

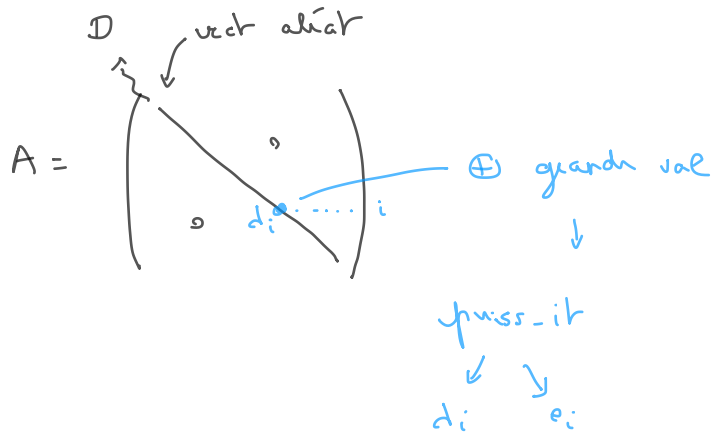
$$\langle u, v \rangle = u^t \cdot v$$

$$\|u\|^2 = \langle u, u \rangle = u^t \cdot u$$

Ex. 4

Puisse-it

test 1 :



test 2 \leadsto A diagonalisable (non diag ...)

D : diag aliatoin

$$A = P D P^{-1} \text{ avec } P \text{ inversible}$$

↓
aliatoin ...

B (rect)
SVD |

$$B = U \Sigma V^t$$

$(u_i, v_i) / \sigma_i$

$$\begin{cases} B \cdot v_i = \sigma_i \cdot u_i \\ \text{et} \\ B^t \cdot u_i = \sigma_i \cdot v_i \end{cases}$$

$$\sigma_i = \sqrt{d_i}$$

\leadsto

$$A = B^t \cdot B$$

sym (positive)
diag en base o.n.

$$A = \underbrace{R}_{\text{diag}} \cdot \underbrace{D}_{\text{orthogonale}} \cdot \underbrace{R^{-1}}_{V^t}$$

$$D = \begin{pmatrix} d_1 & & \\ & \dots & \\ & & d_d \end{pmatrix} \quad (d_i \geq 0)$$

$$u_i = \frac{B v_i}{\sigma_i}$$

vect. propres de A / d_i
 \vec{u}_i

$$A \cdot \vec{u}_i = d_i \cdot u_i$$

\mathbb{R} plus grandes val. sing:

SVDs

σ_1, u_1, v_1 (SVD 1)

$$B = U \Sigma V^t$$

$\begin{pmatrix} \sigma_1 & & \\ & \dots & \\ & & \sigma_m \\ & & & 0 \end{pmatrix}$

↓
lire à l'envers

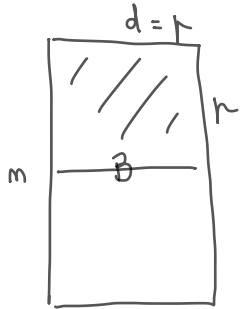
$$B = \sum_{i=1}^{\text{rang } B} \sigma_i u_i v_i^t$$

rang B

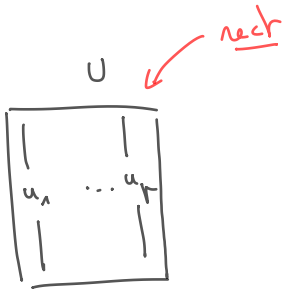
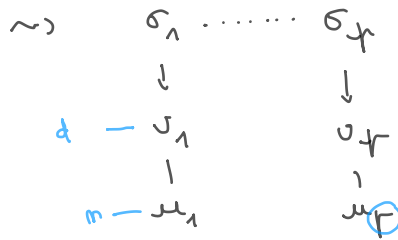
$$B = \sigma_1 u_1 v_1^t + \sum_{i=2}^m \sigma_i u_i v_i^t$$

$$B - \sigma_1 u_1 v_1^t = \sum_{i=2}^m \sigma_i u_i v_i^t$$

recommencer ...



$r = \min(d, m) = d$



d vects de dim m



d vects de dim d



SVDs → décomp. économique



u, v vecteurs

$u^t \times v = u \cdot v$

$$\frac{u^t}{v} \times \frac{v}{v} = \dots$$

$u \times v^t$

