TP4

$\langle u, v \rangle = u^t \times v$

$\|u\|^2 = \langle u, u \rangle = u^t \times u$

Ex. 4

Puiss-it   teor 1 :

$$A = \begin{pmatrix} \ddots & & 0 \\ & d_i & \cdots \ i \\ 0 & & \ddots \end{pmatrix}$$

D — vect aléat

⊕ grande val → puiss-it → d_i, e_i

teor 2 ⇝ A diagonalisable (mon diag ...)

D : diag aléatoire

$A = PDP^{-1}$ avec P inversible → aléatoire ...

B (rect) ⇝ $A = B^t \times B$ — sym (positive) — diag en base o.n.

SVD

$B = U \Sigma V^t$

diag, orthogonale

$A = \cancel{P} \cdot D \cdot \cancel{P}^{-1}$, $P \to V$, $P^{-1} \to V^t$

$$D = \begin{pmatrix} d_1 & & \\ & \ddots & \\ & & d_d \end{pmatrix} \quad (d_i \geq 0)$$

(u_i, v_i) / σ_i

$$\begin{cases} B \times v_i = \sigma_i \cdot u_i \\ \text{et} \\ B^t \times u_i = \sigma_i \cdot v_i \end{cases}$$

$\sigma_i = \sqrt{d_i}$

$u_i = \dfrac{B v_i}{\sigma_i}$

vect. propres de A / d_i

$\vec{u}_i$

$A \times \vec{u}_i = d_i \, u_i$

k plus grandes val. sing: SVDs

σ_1, u_1, v_1 (SVD 1)

$B = U \Sigma V^t$

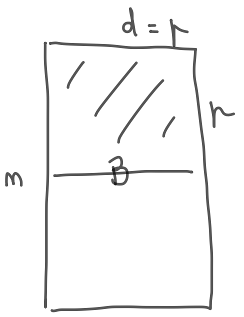
$$\Sigma = \begin{pmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_m \\ & 0 & \end{pmatrix}$$

↓ réécrire

$$\underbrace{B = \sum_{i=1}^{\cancel{m}\ k} \sigma_i u_i v_i^t}_{\text{rang } k}$$

$$B = \sigma_1 u_1 v_1^t + \sum_{i=2}^{m} \sigma_i u_i v_i^t$$

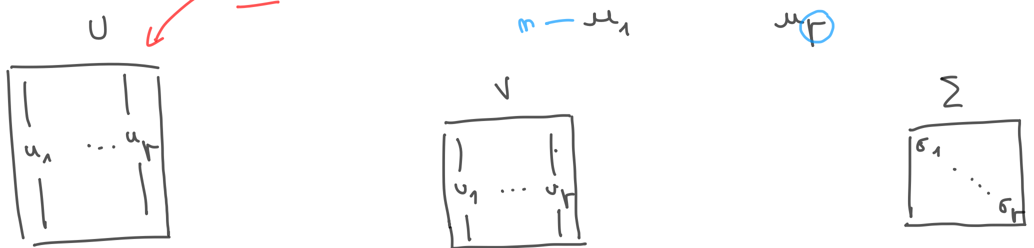
$$B - \sigma_1 u_1 v_1^t = \underbrace{\sum_{i=2}^{m} \sigma_i u_i v_i^t}_{\text{recommencer} \dots}$$



$p = \min(d, m) = d$

$$\rightsquigarrow \quad \begin{array}{ccc} \sigma_1 & \cdots\cdots & \sigma_p \\ \downarrow & & \downarrow \\ d - v_1 & & v_p \\ | & & | \\ m - u_1 & & u_r \end{array}$$

rect

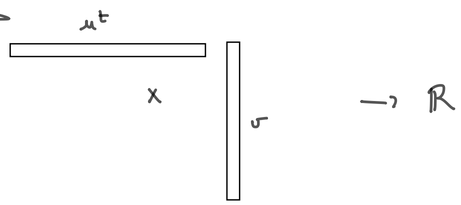


d vects de dim m = r

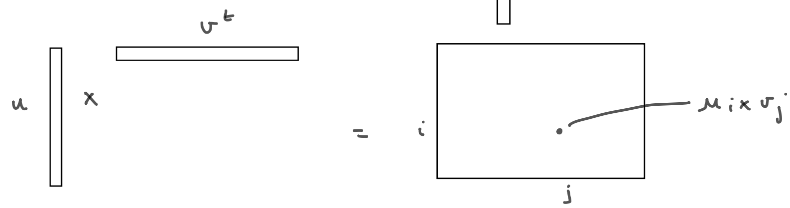
d vects de dim d = p

SVDs ⟶ décomp. économique

u, v vecteurs ⟶ $u^t \times v = u \cdot v$

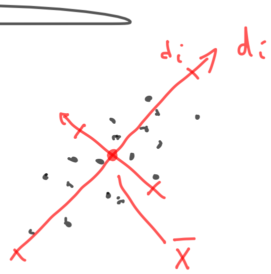


⟶ $u \times v^t$



$$B = U \Sigma V^t = \sum_{i=1}^{d} \sigma_i \underbrace{u_i v_i^t}_{\text{matrice}}$$

PCA $\quad X_i \quad i : 1 \to N$



Vision "stats"	Vision SVD ← ⊕ efficace
Inertie ↓ matrice de corrélation	$Z = \frac{A^t A}{N}$
$Z = \frac{1}{N} \sum_{i=1}^{N} \underbrace{(X_i - \bar{X})}_{A^t} (X_i - \bar{X})^t$	$A = \begin{pmatrix} — (X_i - \bar{X})^t — \\ \vdots \end{pmatrix}$
→ vects propres ⇝ dir. princip. d_i	— vects sing. à droite de A
→ vals propres ⇝ inertie $\sqrt{d_i}$	— σ_i

val. propres de $A^t A$ ⟷ val. sing de A

vects propres .... v_i

$$A v_i = \sigma_i u_i$$

Algo Entrée : X_i

Sortie : directions principales d_i ; inerties α_i

$$\bar{X} = \frac{1}{N} \sum_{i=1}^{N} X_i$$

$$A = \begin{pmatrix} \vdots \\ -(X_i - \bar{X})^t - \\ \vdots \end{pmatrix}$$

$[U, \Sigma, V] \leftarrow$ svd(A, 'econ')

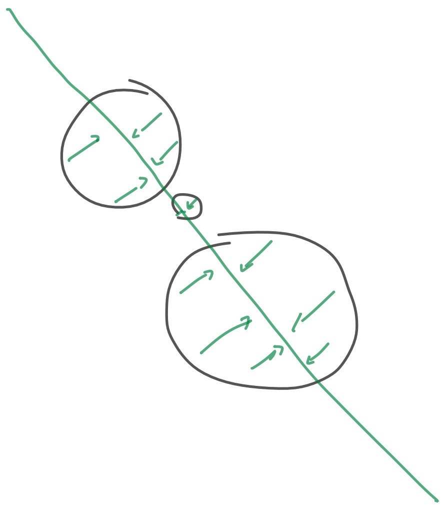
$d_i \leftarrow v_i$

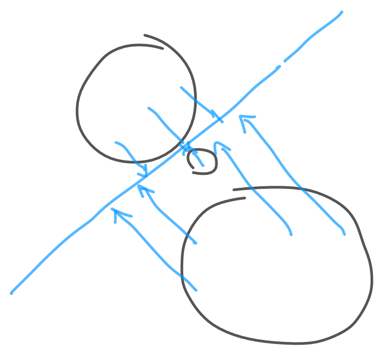
$\alpha_i \leftarrow \sigma_i$

X_i — dim d ~ 17

$\downarrow$

$\mathbb{R}^2$ / $\mathbb{R}^3$





σ_i ↘

|

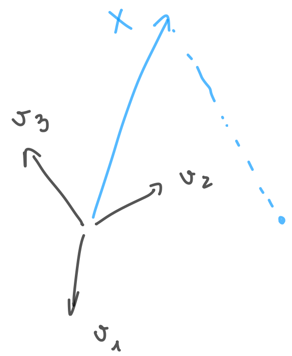
v_i

SVD → dir principales

Projeter en dim 2

|

plan : $(\bar{X}, v_1, v_2) \xrightarrow{\text{vectoriel}} X_i - \bar{X}$ sur plan (v_1, v_2)



$$X = \sum_{j=1}^{N} x_j \cdot v_j \; ?$$

ICI

$x_i = \langle x, v_i \rangle$

car base orthonormale !

$$\langle x, v_i \rangle = \langle \sum_j x_j v_j , v_i \rangle$$

$$\langle v_i, v_j \rangle = \begin{cases} 0 & \text{si } i \neq j \\ 1 & \text{si } i = j \end{cases}$$