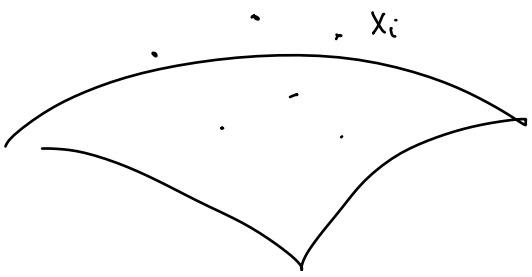


$\rightsquigarrow$ $f(w) = \sum_{i=1}^{N} \left(a_0 x_i^2 + a_1 x_i y_i \cdots - z_i \right)^2$

lin.

$w = \begin{pmatrix} a_0 \\ \vdots \\ a_5 \end{pmatrix}$

$z = \text{poly d° 2}(x,y)$

$a_0 x^2 + a_1 x y + a_2 y^2 + a_3 x + a_4 y + a_5$

Ex 1

$z = a_1 x + a_2 y + a_3$

$w = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$

$X = \begin{pmatrix} \vdots & \vdots & \vdots \\ x_i & y_i & 1 \\ \vdots & \vdots & \vdots \end{pmatrix}$ $\quad b = \begin{pmatrix} \vdots \\ z_i \\ \vdots \end{pmatrix}$

$f(w) = \| X w - b \|^2$

$X = \begin{pmatrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_i^2 & x_i y_i & y_i^2 & x_i & y_i & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$ $\quad b = \begin{pmatrix} \vdots \\ z_i \\ \vdots \end{pmatrix}$

QR pour X

$\tilde{X}_1$: d premières lignes de R (dxd)

$\tilde{y}_1$: $\quad Q^t \cdot y$

↳ résoudre $\tilde{X}_1 w = \tilde{y}_1$

Visu du plan

```
f = @(x,y) w(1)*x + w(2)*y + w(3);
zoom = 1;
[X,Y] = meshgrid(-zoom:0.1:zoom, -zoom:0.1:zoom);
Z = f(X,Y);
mesh(X,Y,Z);
```

(w(1) = a_1, w(2) = a_2, w(3) = a_3)

Ex.3

↳ matrice aléatoire de cond [c] ✓

$$\text{cond}_2(A) \overset{\text{def}}{=} \|A\| \cdot \|A^{-1}\| = \frac{\sigma_1}{\sigma_m}$$

→ Matrices de val. sing. contrôlées

Matrices pour lesquelles σ_i (val. sing évidents), d_i (val. propre)

↓

matrices diagonales

A diag $\begin{bmatrix} d_1 & & \\ & \ddots & \\ & & d_m \end{bmatrix}$ → val. propres : d_i

→ val. sing : $\sqrt{d_i^2} = |d_i|$

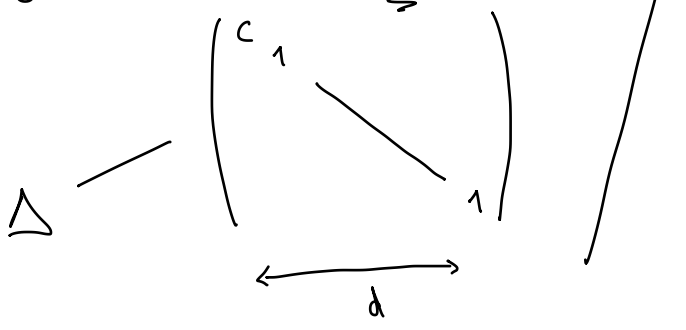
$$A^t A = A^2$$

Rappel SVD

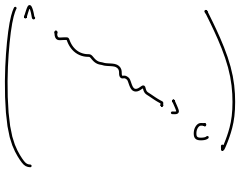
B quelconque

val. sing B — σ_i ↔ val. propres $B^t B$ — d_i ; $\sigma_i = \sqrt{d_i}$

Mat. diag de cond c : $\sigma_1/\sigma_m = c$



diag (linspace (1, c, d))



$U \Delta V^t$

U, V ortogonais aleatórios ?

↓

matriz B aleat.

↓ QR

Q

Meth. 1

A aleat

~~$A^t \cdot A$~~ sym

```
clear
clf
hold on

n = 50 ; M = rand(2,n) - 0.5 ;
alpha = 1 ; beta = 1 ;
epsilon = .01 ;
epsilon2 = .2 ;
f = @(X,Y) 2*X.^2 -  Y.^2 ;
M(1,:) = alpha * M(1,:) ; M(2,:) = beta * M(2,:) ;
M(3,:) = f(M(1,:), M(2,:)) ;
M(3,:) = M(3,:) + epsilon * 2* (rand(size(M(3,:))) - .5) ;
plot3(M(1,:), M(2,:), M(3,:), '.b')

XX = [M(1:2,:)',ones(n,1)] ;
yy = M(3,:)' ;

[Q,R] = qr(XX) ;

d = 3 ;

X1t = R(1:d,:) ;
y = Q'*yy ;
y1t = y(1:d) ;

w = X1t\y1t ;

ff = @(x,y) w(1)*x + w(2)*y +w(3) ;
[X,Y] = meshgrid(-alpha:.1:alpha, -beta:.1:beta) ;
Z = ff(X,Y) ;
mesh(X,Y,Z) ;
```