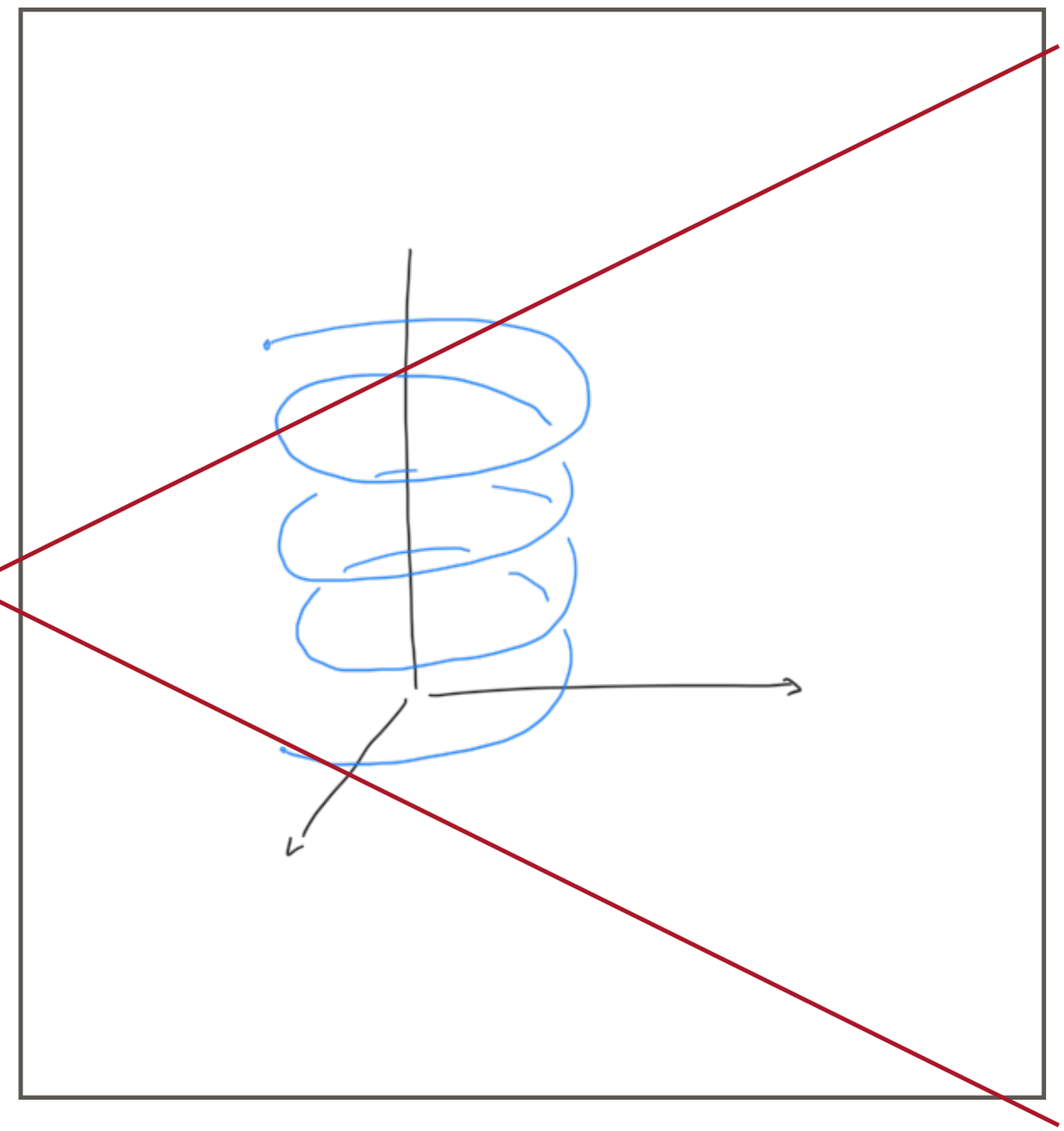
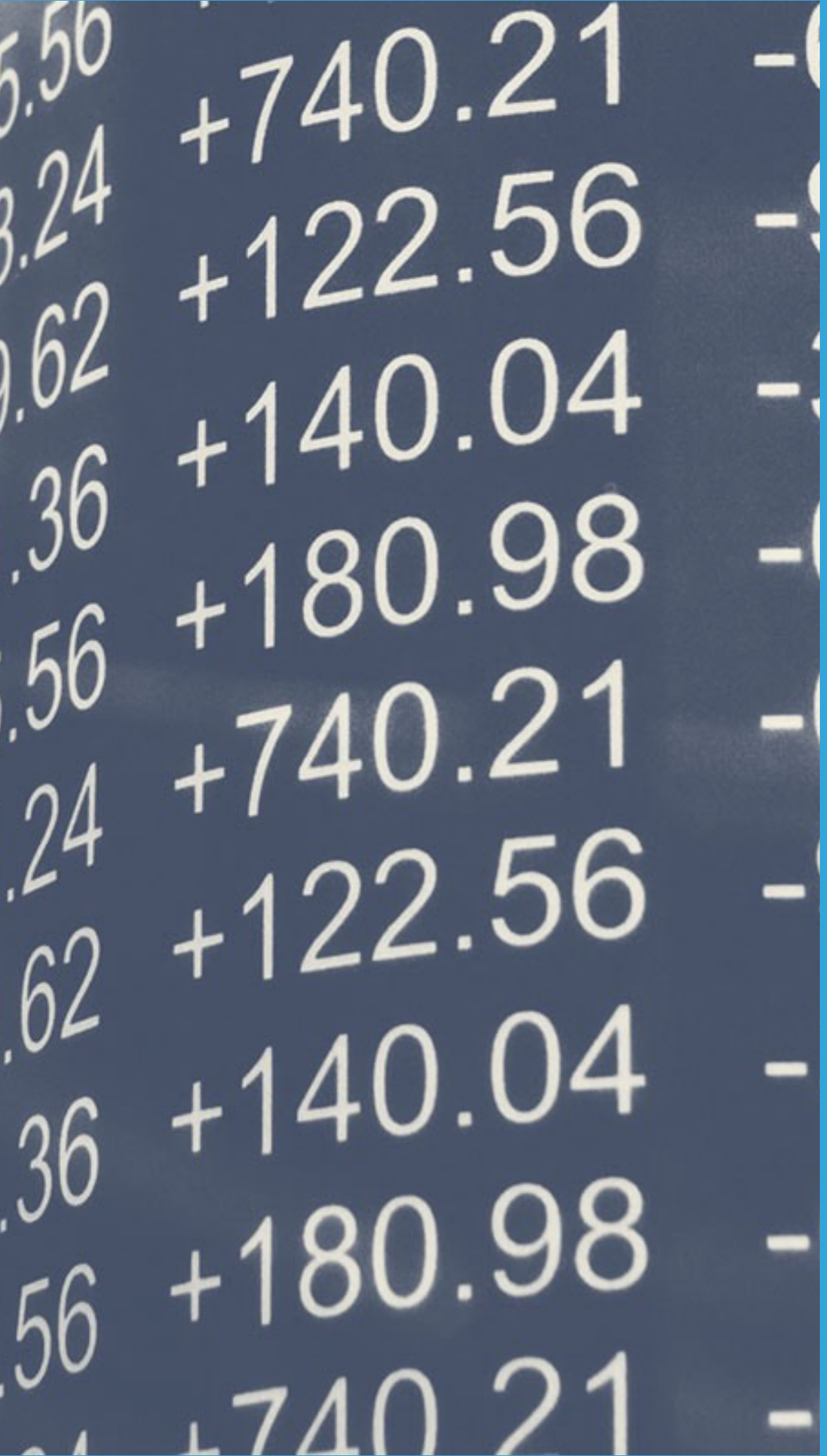


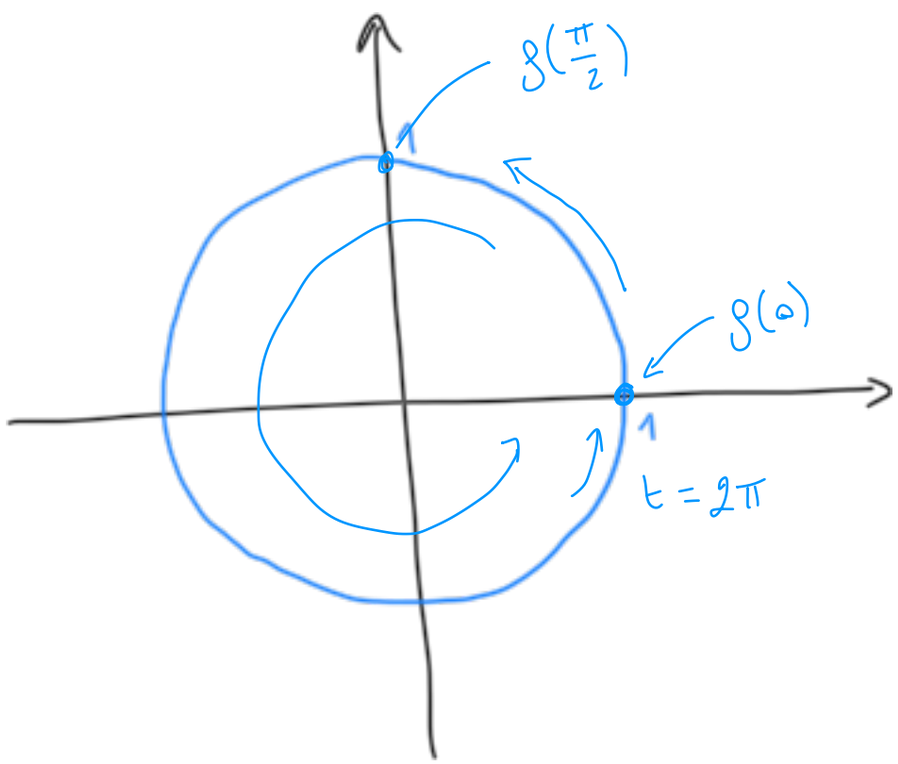




BÉZIERS, B-SPLINES …

MOTIVATION

COMMENT CONTRÔLER LE FORME DE LA COURBE ?

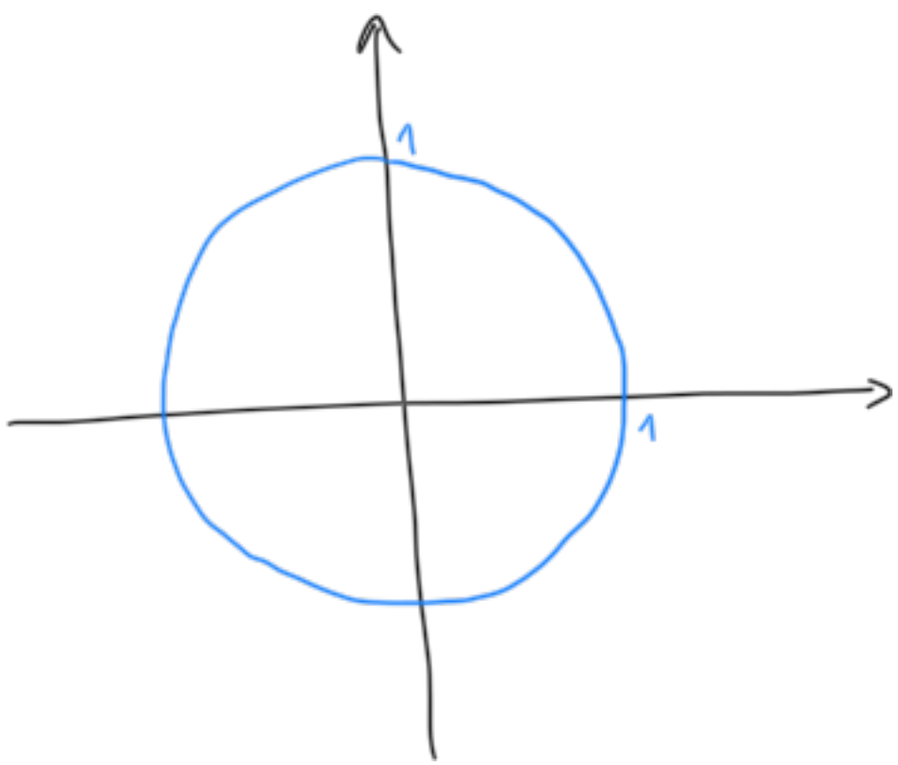


Quelle équation $f : \cancel{\mathbb{R}}\ [0, 2\pi] \to \mathbb{R}^2$?

$$f(t) = \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix}$$

$f(0)$

COMMENT CONTRÔLER LE FORME DE LA COURBE ?



Quelle équation $f : \mathbb{R} \to \mathbb{R}^2$?

Pourtant courbe très simple …

- Question trop générale

Choisir f d'une forme particulière

Dépendant de <u>paramètres simples</u> ✓

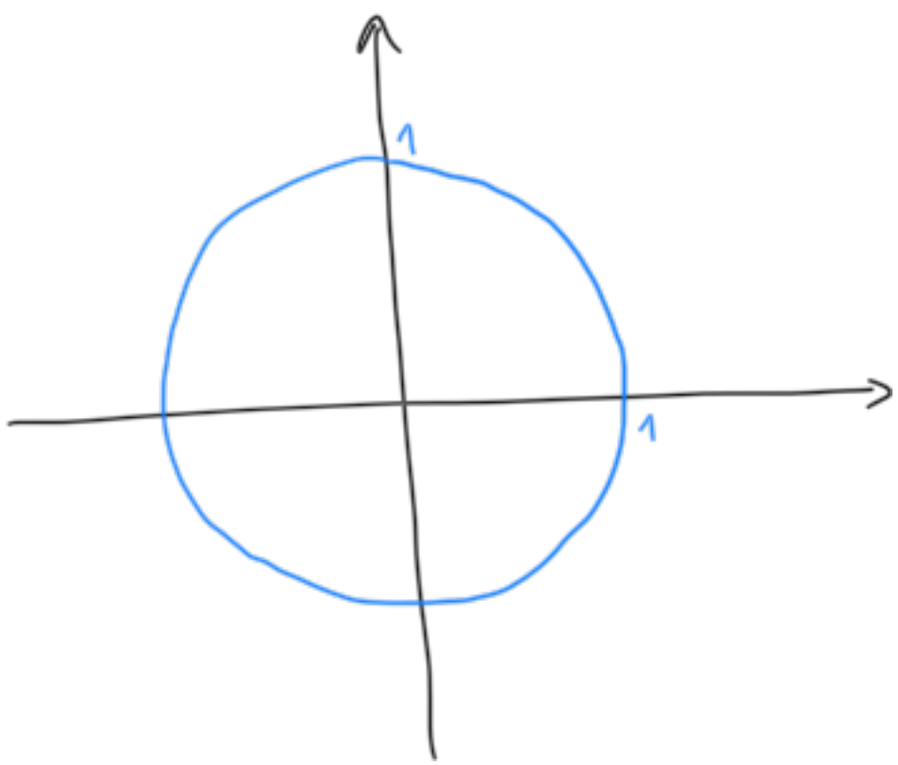
Contrôlent la forme

$\boxed{P - d^o n}$ ⇝ $\mathbb{R}_n[X]$ vect. space

↳ dim. $n+1$

↳ basis : $n+1$ poly ?

COMMENT CONTRÔLER LE FORME DE LA COURBE ?



Quelle équation $f : \mathbb{R} \to \mathbb{R}^2$?

Pourtant courbe très simple …

- Question trop générale

Choisir f d'une forme particulière

Polynômes de degré $\leqslant n$

$n+1$ coefficients

Dépendant de <u>paramètres simples</u>

Contrôlent la forme

…/…

COMMENT CONTRÔLER LE FORME DE LA COURBE ?

$$f(t) = \sum_{i=1}^{n} \varphi_i(t) \cdot P_i$$

Fonctions de base fixes

Paramètres :
points de $\mathbb{R}^k$ (contrôlant la courbe)

(ce cours)

(5A)

COMMENT CONTRÔLER LE FORME DE LA COURBE ?

$$f(t) = \sum_{i=1}^{n} \varphi_i(t) \cdot P_i$$

Fonctions de base fixes

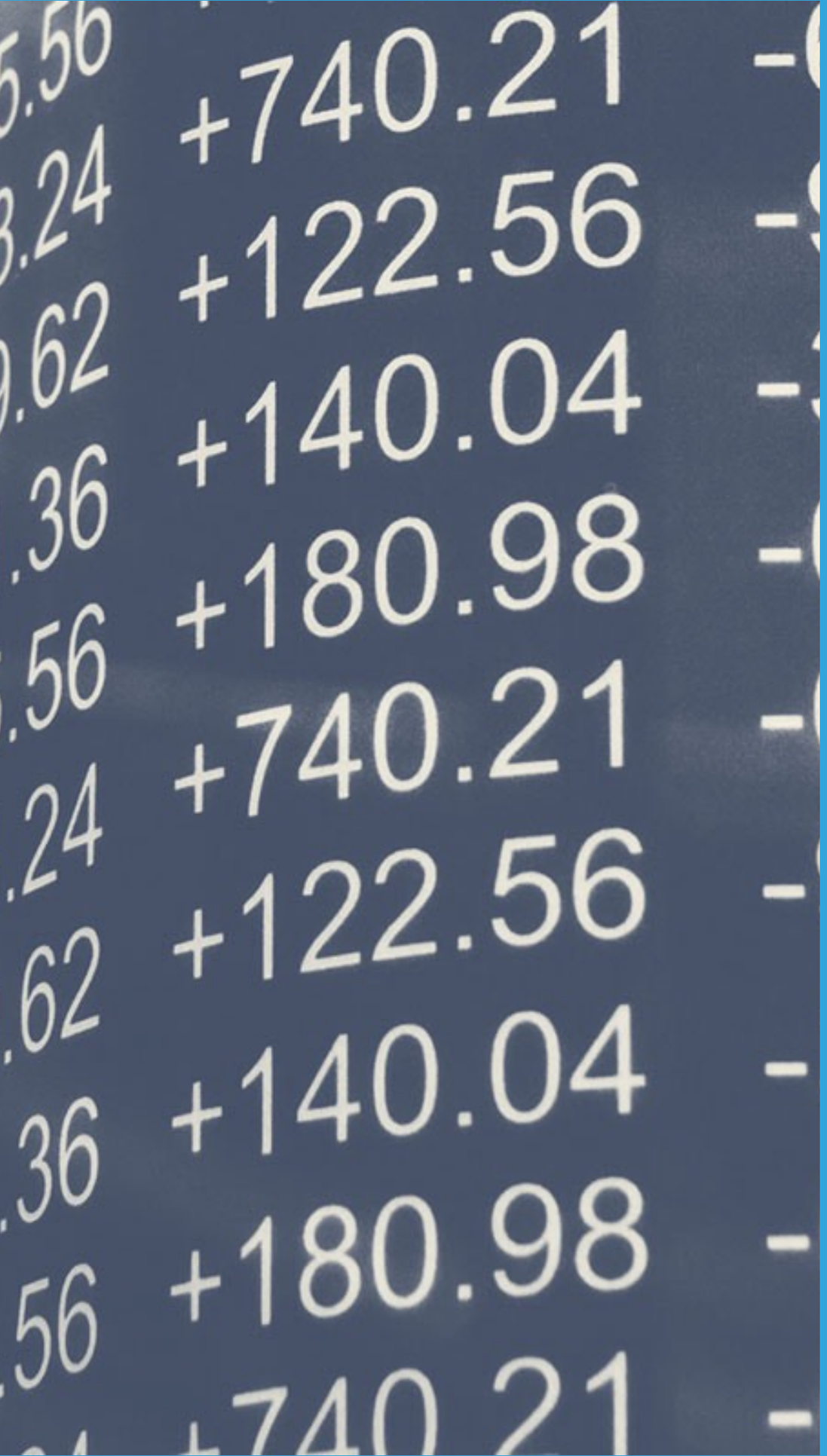
Paramètres :
points de $\mathbb{R}^k$ (contrôlant la courbe)

φ_i polynômes de Bernstein → courbes de Béziers

(ce cours)

φ_i fonctions B-spline → courbes B-spline

(5A)



COURBES DE BÉZIERS

COURBES DE BÉZIERS

$f : \mathbb{R} \to \mathbb{R}^2$

? $f = \sum_i A_i \cdot t^i \rightsquigarrow A_i \in \mathbb{R}^2$

$A_i = (a_i, b_i)$

$$f(t) = \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix}$$

polynômes de degré au plus n

$\to$ basis ??

$\mathbb{R}_n[t]$

$1, t, t^2 \dots t^n$

canonical basis

dans la base canonique :

$\to f_1(t) = a_0 + a_1 t + \cdots a_n t^n$

$\to f_2(t) = b_0 + b_1 t + \cdots b_n t^n$

Parameters :

$A_i \in \mathbb{R}^2$

COURBES DE BÉZIERS

$$f(t) = \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix}$$

polynômes de degré au plus n

dans la base canonique :

$$f_1(t) = a_0 + a_1 t + \cdots a_n t^n$$

$$f_2(t) = b_0 + b_1 t + \cdots b_n t^n$$

$$f(t) = \sum_{i=0}^{n} t^i \cdot \begin{pmatrix} a_i \\ b_i \end{pmatrix}$$

COURBES DE BÉZIERS

$$f(t) = \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix}$$

polynômes de degré au plus n

dans la base canonique :

$$f_1(t) = a_0 + a_1 t + \cdots a_n t^n$$

$$f_2(t) = b_0 + b_1 t + \cdots b_n t^n$$

$$f(t) = \sum_{i=1}^{n} \varphi_i(t) \cdot P_i$$

Base

Paramètre de contrôle

$$f(t) = \sum_{i=0}^{n} t^i \begin{pmatrix} a_i \\ b_i \end{pmatrix}$$

COURBES DE BÉZIERS

$$f(t) = \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix}$$

polynômes de degré au plus n

dans la base canonique :

$$f_1(t) = a_0 + a_1 t + \cdots a_n t^n$$

$$f_2(t) = b_0 + b_1 t + \cdots b_n t^n$$

$$f(t) = \sum_{i=1}^{n} \varphi_i(t) \cdot P_i \quad \in \mathbb{R}^2$$

?

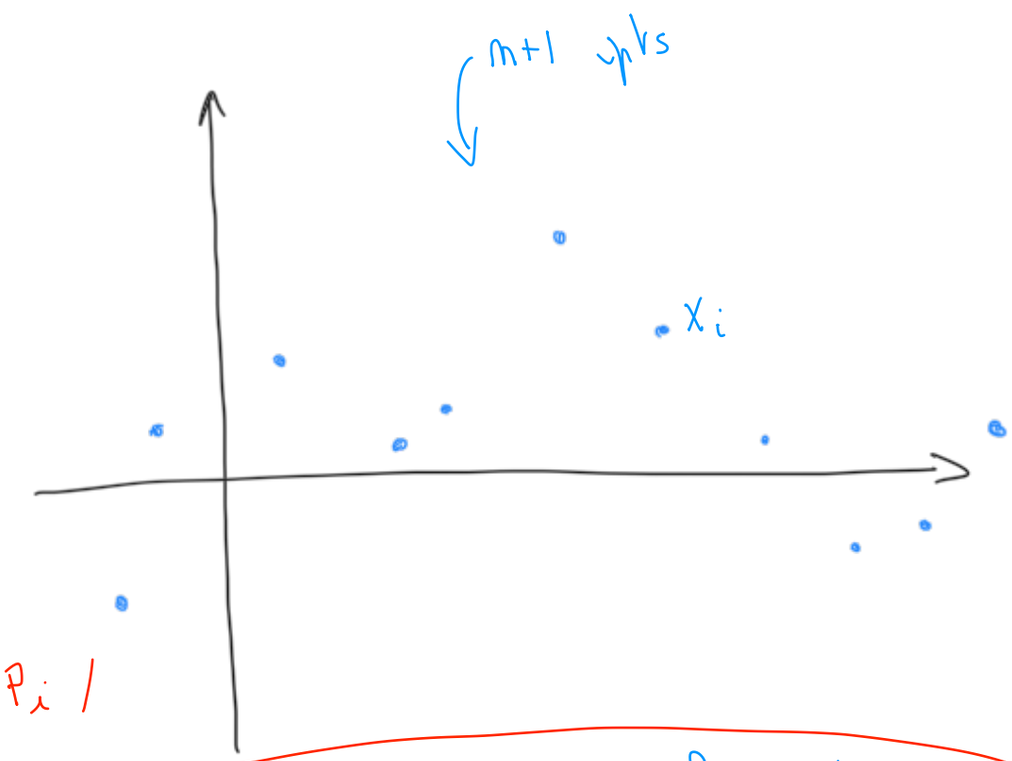
Base

Paramètre de contrôle

$$f(t) = \sum_{i=0}^{n} t^i \cdot \begin{pmatrix} a_i \\ b_i \end{pmatrix}$$

linear comb.

$f(t_i) = x_i \quad \forall i$



(paramètres P_i / pts x_i)

goes through the pts

$\{P_i\}$ tq la ~~courbe passe~~ près de ces points ?

at what time ?

no relation intuitive

COURBES DE BÉZIERS

$$f(t) = \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix}$$

polynômes de degré au plus n

dans la ~~base canonique~~ : Pas la bonne base …

$$f_1(t) = a_0 + a_1 t + \cdots a_n t^n$$

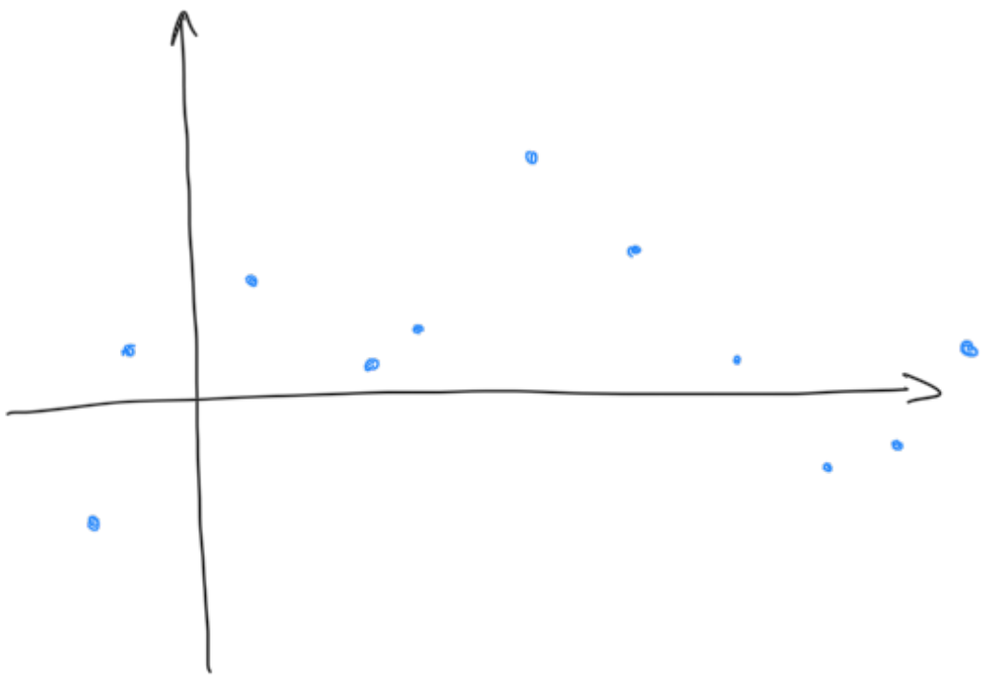
$$f_2(t) = b_0 + b_1 t + \cdots b_n t^n$$

$$f(t) = \sum_{i=1}^{n} \varphi_i(t) \cdot P_i$$

Base

Paramètre de contrôle

$$f(t) = \sum_{i=0}^{n} t^i \begin{pmatrix} a_i \\ b_i \end{pmatrix}$$



$\{P_i\}$ tq la courbe passe près de ces points ?

POLYNÔMES DE BERNSTEIN

Base de Bernstein de $\mathbb{R}_n[t]$

- $n+1$ polynômes
- $\varphi_{n,i} : [0,1] \to \mathbb{R}$ pour le $i = 0 \ldots n$
  $\varphi_{n,i}(t) = C_n^i \cdot t^i \cdot (1-t)^{n-i}$

Parenté avec le binôme de Newton

$$(a+b)^n = \sum_{k=0}^{n} C_n^k a^k b^{n-k}$$

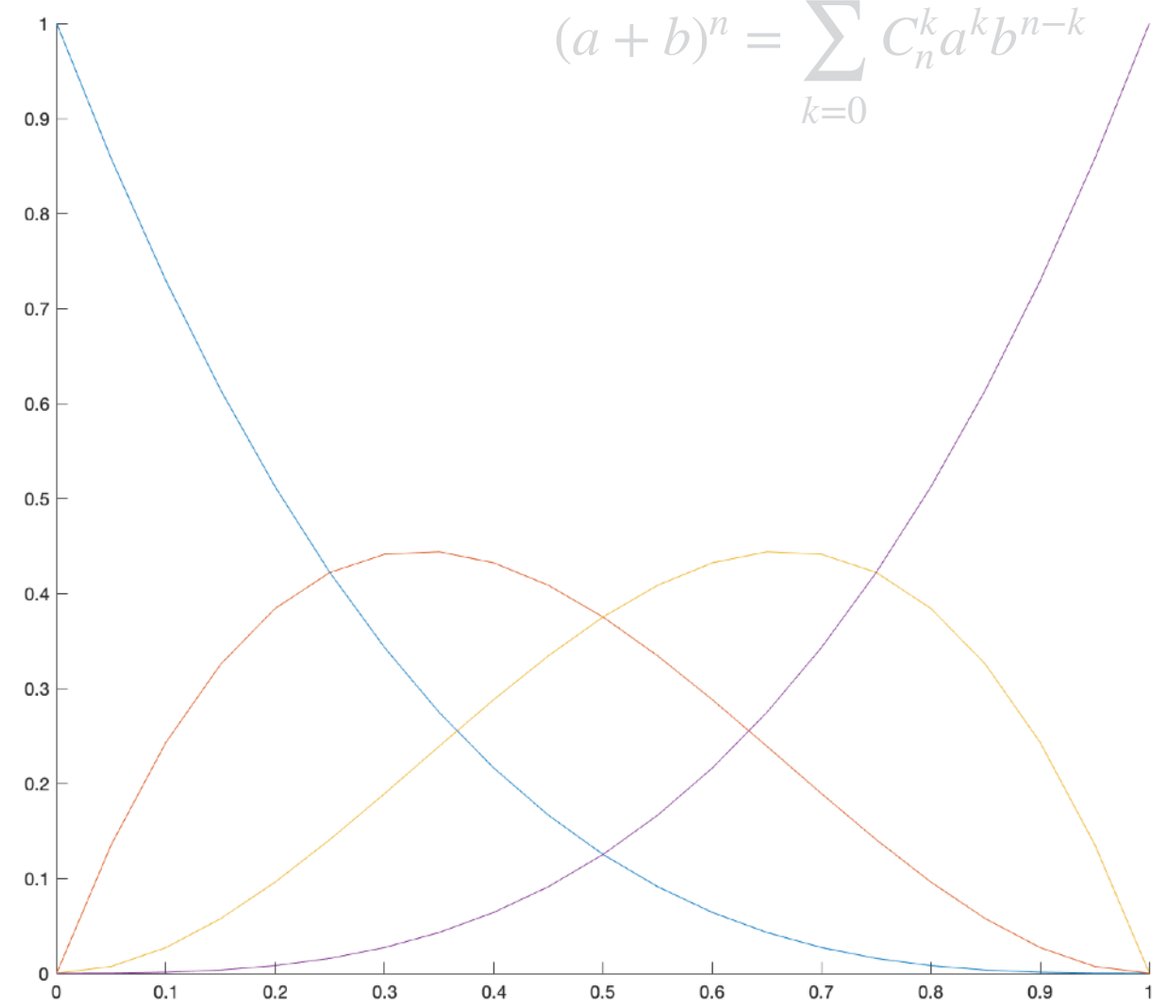
Ex : pour $n = 3$

$\varphi_{3,0}(t) = (1-t)^3$

$\varphi_{3,1}(t) = 3t(1-t)^2$

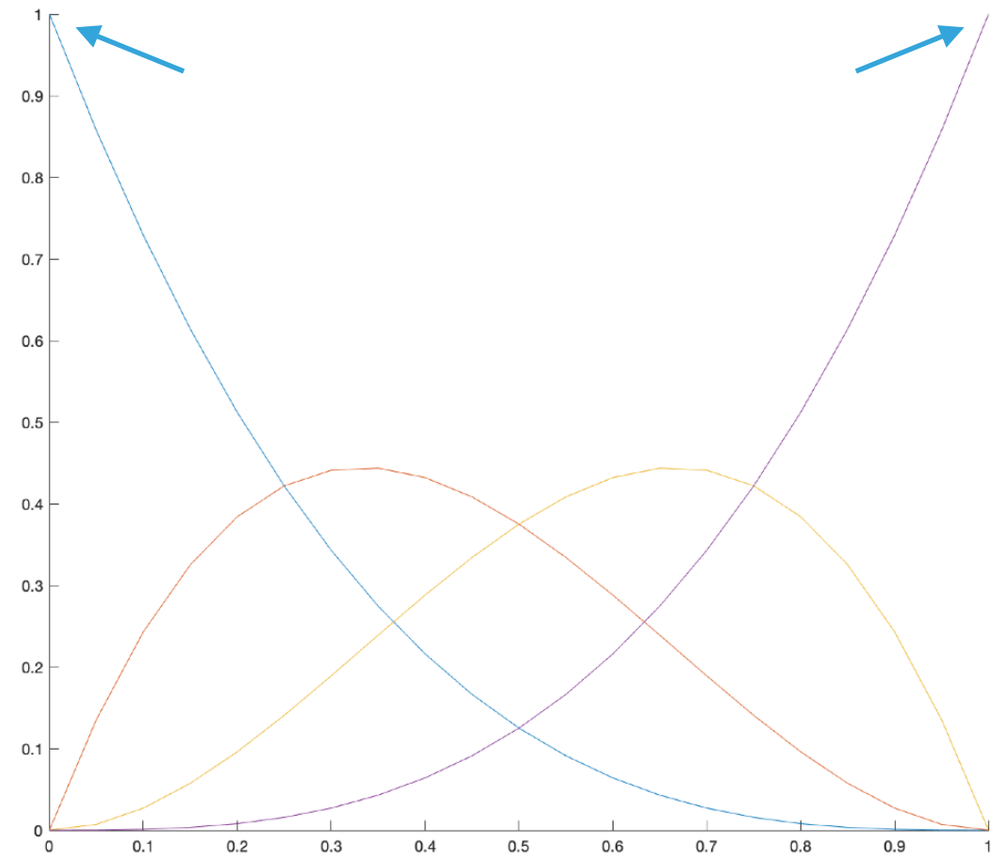
$\varphi_{3,2}(t) = 3t^2(1-t)$

$\varphi_{3,3}(t) = t^3$



POLYNÔMES DE BERNSTEIN – PROPRIÉTÉS

$$\varphi_{n,0}(0) = 1$$
$$\varphi_{n,n}(1) = 1$$



POLYNÔMES DE BERNSTEIN – PROPRIÉTÉS

$$\varphi_{n,0}(0) = 1$$
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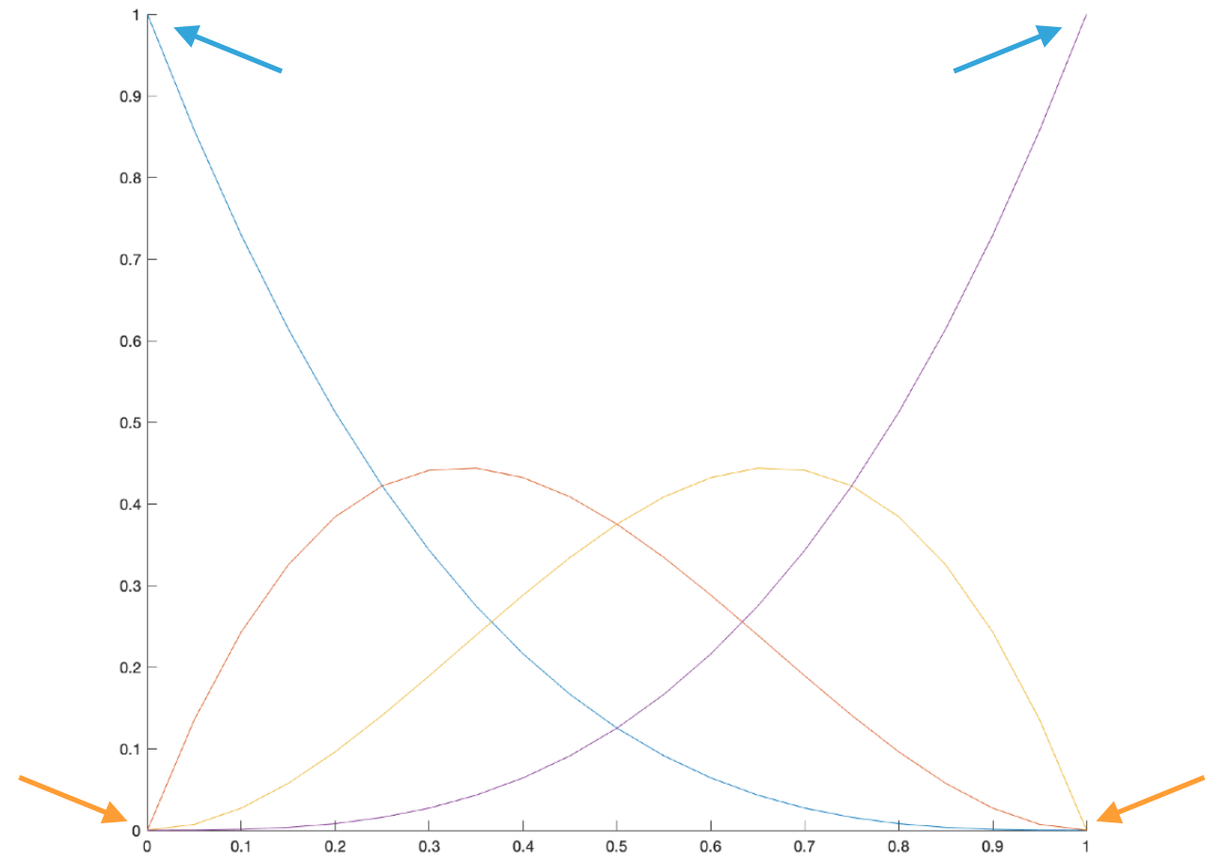
$$\varphi_{n,i}(0) = 0$$
$$0 < i < n$$



POLYNÔMES DE BERNSTEIN – PROPRIÉTÉS

$$\varphi_{n,0}(0) = 1$$
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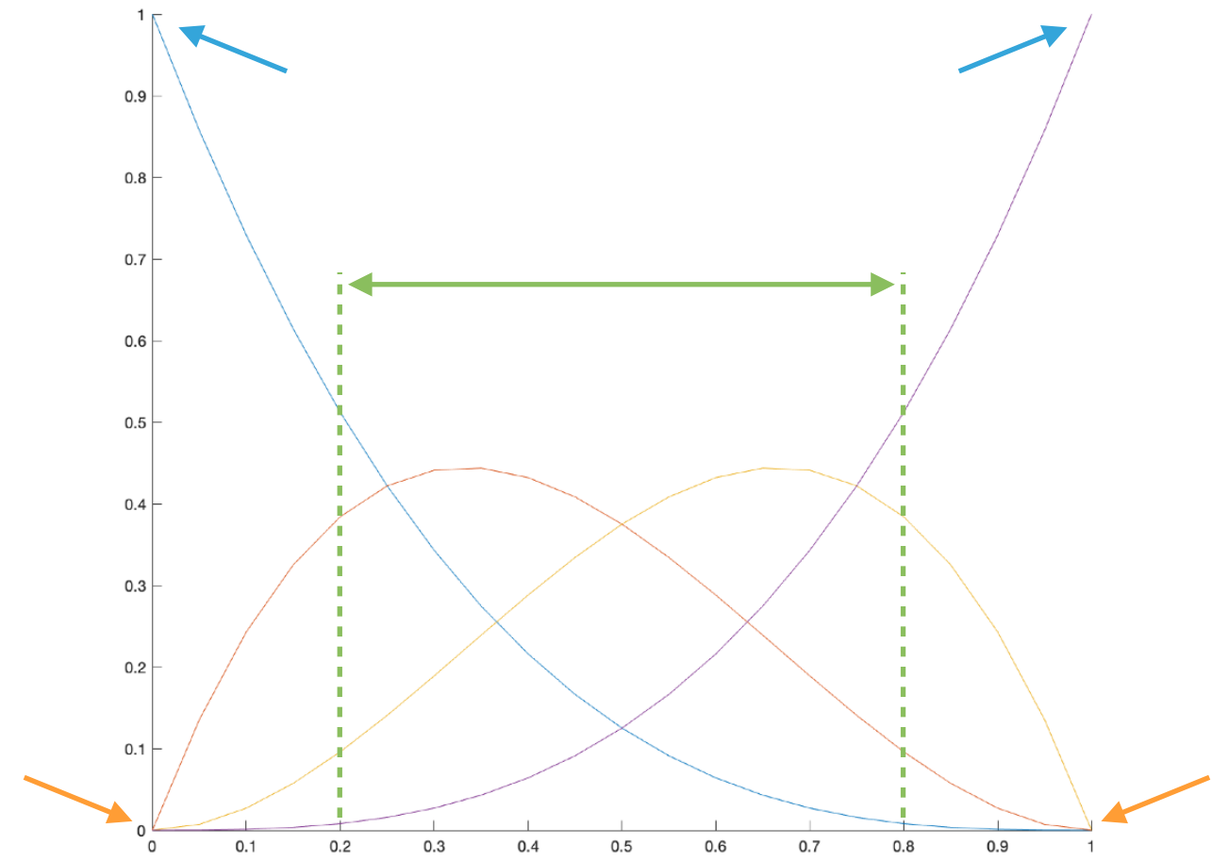
<u>Symétrie</u>

$$\varphi_{n,i}(t) = \varphi_{n,n-i}(1-t)$$



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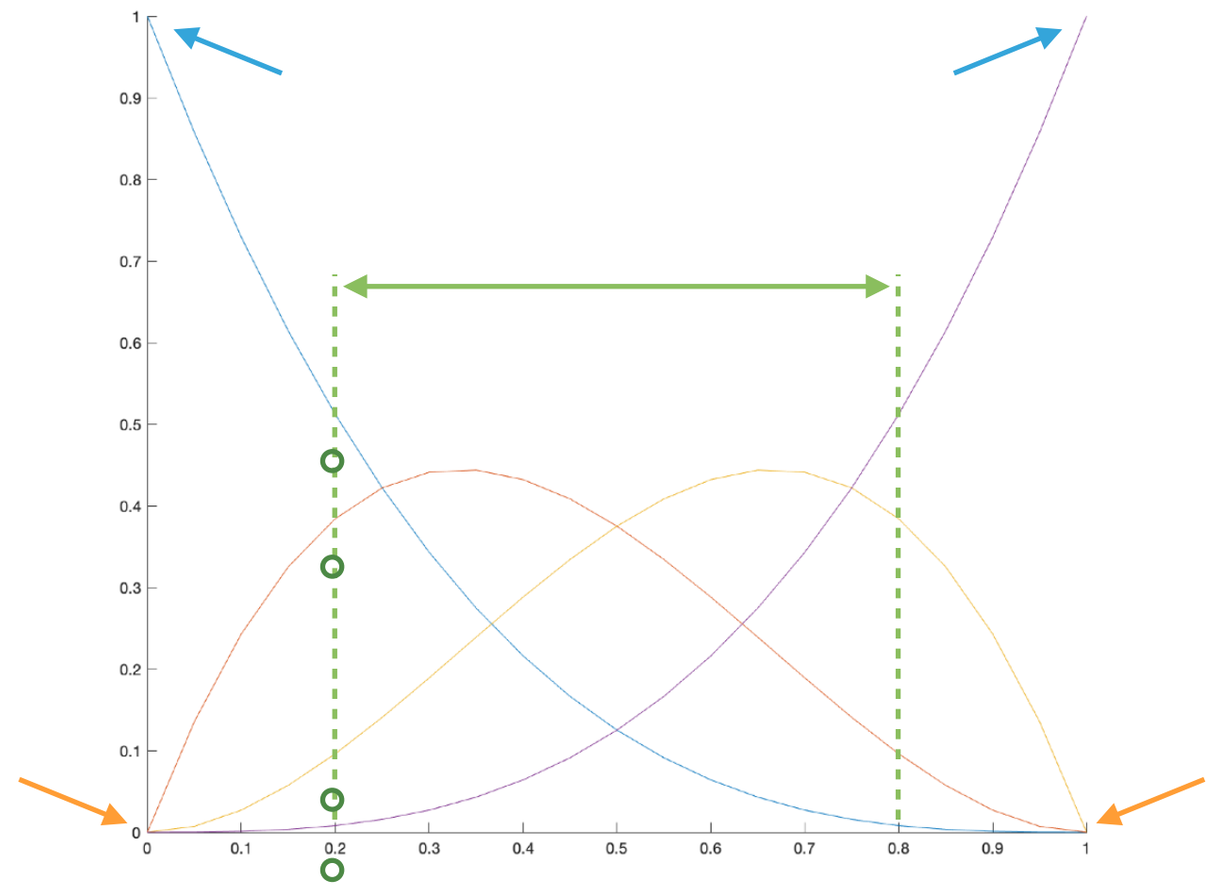

Symétrie

$$\varphi_{n,i}(t) = \varphi_{n,n-i}(1-t)$$

Partition de l'unité

$$\sum_{i=0}^{n} \varphi_{n,i}(t) = 1 \qquad \forall t \in [0,1]$$

POLYNÔMES DE BERNSTEIN – PROPRIÉTÉS

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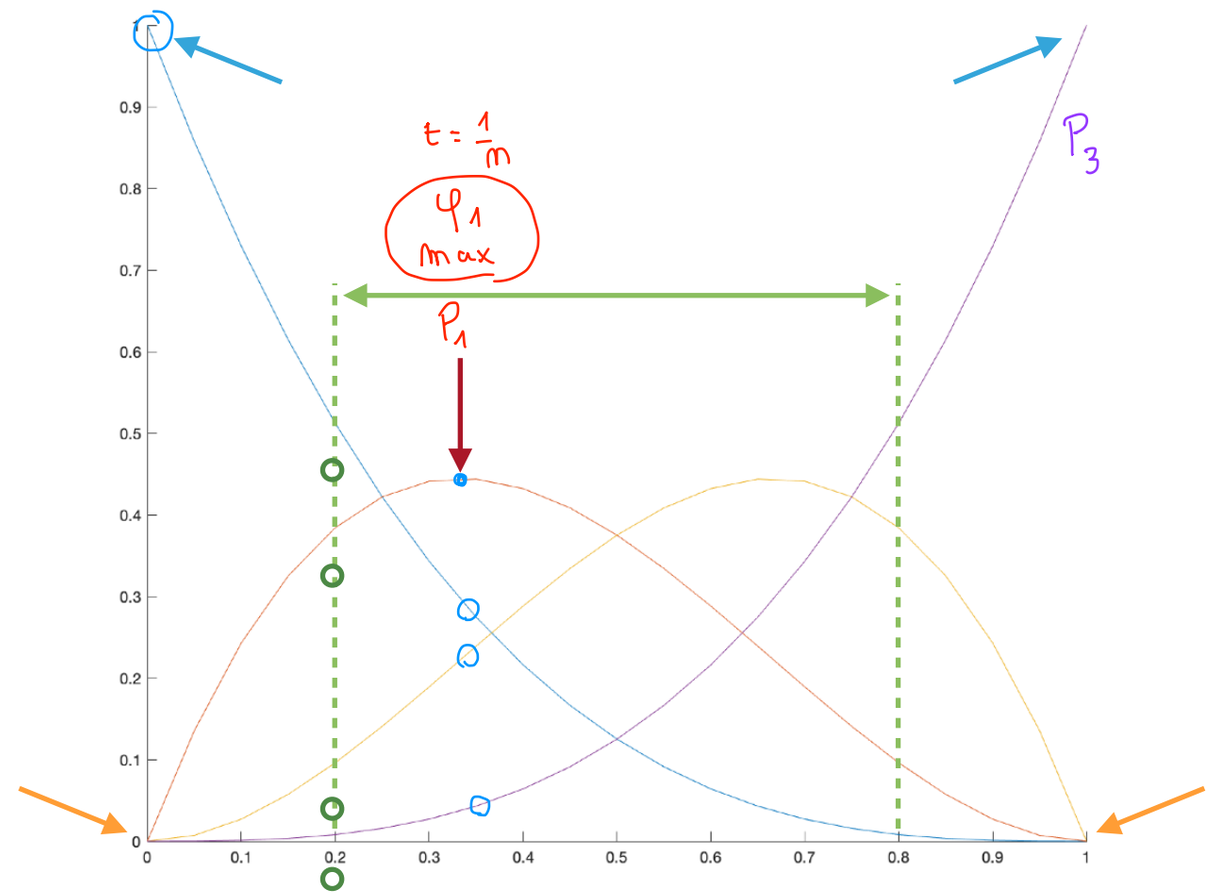

Symétrie

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Partition de l'unité

$$\sum_{i=0}^{n} \varphi_{n,i}(t) = 1 \qquad \forall t \in [0,1]$$

Maximum de $\varphi_{n,i}(t)$ en $\dfrac{i}{n}$

POLYNÔMES DE BERNSTEIN – PROPRIÉTÉS

$$\varphi_{n,0}(0) = 1$$
$$\varphi_{n,n}(1) = 1$$

$$\varphi_{n,i}(0) = 0$$
$$0 < i < n$$

Proche triangle Pascal

Formule récursive

en $\varphi_{n,i}(t) = t\varphi_{n-1,i-1}(t) + (1-t)\varphi_{n-1,i}(t)$

Symétrie

$$\varphi_{n,i}(t) = \varphi_{n,n-i}(1-t)$$

Partition de l'unité

$$\sum_{i=0}^{n} \varphi_{n,i}(t) = 1 \qquad \forall t \in [0,1]$$

Maximum de $\varphi_{n,i}(t)$

en $\dfrac{i}{n}$

COURBES DE BÉZIERS

Courbe de Béziers de degré n

$$f(t) = \sum_{i=0}^{n} \varphi_{n,i}(t) \cdot P_i$$

$P_0, \ldots, P_n$ points de contrôle

COURBES DE BÉZIERS

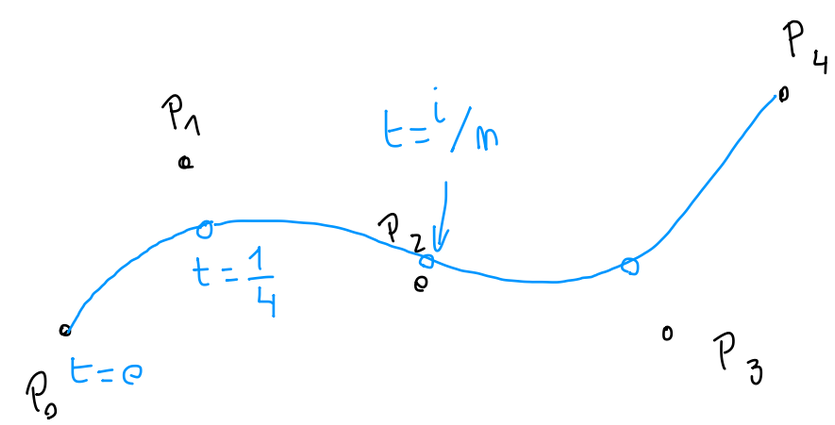
Courbe de Béziers de degré n

$$f(t) = \sum_{i=0}^{n} \varphi_{n,i}(t) \cdot P_i$$

d° n poly.

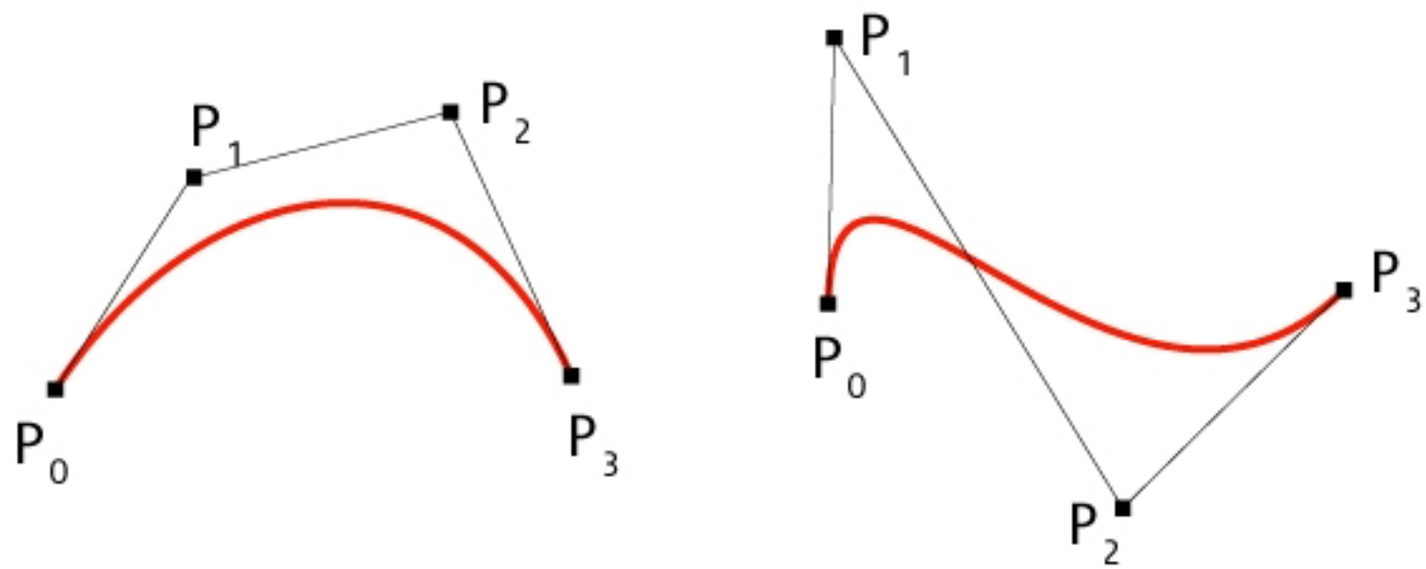
$P_0, \ldots, P_n$ points de contrôle

- $f(0) = P_0$
- $f(1) = P_n$
- Influence maximale de P_i en $\dfrac{i}{n}$



↳ d° 4 poly.

COURBES DE BÉZIERS



- $f(t)$ barycentre des P_i pondérés par $\varphi_{n,i}(t)$
- Courbe dans l'enveloppe convexe des points de contrôle

DÉRIVÉES / TANGENTES DES COURBES DE BÉZIERS

$$\varphi'_{n,i}(t) =$$

ToDo

$$\varphi'_{n,i}(t) = n(\varphi_{n-1,i-1}(t) - \varphi_{n-1,i}(t))$$

Bezier d° n

$$g(t) = \sum_{i=0}^{n} \varphi_{n,i}(t) \cdot \overset{\text{constant}}{P_i}$$

$$f'(t) = \sum_i \varphi_{n,i}(t)' \cdot P_i$$

$$= \sum_i n\left(\varphi_{n-1,i-1}(t) - \varphi_{n-i,i}(t)\right) \cdot P_i$$

variable changes...

ToDo

$$\boxed{f'(t) = n \sum_{i=0}^{n-1} \varphi_{n-1,i}(t) \cdot (P_{i+1} - P_i)}$$

Bezier curve of degree $n-1$

control pts : $P_{i+1} - P_i$: $\overrightarrow{P_i P_{i+1}}$

$$g'(0) = n\,\overrightarrow{P_0 P_1} = n(P_1 - P_0)$$

$$g'(1) = n(P_n - P_{n-1}) = n\,\overrightarrow{P_{n-1} P_n}$$

DÉRIVÉES / TANGENTES DES COURBES DE BÉZIERS

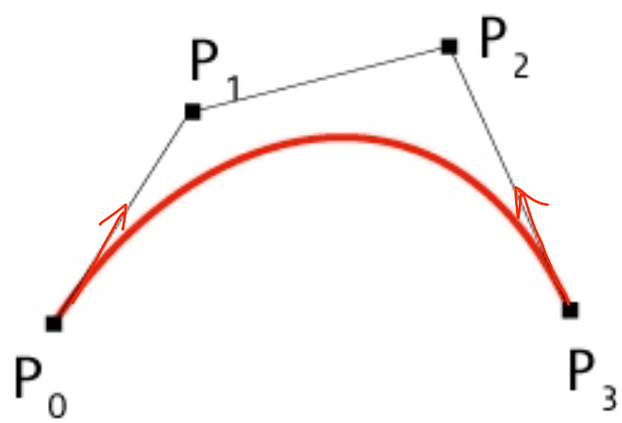
$$\boxed{f'(t) = n \sum_{i=0}^{n-1} \varphi_{n-1,i}(t) \cdot (P_{i+1} - P_i)}$$

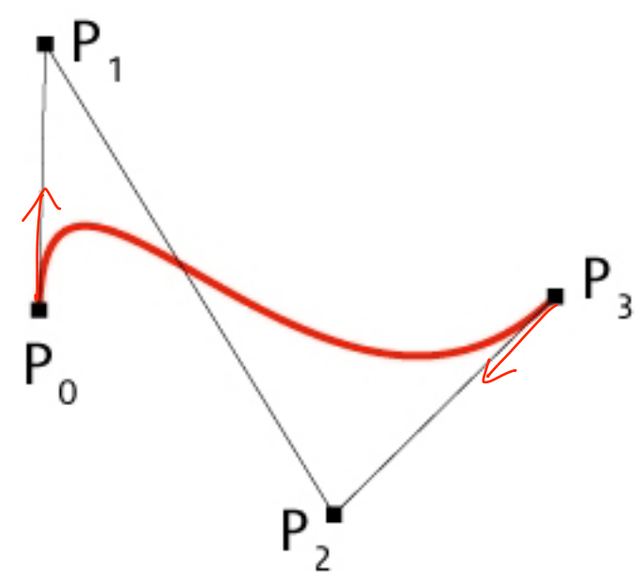
Courbe de Béziers

- Degré $n-1$
- Points de contrôle $P_1 - P_0, \ldots, P_n - P_{n-1}$

Tangente à la courbe en 0 :
$n \cdot \overrightarrow{P_0 P_1}$

Tangente à la courbe en 1 :
$n \cdot \overrightarrow{P_{n-1} P_n}$





DÉRIVÉES / TANGENTES DES COURBES DE BÉZIERS

$$\boxed{f'(t) = n \sum_{i=0}^{n-1} \varphi_{n-1,i}(t) \cdot (P_{i+1} - P_i)}$$

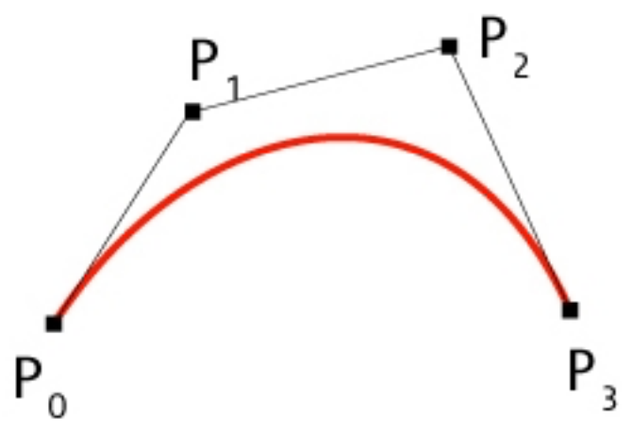
Courbe de Béziers

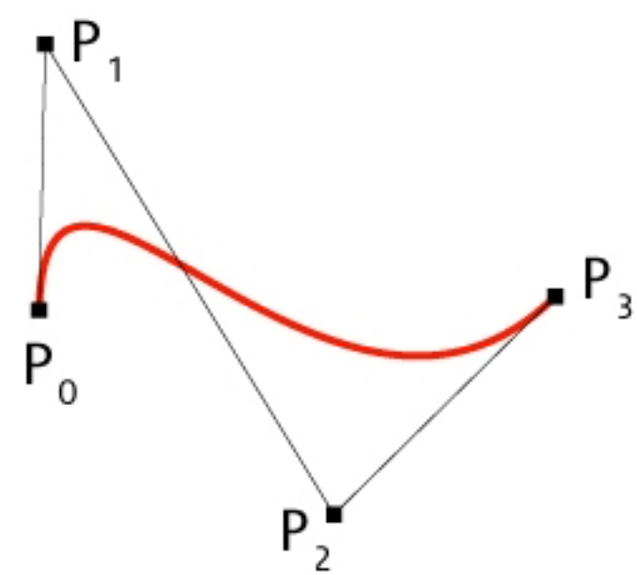
- Degré $n - 1$
- Points de contrôle $P_1 - P_0, \ldots, P_n - P_{n-1}$

Tangente à la courbe en 0 :
$n \cdot \overrightarrow{P_0 P_1}$

Tangente à la courbe en 1 :
$n \cdot \overrightarrow{P_{n-1} P_n}$

Permet de raccorder des courbes de Béziers





CALCUL DE POINT : ALGORITHME DE DE CASTELJAU

evaluate $f(t)$

$$= \sum_i \underbrace{C_m^i \, t^i (1-t)^{m-i}}_{\varphi_{m,i}(t)} \cdot P_i$$

Algorithme similaire déjà connu : calcul de $\boxed{C_n^k}$ ...

$$C_n^k = \frac{n!}{k!(n-k)!}$$

CALCUL DE POINT : ALGORITHME DE DE CASTELJAU

Algorithme similaire déjà connu : calcul de $\boxed{C_n^k}$ ...

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Débordements de capacité ...

CALCUL DE POINT : ALGORITHME DE DE CASTELJAU

Algorithme similaire déjà connu : calcul de $\boxed{C_n^k}$ ...

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Débordements de capacité ...

$$C_n^k = C_{n-1}^{k-1} + C_{n-1}^k$$

Fonction récursive :

CALCUL DE POINT : ALGORITHME DE DE CASTELJAU

Algorithme similaire déjà connu : calcul de $\boxed{C_n^k}$ ...

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~~Fonction récursive :~~

duplication des calculs ...

CALCUL DE POINT : ALGORITHME DE DE CASTELJAU

Algorithme similaire déjà connu : calcul de $\boxed{C_n^k}$ ...

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$$C_n^k = C_{n-1}^{k-1} + C_{n-1}^k$$

Débordements de capacité ...

~~Fonction récursive :~~

duplication des calculs ...

Calcul itératif via un tableau :

Triangle de Pascal

CALCUL DE POINT : ALGORITHME DE DE CASTELJAU

1) Formule récursive

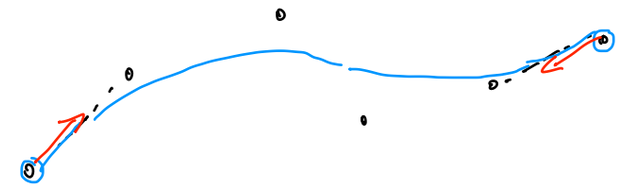
$$\varphi_{n,i}(t) = t\varphi_{n-1,i-1}(t) + (1-t)\varphi_{n-1,i}(t)$$

$C_m^i t^i (1-t)^{m-i}$

$$f(t) = \sum_{i=0}^{n} \varphi_{n,i}(t) \cdot P_i =$$

m : dimension of poly
degree

m+1 control pts P_i



$f(t) \quad t \in [0,1]$

CALCUL DE POINT : ALGORITHME DE DE CASTELJAU

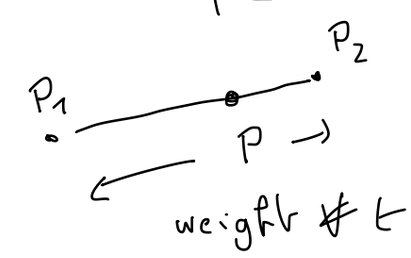
1) Formule récursive

$$\varphi_{n,i}(t) = t\varphi_{n-1,i-1}(t) + (1-t)\varphi_{n-1,i}(t)$$

$t \in [0,1]$

$P = t \cdot P_1 + (1-t) \cdot P_2$

weighted barycentre of P_1, P_2 (t)



$$f(t) = \sum_{i=0}^{n} \varphi_{n,i}(t) \cdot P_i =$$

$t \cdot P_1^{n-1} \quad + (1-t) \cdot P_0^{n-1}$

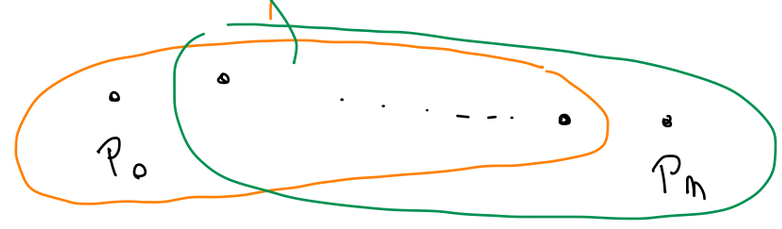
$$= t\left(\sum_{i=0}^{n-1} \varphi_{n-1,i}(t) P_{i+1}\right) + (1-t)\left(\sum_{i=0}^{n-1} \varphi_{n-1,i}(t) P_i\right)$$

P_1^{n-1} $\qquad$ P_0^{n-1}

Points de courbes intermédiaires de degré $n-1$

On pose :

$$P_i^j = f_{P_i,\ldots,P_{i+j}}(t)$$



CALCUL DE POINT : ALGORITHME DE DE CASTELJAU

1) Formule récursive

$$\boxed{P_i^j = tP_{i+1}^{j-1} + (1-t)P_i^{j-1}}$$

avec $P_i^0 = P_i$

Ressemble fortement au triangle de Pascal

CALCUL DE POINT : ALGORITHME DE DE CASTELJAU

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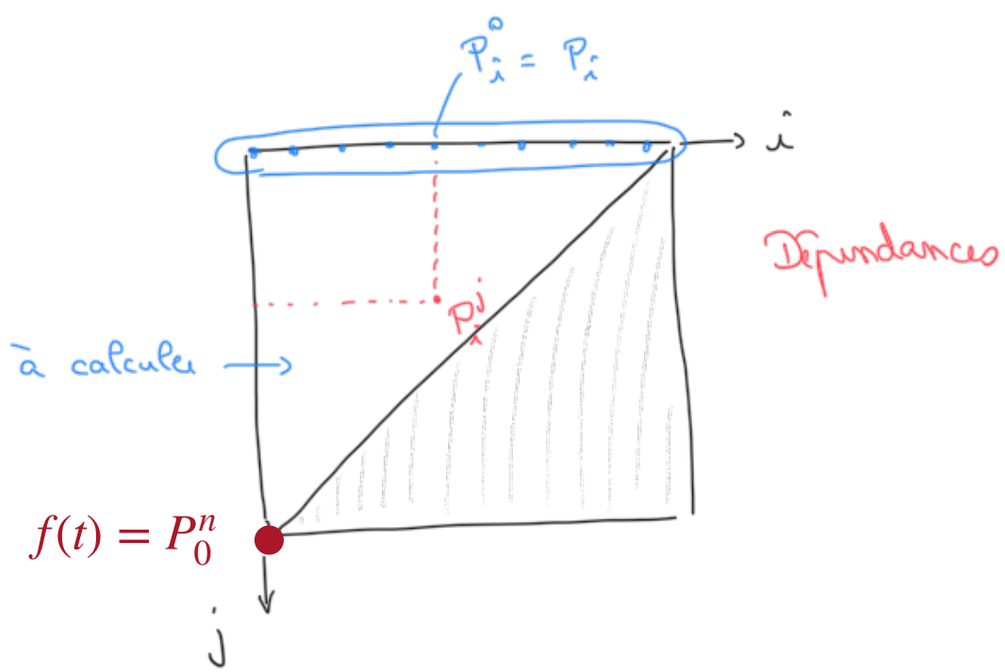
2) Dépendances de calcul



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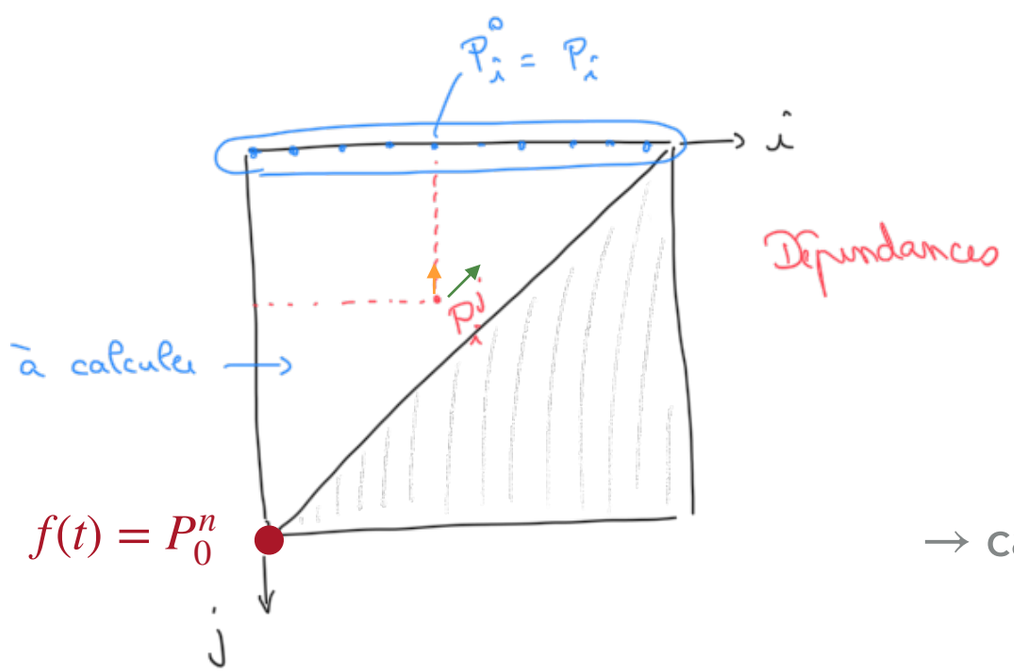


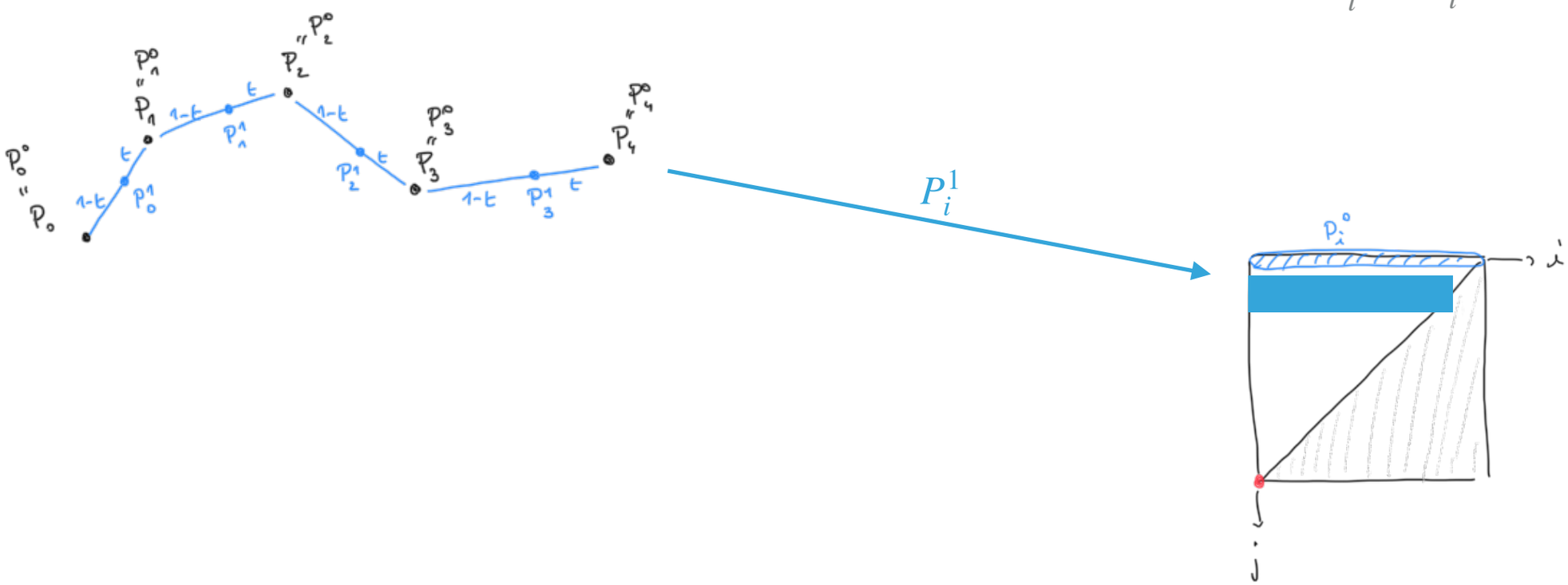
→ calcul par lignes

CALCUL DE POINT : ALGORITHME DE DE CASTELJAU

$$\boxed{P_i^j = tP_{i+1}^{j-1} + (1-t)P_i^{j-1}}$$

avec $P_i^0 = P_i$

Interprétation géométrique



CALCUL DE POINT : ALGORITHME DE DE CASTELJAU

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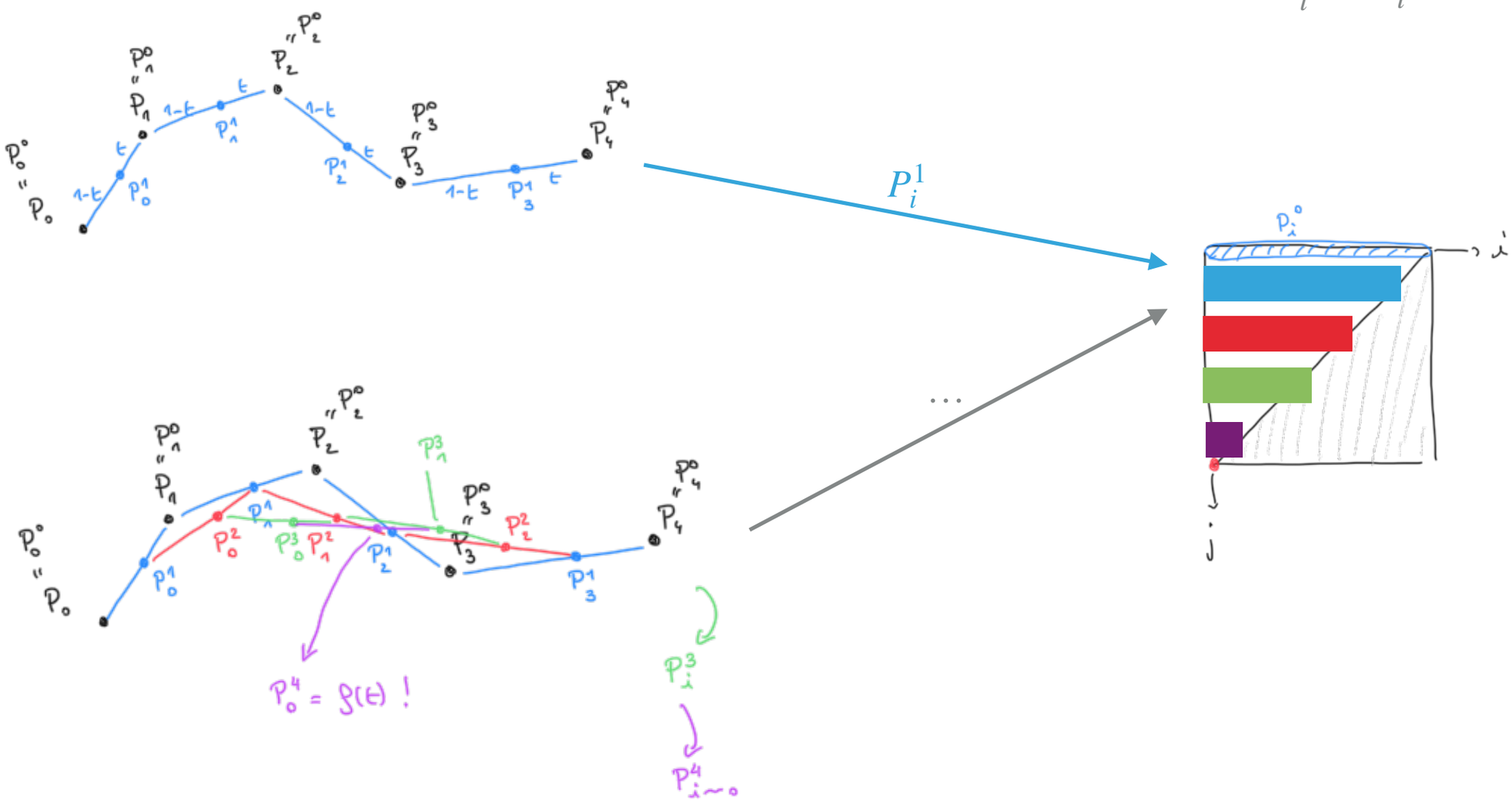
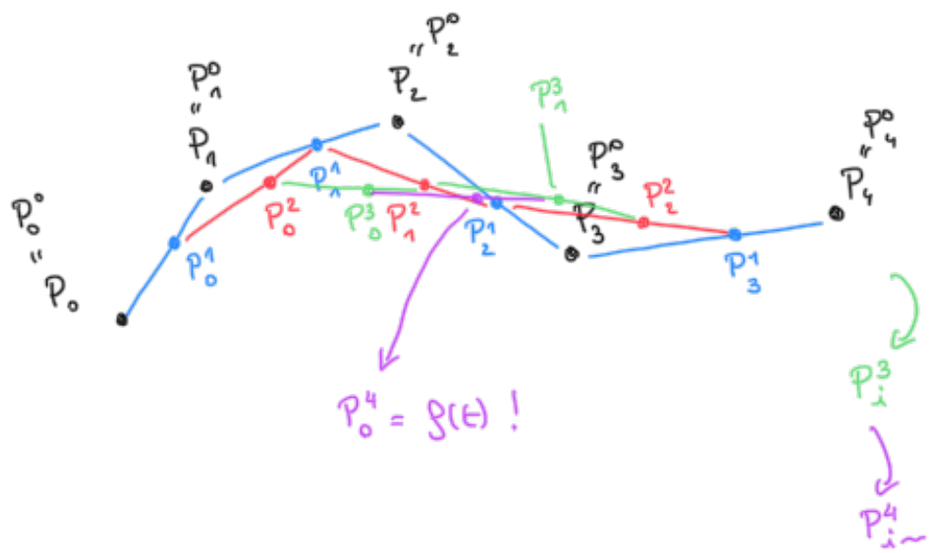


DE CASTELJAU ET SUBDIVISION

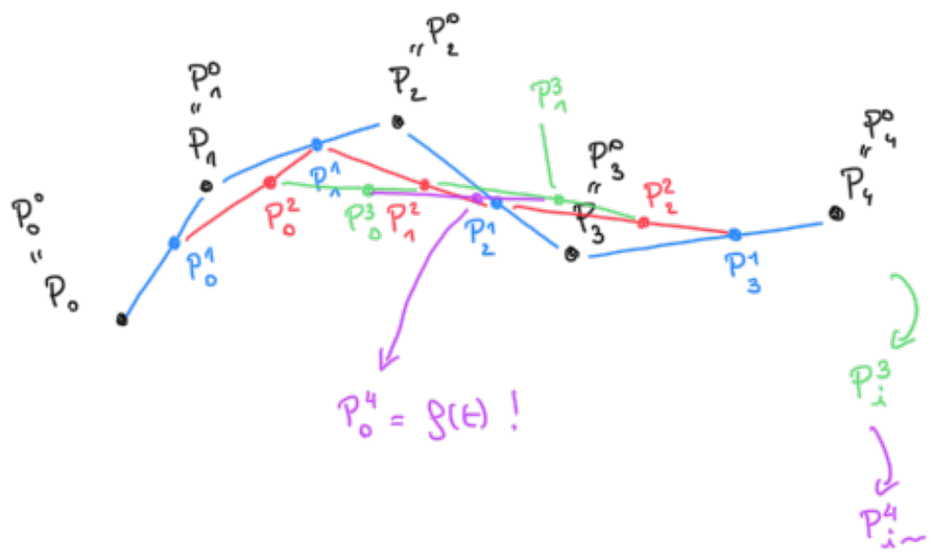


$$
\begin{array}{ccccc}
P_0^0 & P_1^0 & P_2^0 & P_3^0 & P_4^0 \\
P_0^1 & P_1^1 & P_2^1 & P_3^1 & \\
P_0^2 & P_1^2 & P_2^2 & & \\
P_0^3 & P_1^3 & & & \\
P_0^4 & & & &
\end{array}
$$

Créent la courbe $f|_{[0,t]}$

Créent la courbe $f|_{[t,1]}$

DE CASTELJAU ET SUBDIVISION



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P_0^2 & P_1^2 & P_2^2 & & \\
P_0^3 & P_1^3 & & & \\
P_0^4 & & & &
\end{array}
$$

Créent la courbe $f|_{[0,t]}$

Créent la courbe $f|_{[t,1]}$

→ Augmenter le nombre de points de contrôle

→ Subdivision de la courbe en deux courbes de même degré

ÉLÉVATION DE DEGRÉ

$$f(t) = \sum_{i=0}^{n} \varphi_{n,i}(t) \cdot P_i$$

$$\varphi_{n,i}(t) = t\varphi_{n,i}(t) + (1-t)\varphi_{n,i}(t)$$

Evident ...

$$f(t) = \sum_{i=0}^{n+1} \varphi_{n+1,i}(t) \cdot P'_i$$

$$\begin{cases} P'_0 = P_0 \\ P'_{n+1} = P_n \\ P'_i = \dfrac{i}{n+1} P_{i-1} + \dfrac{n+1-i}{n+1} P_i \end{cases}$$

ÉLÉVATION DE DEGRÉ

$$f(t) = \sum_{i=0}^{n} \varphi_{n,i}(t) \cdot P_i$$

$$\varphi_{n,i}(t) = t\varphi_{n,i}(t) + (1-t)\varphi_{n,i}(t)$$

Evident ...

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$$\begin{cases} P'_0 = P_0 \\ P'_{n+1} = P_n \\ P'_i = \dfrac{i}{n+1} P_{i-1} + \dfrac{n+1-i}{n+1} P_i \end{cases}$$

→ Augmenter le degré à $n+1$

→ Courbe inchangée

ÉLÉVATION DE DEGRÉ

$$f(t) = \sum_{i=0}^{n} \varphi_{n,i}(t) \cdot P_i$$

$$\varphi_{n,i}(t) = t\varphi_{n,i}(t) + (1-t)\varphi_{n,i}(t)$$

Evident ...

$$t\varphi_{n,i}(t) = \frac{i+1}{n+1} t\varphi_{n+1,i+1}(t)$$

$$(1-t)\varphi_{n,i}(t) = \frac{n+1-i}{n+1} t\varphi_{n+1,i}(t)$$

$$f(t) = \sum_{i=0}^{n+1} \varphi_{n+1,i}(t) \cdot P'_i$$

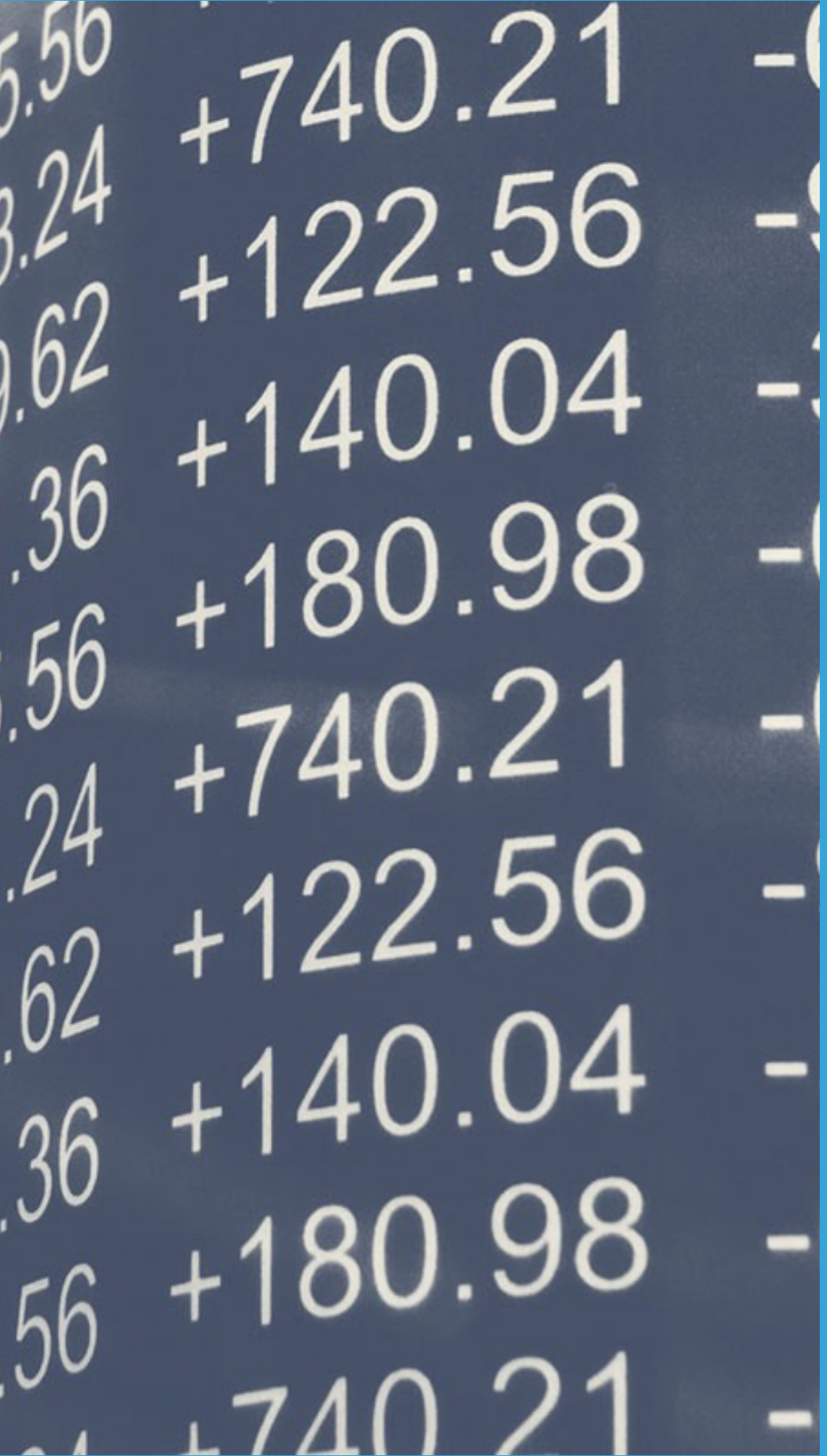
$$P'_0 = P_0$$

$$P'_{n+1} = P_n$$

$$P'_i = \frac{i}{n+1} P_{i-1} + \frac{n+1-i}{n+1} P_i$$

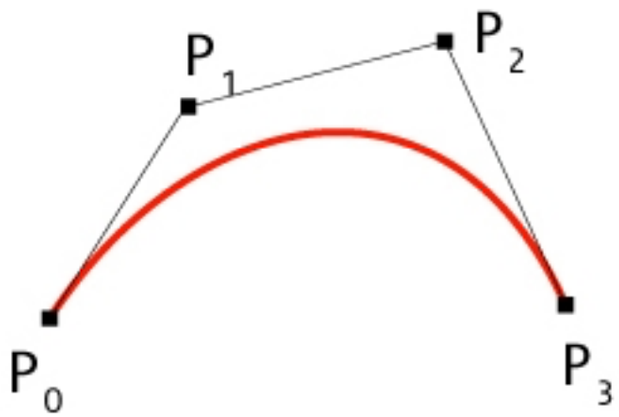
→ Augmenter le degré à $n+1$

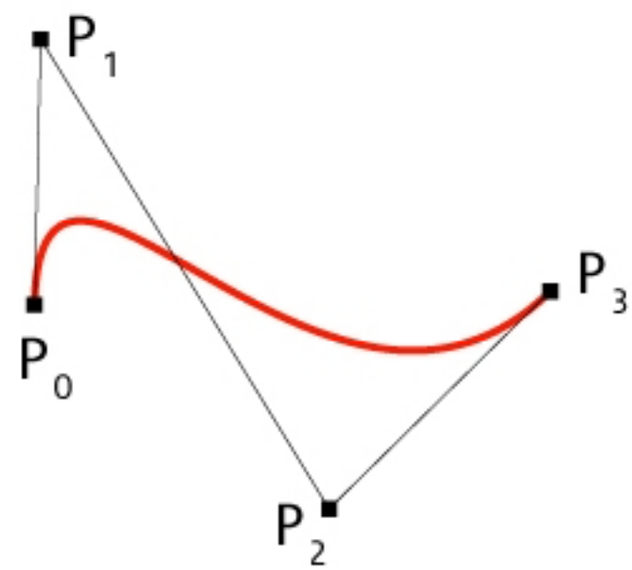
→ Courbe inchangée



CONCLUSION

CONCLUSION





- $f(t)$ s'approche de P_i en $t = \dfrac{i}{n}$
- Pas de contrôle plus précis
- Paramétrisation « cachée »

PLUS DE … ?

Base B-spline

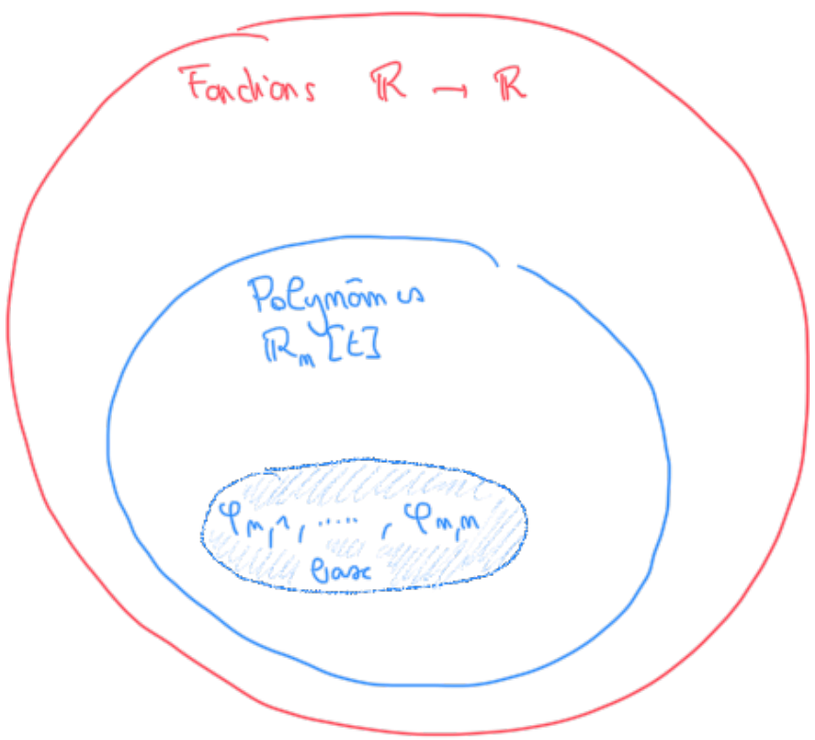
A suivre en 5A …

Sera aussi beaucoup plus technique (paramétrisation explicite)

CONCLUSION / RACCORDEMENT ?

Surfaces paramétriques (Béziers)

Base de Bernstein …



Courbes (Béziers)

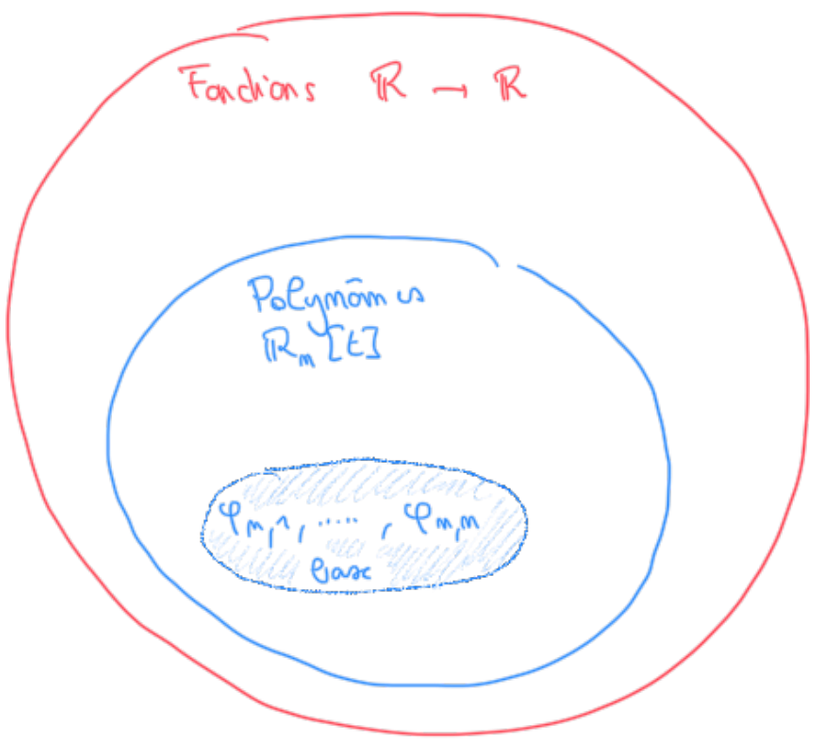
$$f(t) = \sum_{i=0}^{m} \varphi_{m,i}(t) \cdot P_i$$

Surfaces paramétriques

$f : \mathbb{R}^2 \to \mathbb{R}$

$f(u, v) =$

Base de Bernstein …



Base de fonctions
$f_i : \mathbb{R}^2 \to \mathbb{R}$

Courbes (Béziers)

$$f(t) = \sum_{i=0}^{m} \varphi_{m,i}(t) \cdot P_i$$

Surfaces paramétriques

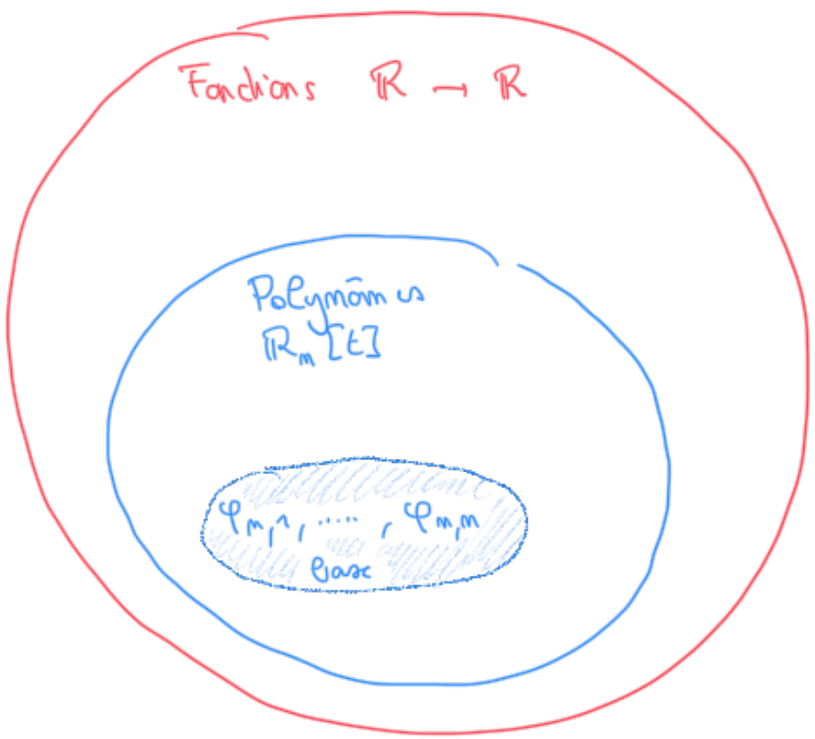
$$f : \mathbb{R}^2 \to \mathbb{R}^3$$

$$f(u,v) = \sum_i f_i(u,v) \cdot P_i$$

Base de Bernstein …

$$f, g \,:\, \mathbb{R} \to \mathbb{R} \quad \longrightarrow \quad f \otimes g \,:\, \mathbb{R}^2 \to \mathbb{R}$$

$$(f \otimes g)(u, v) = f(u)g(v)$$



Base de fonctions
$g_i : \mathbb{R}^2 \to \mathbb{R}$

Courbes (Béziers)

$$f(t) = \sum_{i=0}^{m} \varphi_{m,i}(t) \cdot P_i$$

Surfaces paramétriques

$$f \,:\, \mathbb{R}^2 \to \mathbb{R}$$

$$f(u, v) = \sum_i f_i(u, v) \cdot P_i$$

Base de Bernstein ...

$$f, g \,:\, \mathbb{R} \to \mathbb{R} \quad \longrightarrow \quad f \otimes g \,:\, \mathbb{R}^2 \to \mathbb{R}$$

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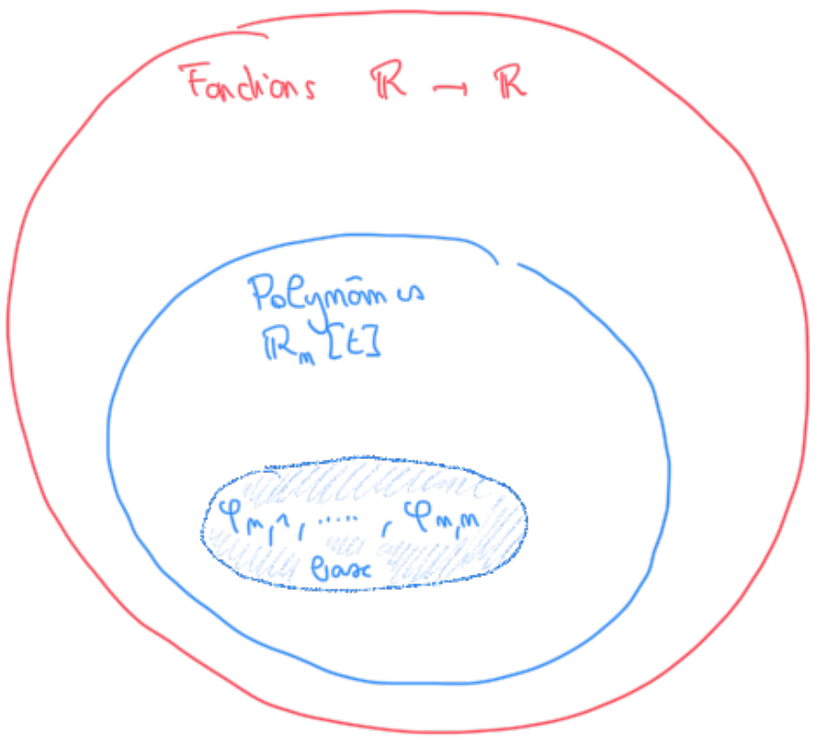
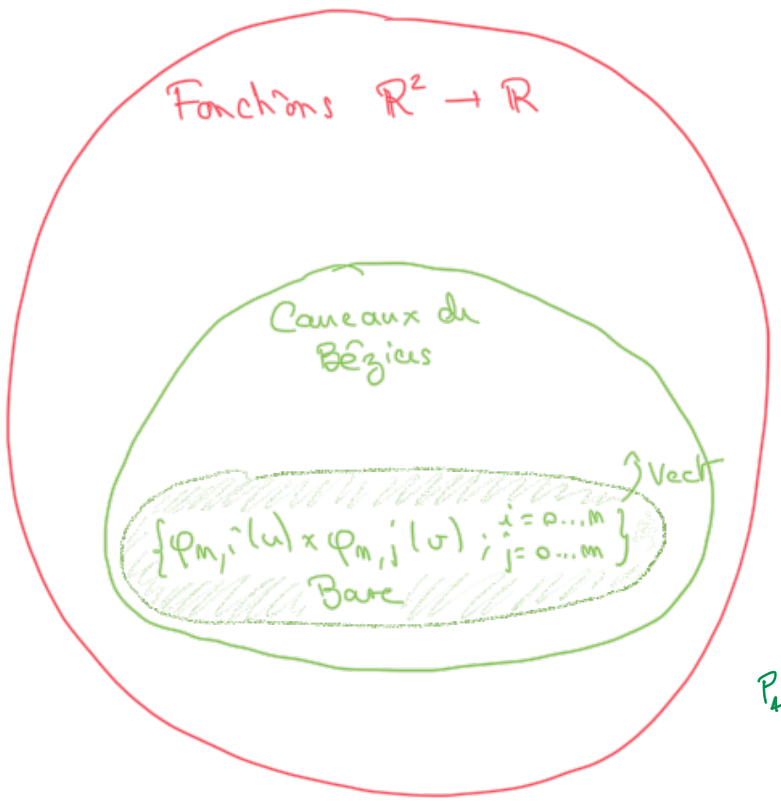
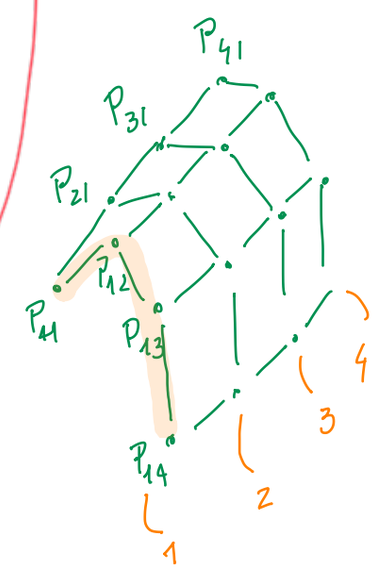




Courbes (Béziers)

$$f(t) = \sum_{i=0}^{m} \varphi_{m,i}(t) \cdot P_i$$



Surfaces paramétriques

$$f \,:\, \mathbb{R}^2 \to \mathbb{R}^3$$

$$f(u, v) = \sum_{i,j} \varphi_{n,i}(u)\varphi_{m,j}(v) \cdot P_{i,j}$$

2 params

2 indices pour control pts

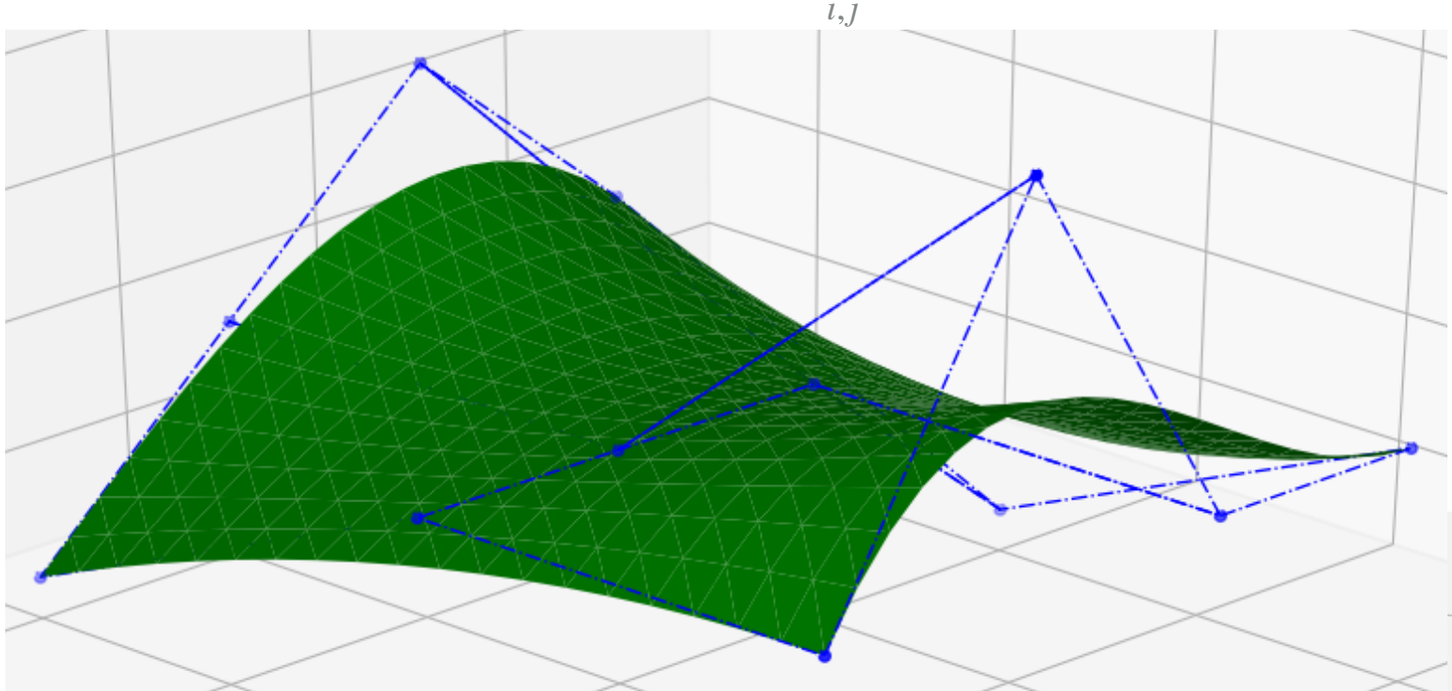
Param. surface

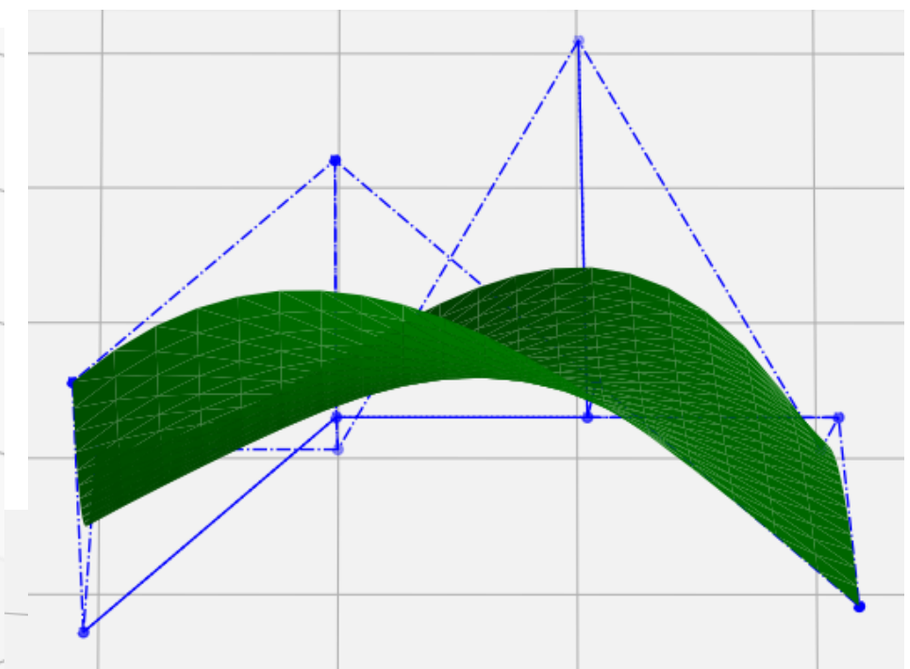
CARREAUX DE BÉZIERS

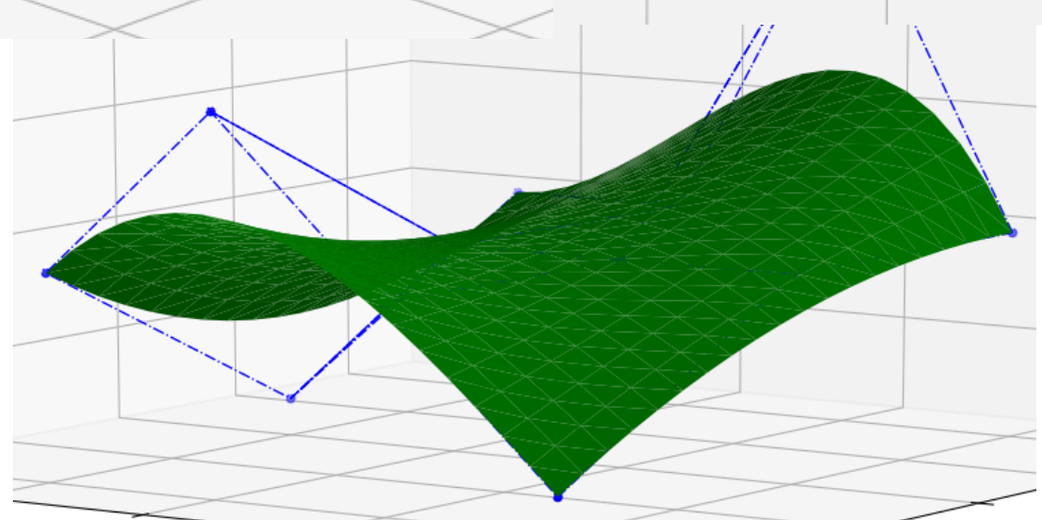
polygone de contrôle

Surface produit tensoriel - carreaux de Béziers

$$f(u,v) = \sum_{i,j} \varphi_{n,i}(u)\varphi_{m,j}(v) \cdot P_{i,j}$$







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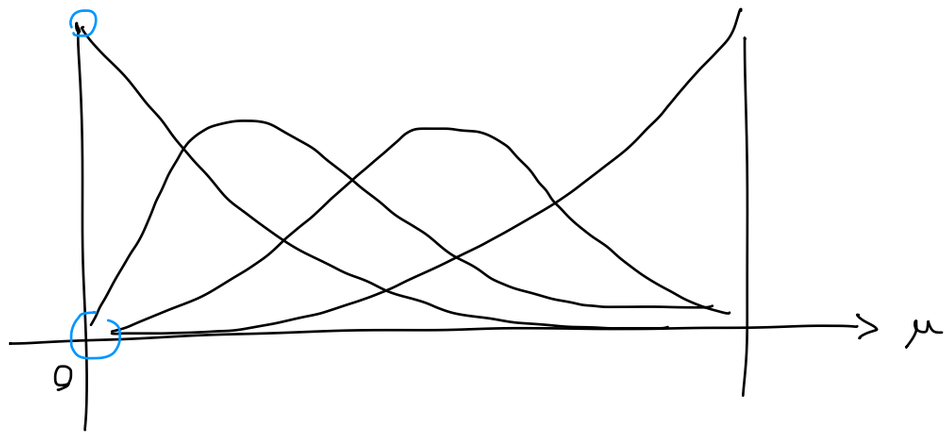
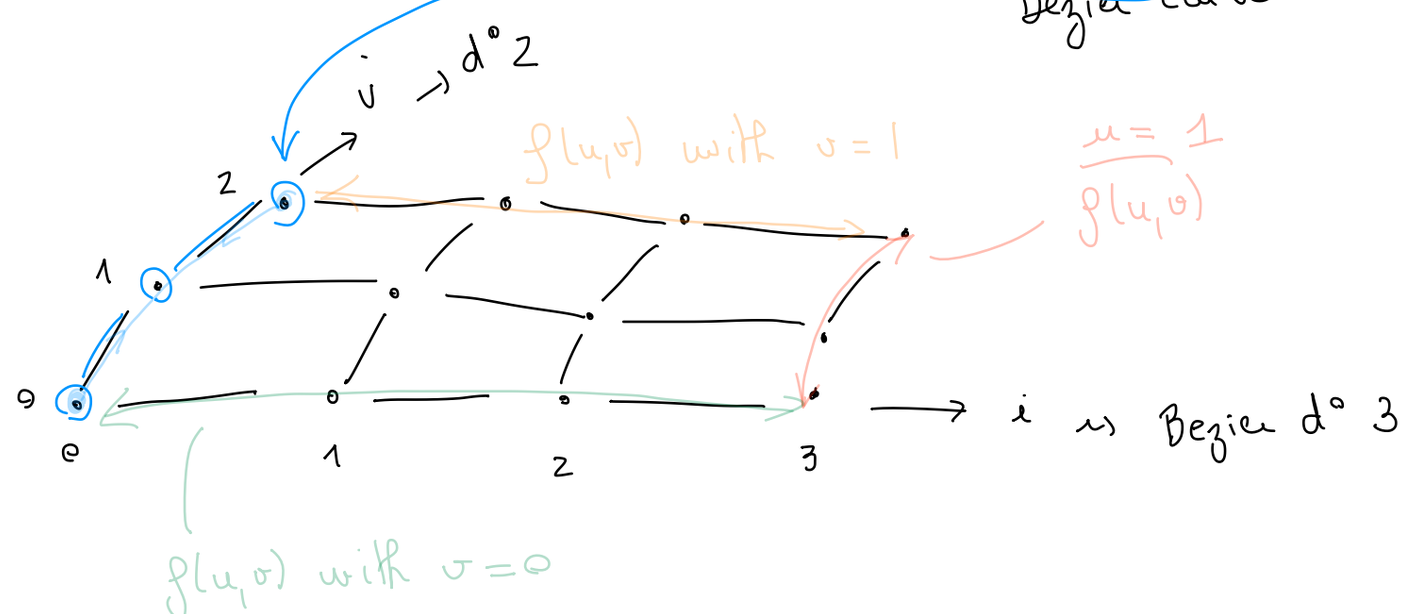


$\varphi_{..}(u)$

$$\varphi_{m,i}(u) = \begin{cases} 0 & \text{if } i \geq 1 \\ 1 & i = 0 \end{cases}$$

$$f(0,v) = \sum_j \underbrace{\cancel{\varphi_{m,0}(0)}}_{1} \varphi_{m,j}(v)\, P_{0,j} = \underbrace{\sum_j \varphi_{m,j}\, P_{0,j}}_{\text{Bézier curve}}$$

$f(u,v)$

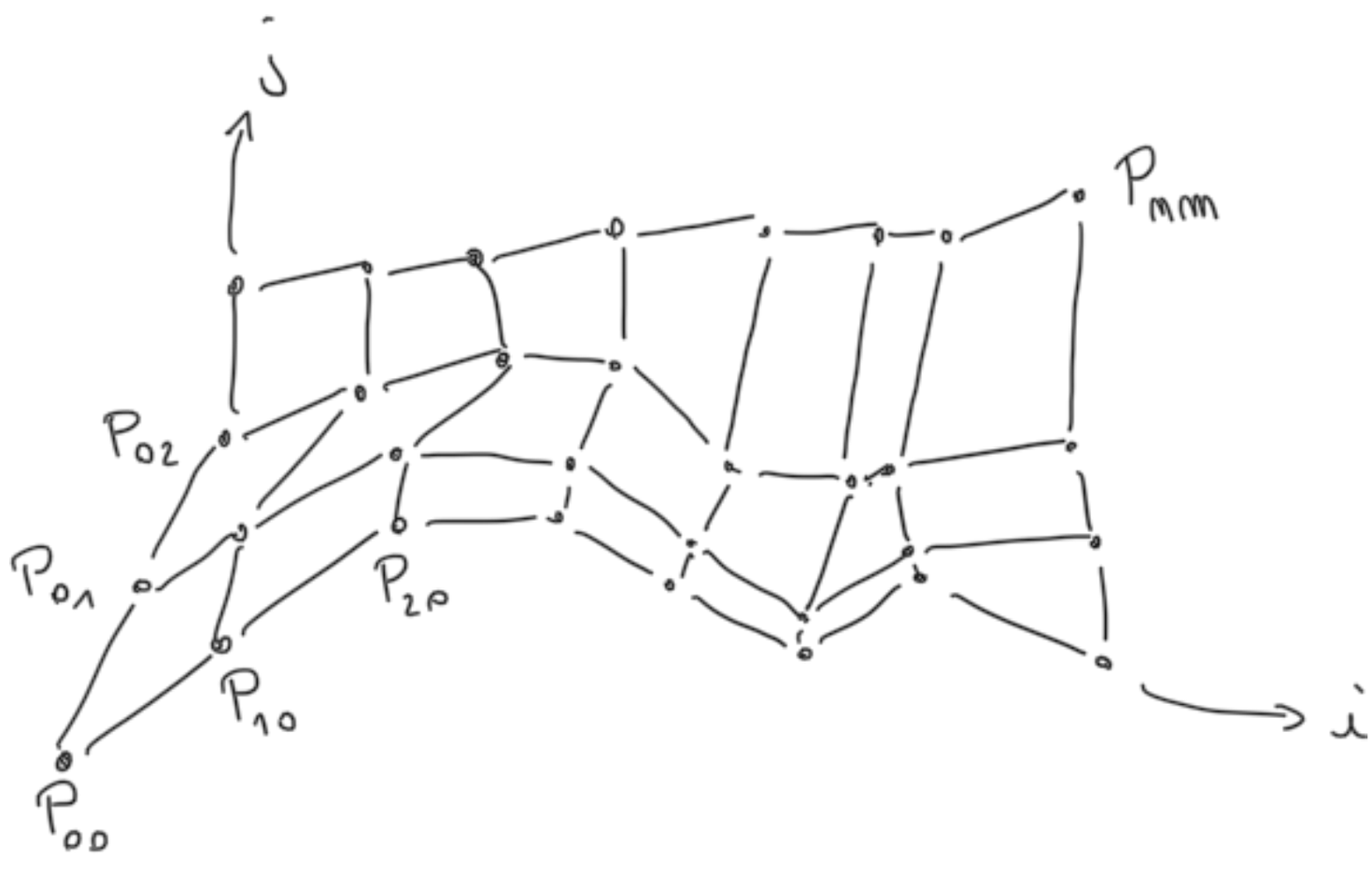


CARREAUX DE BÉZIERS

polygone de contrôle

Surface produit tensoriel - carreaux de Béziers

$$f(u,v) = \sum_{i,j} \varphi_{n,i}(u)\varphi_{m,j}(v) \cdot P_{i,j}$$

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conda install -c orbingol geomdl
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CARREAUX DE BÉZIERS

Surface produit tensoriel - carreaux de Béziers

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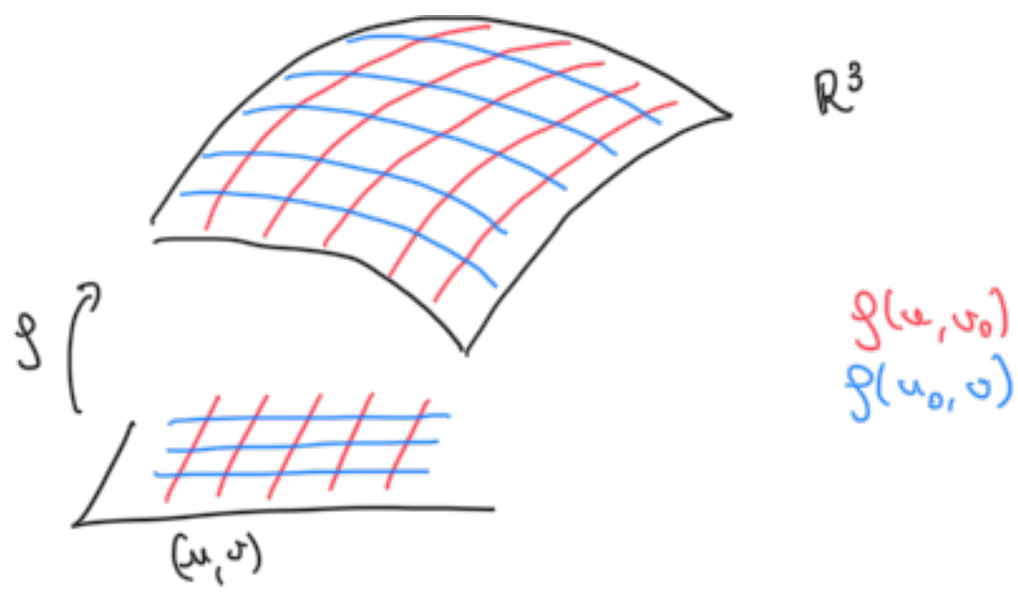
Courbes isoparamétriques



CARREAUX DE BÉZIERS

polygone de contrôle

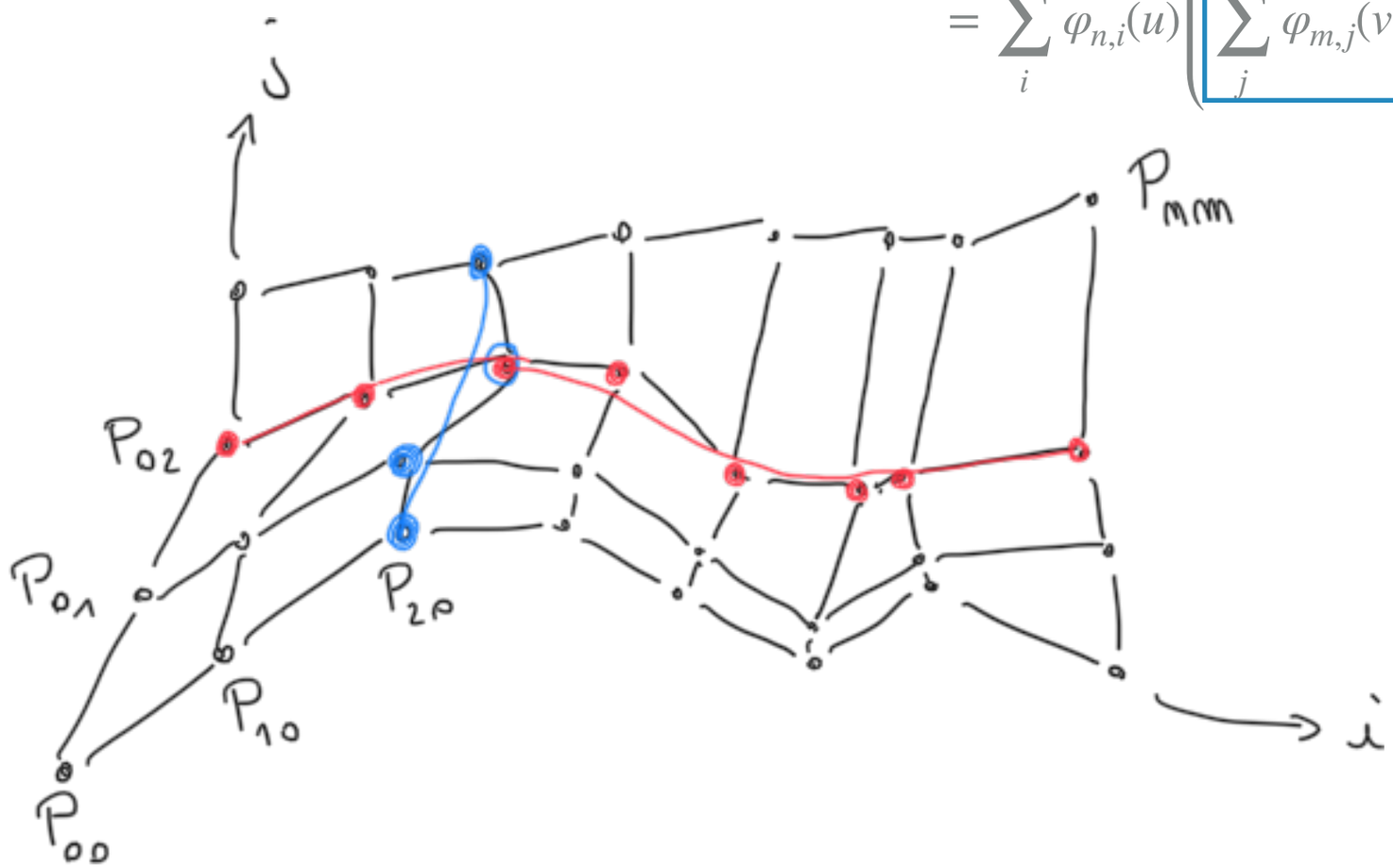
Surface produit tensoriel - carreaux de Béziers

$$f(u,v) = \sum_{i,j} \varphi_{n,i}(u)\varphi_{m,j}(v) \cdot P_{i,j}$$

$$= \sum_j \varphi_{m,j}(v)\left(\boxed{\sum_i \varphi_{n,i}(u) \cdot P_{i,j}}\right)$$

$$= \sum_i \varphi_{n,i}(u)\left(\boxed{\sum_j \varphi_{m,j}(v) \cdot P_{i,j}}\right)$$

Courbes de Bézier



CARREAUX DE BÉZIERS

polygone de contrôle

Surface produit tensoriel - carreaux de Béziers

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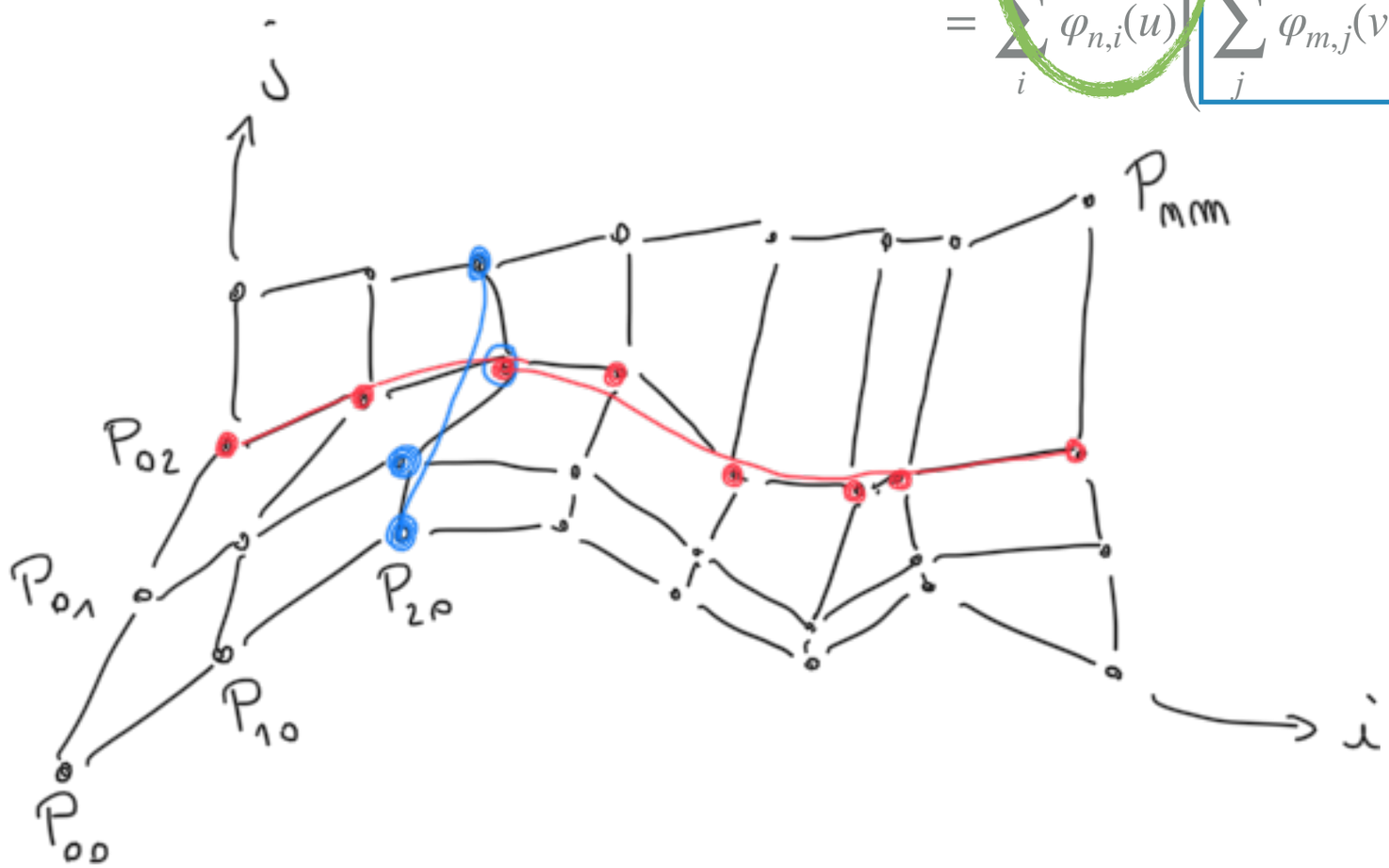
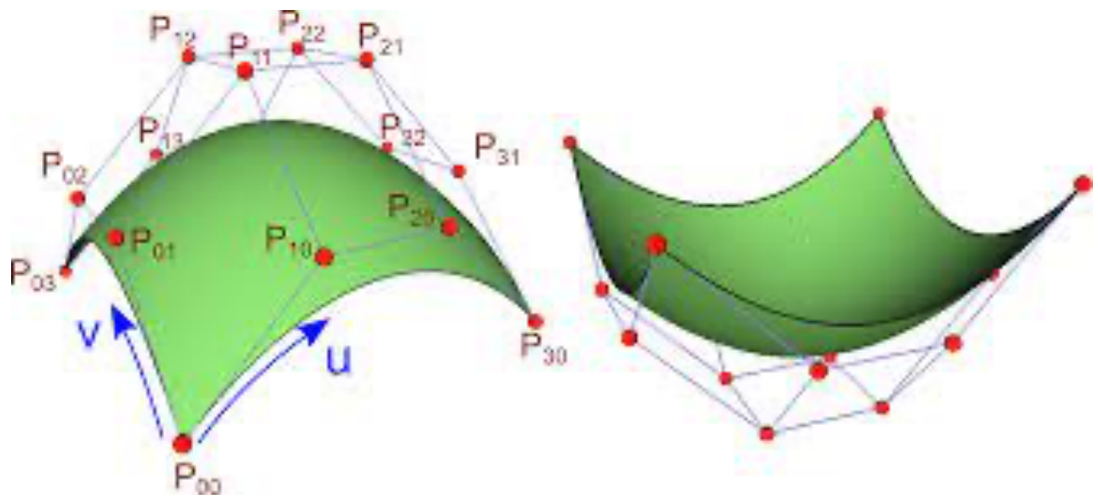
Courbes de Bézier

Pas sur la surface !

PAS COURBES ISOPARA-MÉTRIQUES



CARREAUX DE BÉZIERS / RACCORDEMENT