Two new measures for the homology groups of discrete objects

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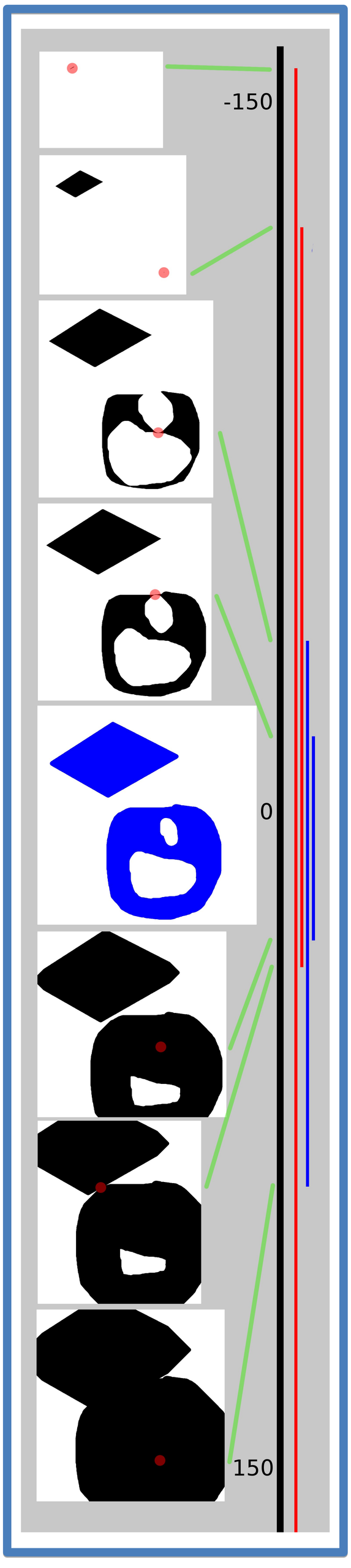
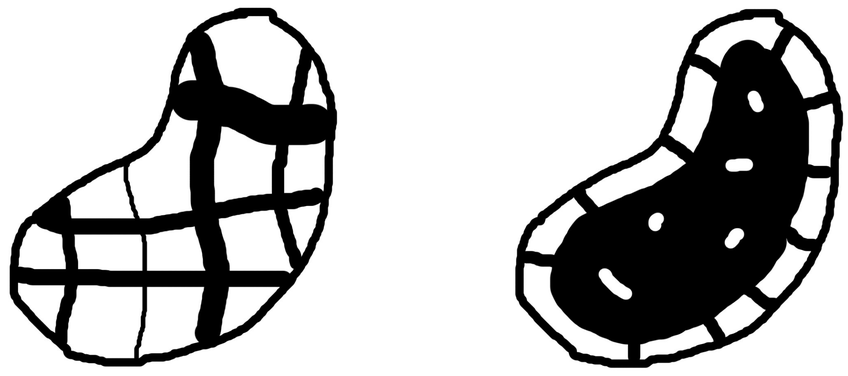
Abstract: Given a binary object (2D or 3D), its Betti numbers characterize the number of holes in each dimension. They are obtained algebraically, and even though they are perfectly defined, there is no unique way to display these holes. We propose two geometric measures for the holes, which are uniquely defined and try to compensate the loss of geometric information during the homology computation: the *thickness* and the *breadth*. They are obtained by filtering the information of the persistent homology computation of a filtration defined through the signed distance transform of the binary object.
Keywords: *binary object · distance transform · persistent homology · geometric information · holes*



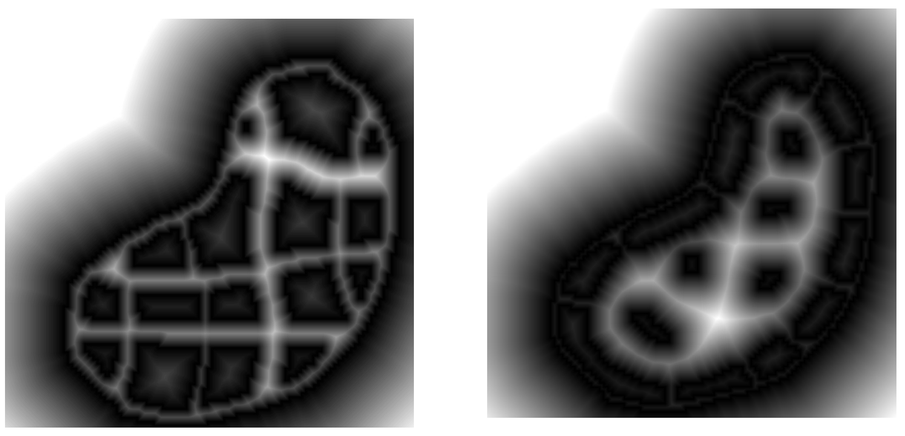
Motivation

The number of holes is not sufficient to understand binary objects. Their “size” is also important.

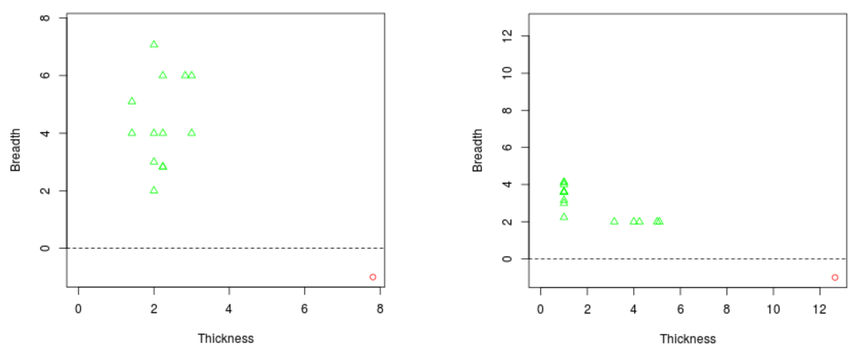


Pipeline

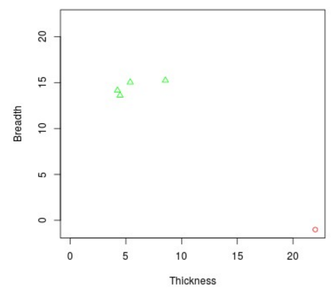
Signed distance transform



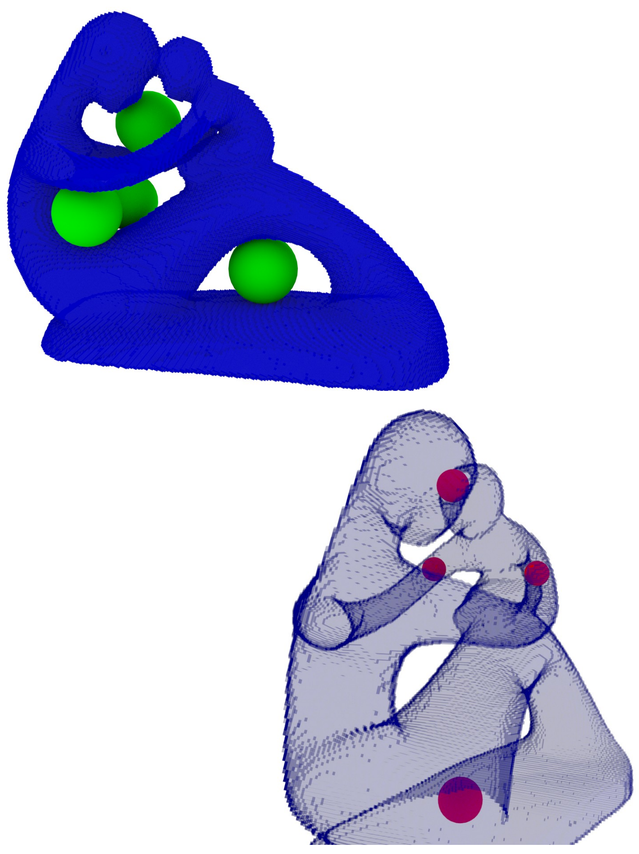
Persistent homology



Validation



The measures can be visualized as balls



CONCLUSION
- Definition of two measures for holes
- Alternative visualization of holes
- Any dimension, but only Z_2 homology
- Useful for classification or understanding

FUTURE WORK
- Proof of regularity
- Real world applications: medical context, geostatistics, planetology, etc.

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