Cellular skeletons or: how to combine topological skeletons with homology computation

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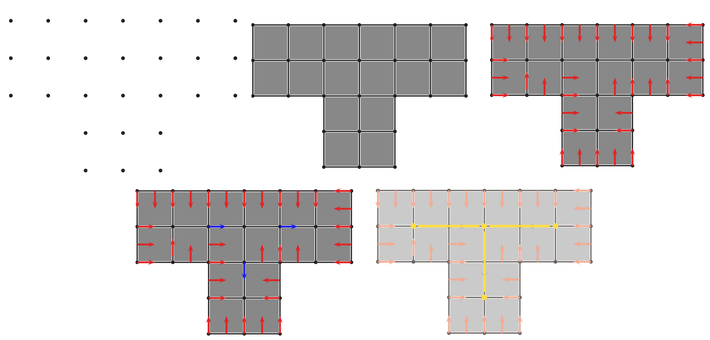
Motivation

HC → TS: Betti numbers, which measure the number of holes in an object, give the number of pieces (cells) necessary to build a homotopically equivalent object

HC ← TS: Existing methods for skeletonization can be used for obtaining well-shaped homology generators

Objective

Given a 3D binary volume with its connectivity relation (6 o 26), we obtain a reduction (f,g,h) which encodes a topological skeleton preserving its geometric features. This algebraic object can be reused for computing the exact homology information of the original object

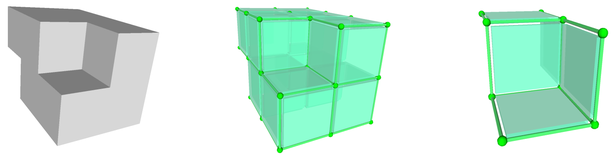


Method

There are three steps:

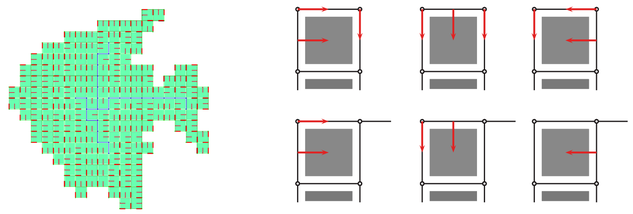
1.- Construction of the cubical complex: choosing the connectivity

We can build the cubical complex by substituing each point by a cube (26-connectivity) or a point (6-connectivity)



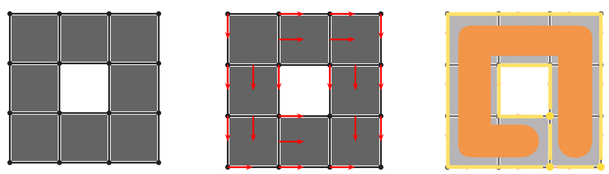
2.- Homotopic thinning algorithm

We perform a homotopic thinning by elementary collapses. There are different methods [Liu10, Couprie13, Dlotko14] for keeping the geometrical features

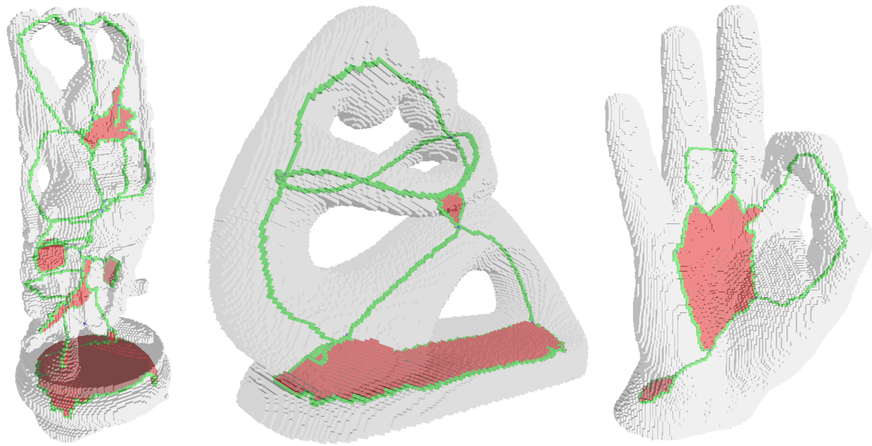


3.- Cell clustering: minimizing the number of cells

We extend the reduction given by the previous step, reducing the number of cells in the complex but maintaining the shape of the skeleton.



Validation



Conclusion

- A new kind of skeleton for binary volumes which is a chain complex together with a reduction
- It works for different connectivity relations and it does not make use of look-up tables
- It can be extended for further homology computations

References

[Liu10] L. Liu, E. W. Chambers, D. Letscher, and T. Ju. A simple and robust thinning algorithm on cell complexes. *Computer Graphics Forum*, 2010.
[Dlotko14] P. Dlotko and R. Specogna. Topology preserving thinning of cell complexes. *Image Processing, IEEE Transactions on Image Processing*, 2014.
[Couprie13] Michel Couprie. Topological maps and robust hierarchical euclidean skeletons in cubical complexes. *Computer Vision and Image Understanding*, 2013.

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Additional info

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