Opening holes in Discrete Objects with Digital Homotopy

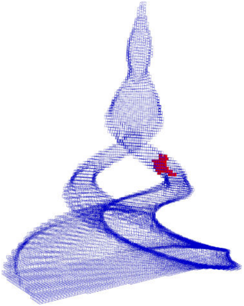
Aldo Gonzalez-Lorenzo   Alexandra Bac   Jean-Luc Mari

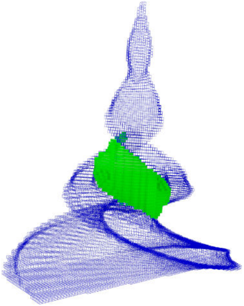
Aix-Marseille Université, CNRS, LSIS UMR 7296 (France)

20 september 2017

Structure

1 Opening and closing holes with homology





2 Closing holes in discrete objects

1. Opening and closing holes with homology
2. Closing holes in discrete objects

ELSEVIER

Pattern Recognition Letters 23 (2002) 523–531

Pattern Recognition Letters

www.elsevier.com/locate/patrec

A three-dimensional holes closing algorithm

Zouina Aktouf, Gilles Bertrand, Laurent Perroton *

Pattern Recognition 43 (2010) 3548–3559

Contents lists available at ScienceDirect

Pattern Recognition

journal homepage: www.elsevier.com/locate/pr

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Hole filling in 3D volumetric objects

Marcin Janaszewski [a,*], Michel Couprie [b], Laurent Babout [a]

Pattern Recognition Letters 32 (2011) 2231–2238

Contents lists available at ScienceDirect

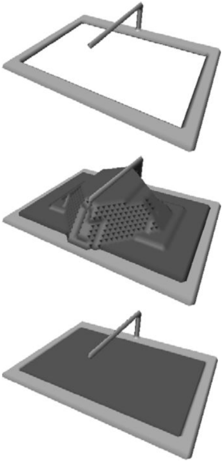
Pattern Recognition Letters

journal homepage: www.elsevier.com/locate/patrec

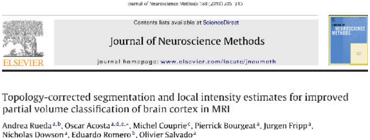
ELSEVIER

Robust algorithm for tunnel closing in 3D volumetric objects based on topological characteristics of points

Marcin Janaszewski [a,*], Michał Postolski [a,b], Laurent Babout [a]



A little bit of opening holes


Journal of Neuroscience Methods 188 (2010) 305–315

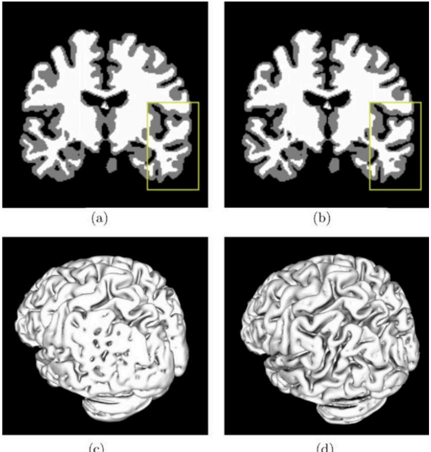
Contents lists available at ScienceDirect

Journal of Neuroscience Methods

journal homepage: www.elsevier.com/locate/jneumeth

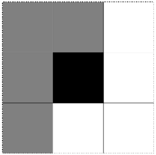
Topology-corrected segmentation and local intensity estimates for improved partial volume classification of brain cortex in MRI

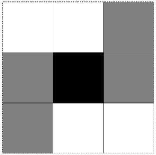
Andrea Rueda[a,b], Oscar Acosta[a,d,e,*], Michel Couprie[c], Pierrick Bourgeat[a], Jurgen Fripp[a], Nicholas Dowson[a], Eduardo Romero[b], Olivier Salvado[a]



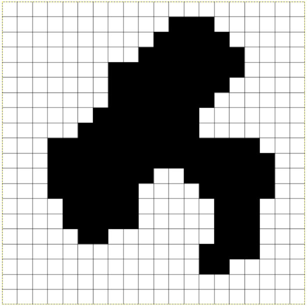
Structure

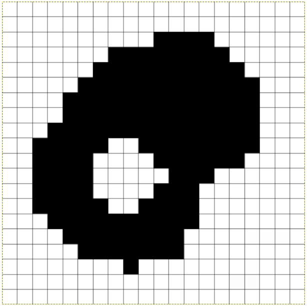
- Simple point





- Contractible object





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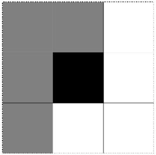

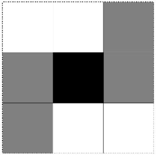

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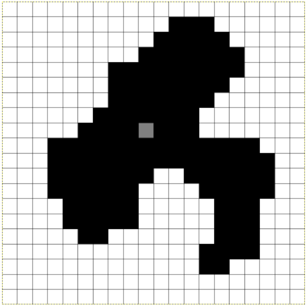
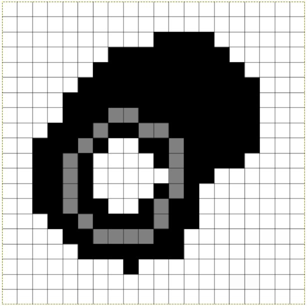
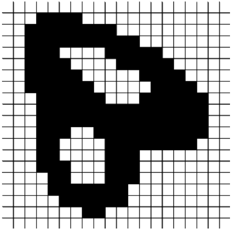
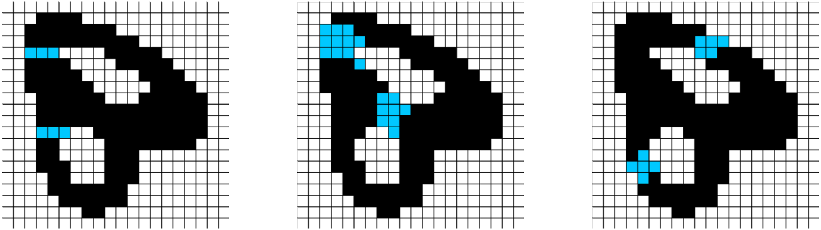




Homotopic opening

Let X be a discrete object.
$Y \subset X$ is a *homotopic opening* if $X - Y$ is contractible





Structure

KISS algorithm

Input: $X \subset \mathbb{Z}^d$
Output: Homotopic opening for X
$C \leftarrow \{x\}$, for $x \in X$;
$S \leftarrow N^*_\alpha(x) \cap (X - C)$;
while $S \neq \emptyset$ **do**
  $x \leftarrow$ some point in S;
  $S \leftarrow S - \{x\}$;
  if x *is simple for* C **then**
    $C \leftarrow C \cup \{x\}$;
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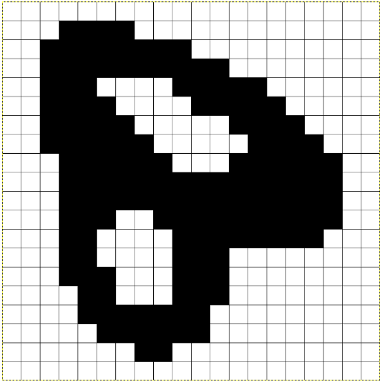
return $X - C$;



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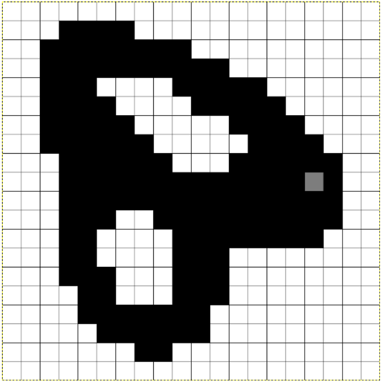

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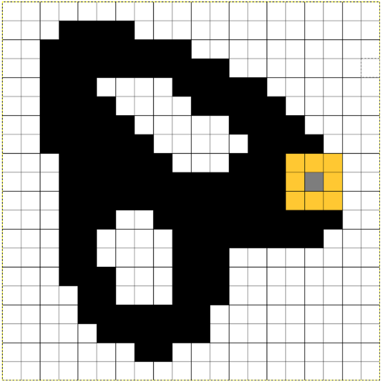

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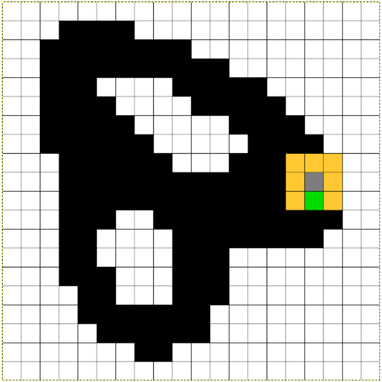

KISS algorithm

Input: $X \subset \mathbb{Z}^d$
Output: Homotopic opening for X
$C \leftarrow \{x\}$, for $x \in X$;
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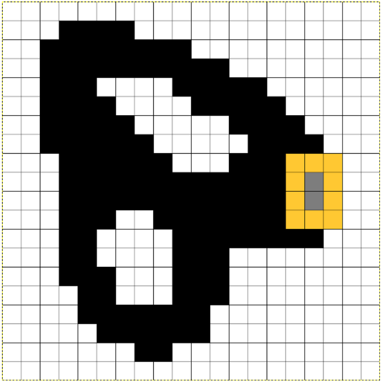

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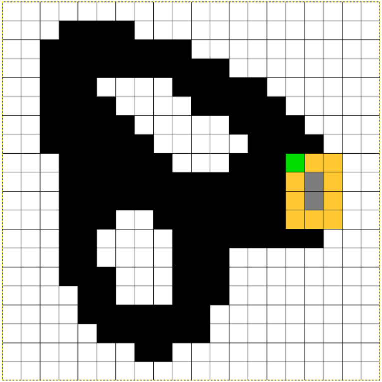

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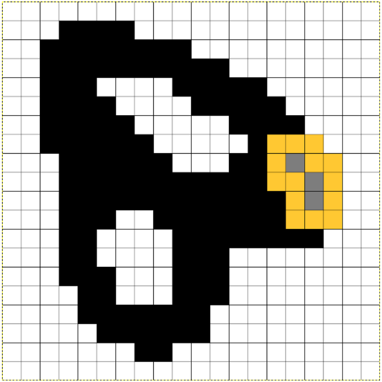

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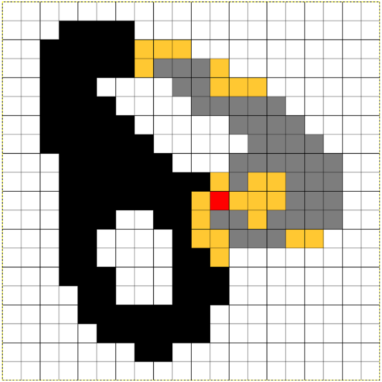

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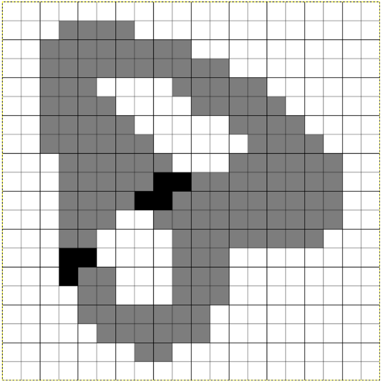

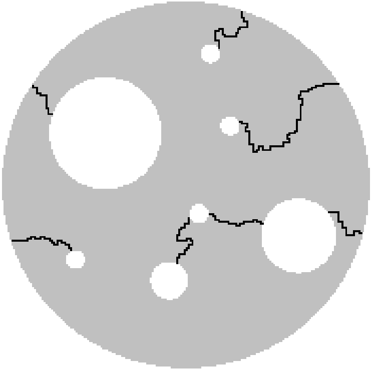

Better use the distance transform

Algorithm 1: random propagation

$C \leftarrow \{x\}$, for random $x \in X$ such that $dt_X(x)$ is maximal;
$S \leftarrow N^*_\alpha(x) \cap (X - C)$;
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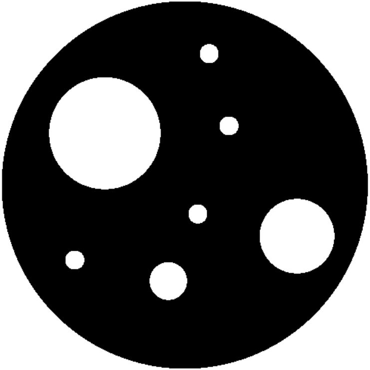

Complexity

dD: $3^d n(\lg n + f(d))$
3D: $n \lg n$

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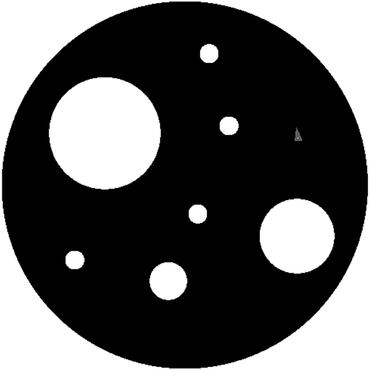

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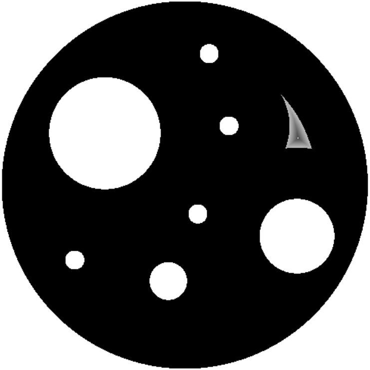

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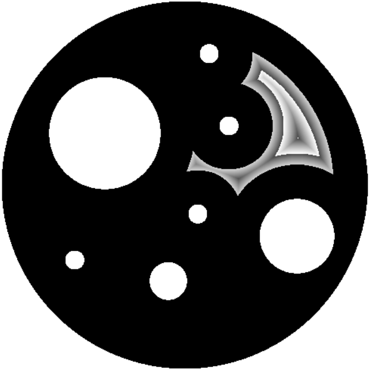

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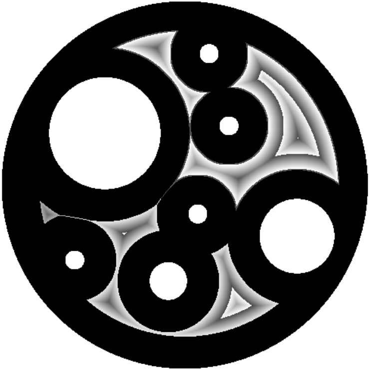

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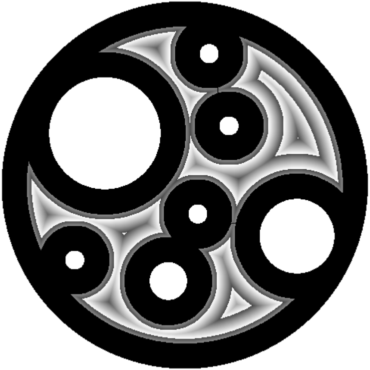

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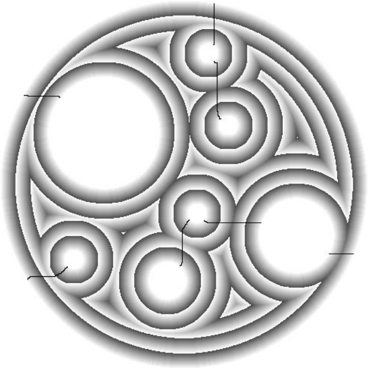

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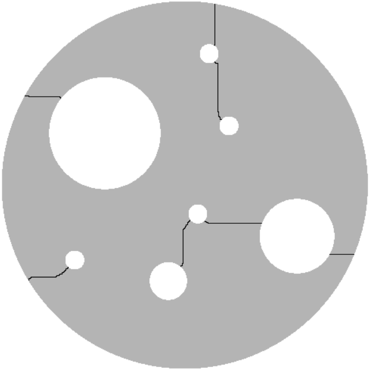

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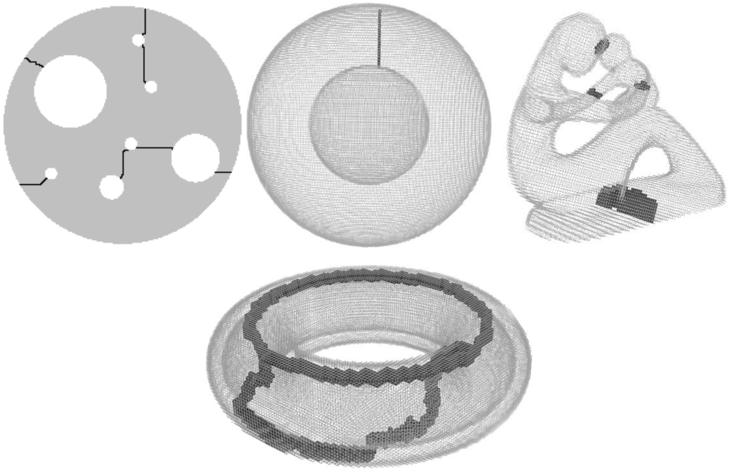
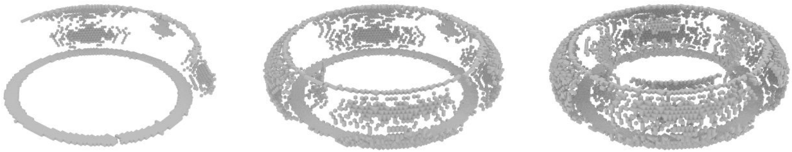


Problems:

- Not straight lines → later
- Torus → next

What happens to the torus?

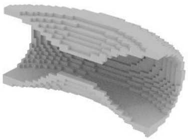


Too many points with equal distance

Solution: propagate by layers (Algorithm 2)

Algorithm 2: propagation by layers

$C \leftarrow \{x\}$, for random $x \in X$ with highest dt_X value;
repeat
  $m \leftarrow \max\{dt_X(x) \mid x \in N^*_\alpha(C) \cap X,$ x simple for $C\}$;
  $L \leftarrow dt_X^{-1}([m-1, m]) \cap N^*_\alpha(C)$;
  foreach $x \in L$ **do**
    if x *is simple for* C **then**
      $C \leftarrow C \cup \{x\}$;
until *idempotency*;
return $X - C$;



Complexity

dD: $3^d n^2 f(d)$
3D: n^2

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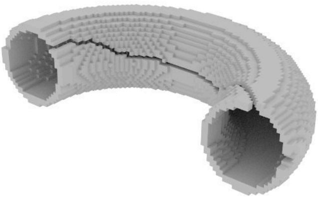
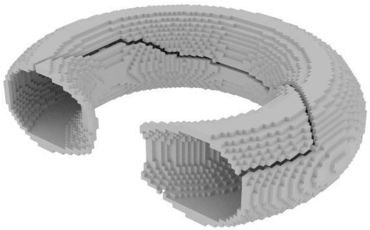


Complexity

dD: $3^d n^2 f(d)$
3D: n^2

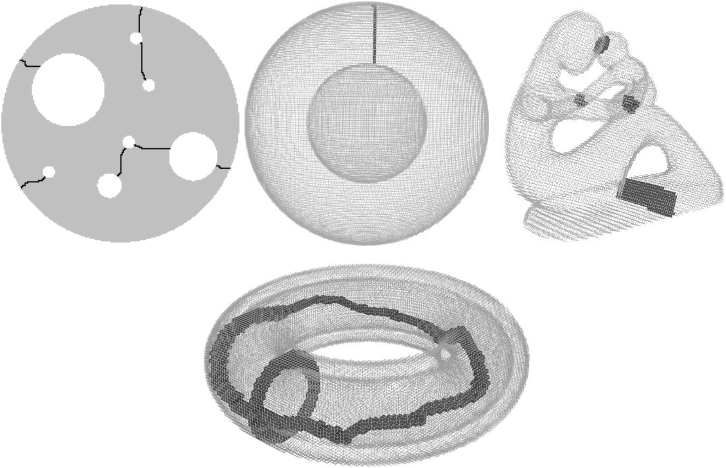
Algorithm 2: propagation by layers

$C \leftarrow \{x\}$, for random $x \in X$ with highest dt_X value;
repeat
  $m \leftarrow \max\{dt_X(x) \mid x \in N^*_\alpha(C) \cap X,\ x \text{ simple for } C\}$;
  $L \leftarrow dt_X^{-1}([m-1, m]) \cap N^*_\alpha(C)$;
  foreach $x \in L$ **do**
    if x *is simple for* C **then**
      $C \leftarrow C \cup \{x\}$;
until *idempotency*;
return $X - C$;



Complexity

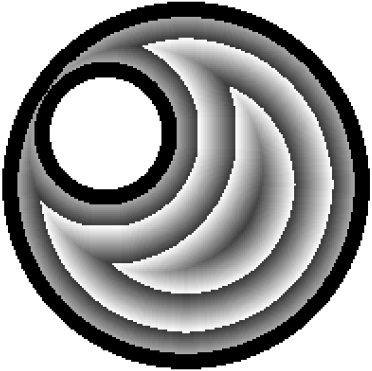
dD: $3^d n^2 f(d)$
3D: n^2



Problems:

- No straight lines → next
- ~~Torus~~

Why do we obtain those segments?



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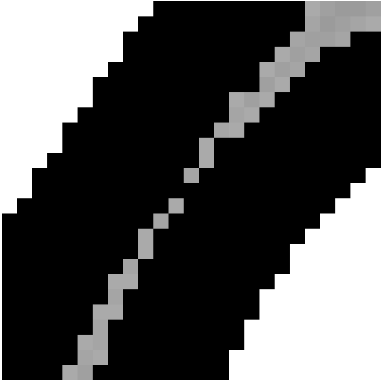

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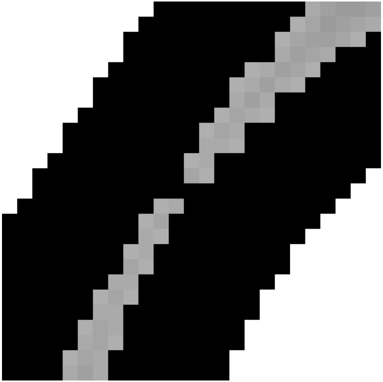

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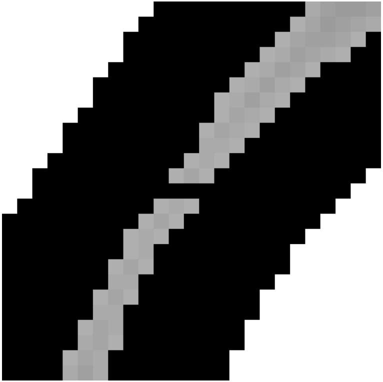

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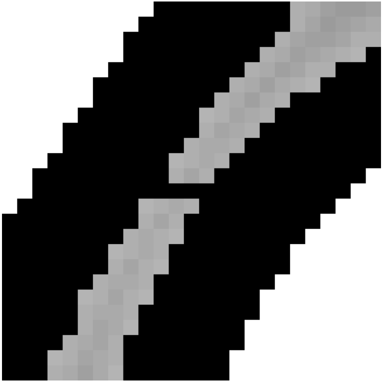

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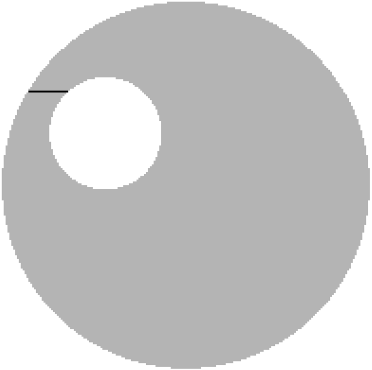


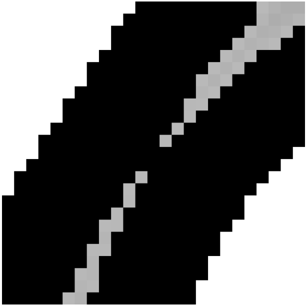
Solution: keep fronts separated (Plugin)

Plugin: simple balls

Input: $C \subset X$, $x \in x$, $r \geq 0$
Output: Can we add point x to C?

foreach $y \in N_\alpha(x)$ **do**
  if *$C \cup (B(y, r) \cap X)$ is collapsible to C* **then**
    return true;

return false;



Complexity

dD: $f(d) \rightarrow |N_\alpha| r^{2d} f(d)$
3D: $f(d) \rightarrow r^6$

Plugin: simple balls

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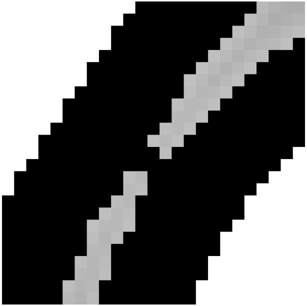

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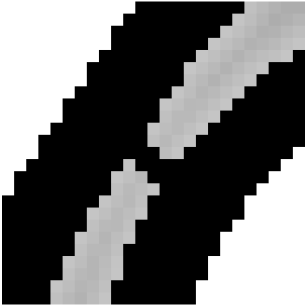

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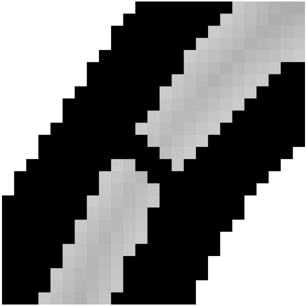

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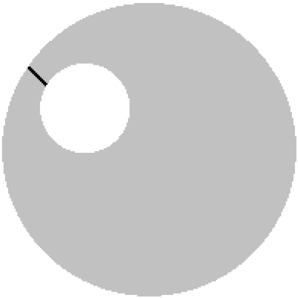
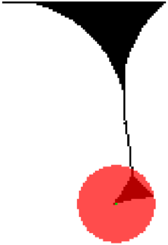
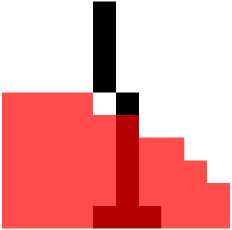


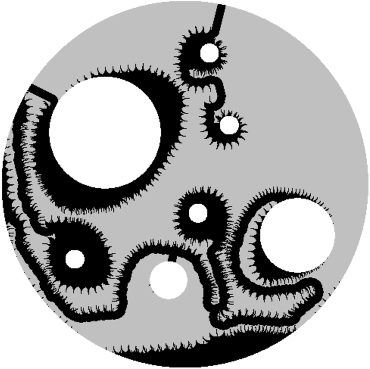
Complexity

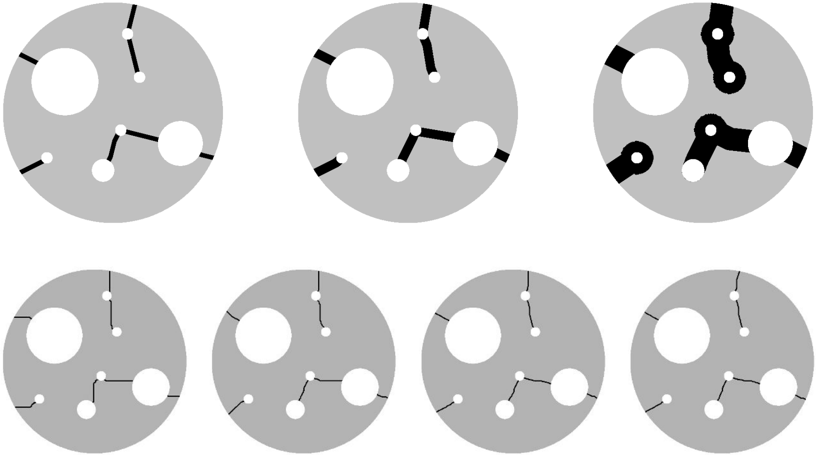
dD: $f(d) \to |N_\alpha| r^{2d} f(d)$
3D: $f(d) \to r^6$

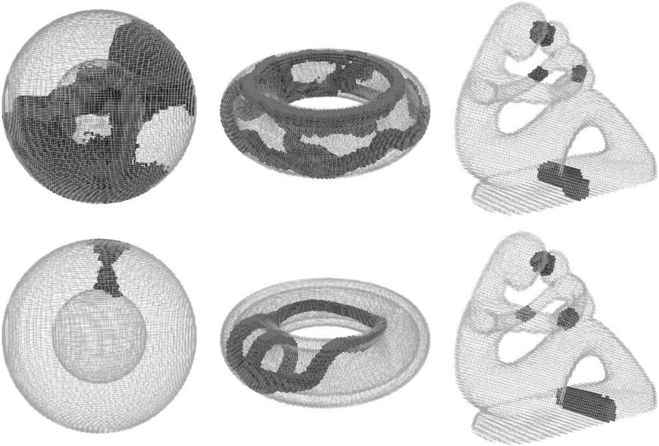
Why the neighborhood? The *tangency problem*











Problems:

- 3D isn't great $\rightarrow$ some day

Structure

- 2 algorithms + 1 parameter for ball radius
- Algorithm 1: faster ($n \lg n$) but worse output
- Algorithm 2: slower (n^2) but better output
- Study the *tangency problem* in 3D

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- Algorithm 1: faster ($n \lg n$) but worse output
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Thank you for your attention